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EVOLUTION OF HELIUM STARS IN CLOSE BINARY SYSTEMS

Vladimir A. Avila Reese

Observatorio Astronómico Nacional
Academia Nacional de Ciencias de Bolivia

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RESUMEN

Se calculó la evolución de estrellas de helio de 2.0, 2.2, 2.5, 2.9, 3.5, 4.0 y 6.0 $M_\odot$. Con la ayuda de un método simple para la determinación del límite del núcleo convectivo de helio, se tomó en cuenta la influencia de la sobreconvexión en su borde. Si las estrellas de helio con masas de 6 a 2 $M_\odot$ se formaron en sistemas binarios masivos con separaciones entre sus componentes menores que $\sim$ 2–10 $R_\odot$ respectivamente (donde las segundas componentes se supusieron objetos compactos), entonces éstas podrían alcanzar sus lóbulos de Roche antes de que se encienda el carbón en el núcleo y perderían masa en forma no conservativa. Se investiga numéricamente dicha fase de pérdida de masa. Se derivaron las masas restantes, la duración y otras características de esta fase. Las estrellas de helio más masivas que 2.2 $M_\odot$ sufren una explosión de supernova y forman estrellas de neutrón (después de la fase de pérdida de masa). Si el sistema no se desintegra después de la explosión, entonces ésta podría ser el antecedente de un radiopulsar binario. Se estimaron las masas de las estrellas de helio y las separaciones orbitales iniciales de dicho radiopulsar binario.

ABSTRACT

The evolution of the 2.0, 2.2, 2.5, 2.9, 3.5, 4.0, and 6.0 $M_\odot$ helium stars was calculated. With the help of a simple method for the determination of the helium convective core-boundary, the influence of overshooting at its edge was accounted. If the helium stars with masses from 6.0 to 2.0 $M_\odot$ formed in massive binary systems with separations between the components less than $\sim$ 2–10 $R_\odot$ respectively (the second components are assumed to be compact objects), then they might reach their Roche lobes before the carbon ignition in the core and they will nonconservately lose mass. Such a mass loss phase is numerically investigated. The remnant masses, the durations and other characteristics of this phase were obtained. Helium stars more massive than 2.2 $M_\odot$ undergo a SN explosion and form neutron stars (after the mass loss phase). If the system does not disintegrate by the explosion, then it might be the progenitor of a binary radiopulsar. The initial helium-star masses and the orbital separations of such a binary radiopulsar are estimated.

Key words: **PULSARS — STARS-BINARIES — STARS-EVOLUTION — STARS-MASS-LOSS**

1. INTRODUCTION

The existence of stars with high helium abundance (Y > 0.90) and extremely low hydrogen content in the envelope (X < 0.0001) is a fact confirmed by astrophysical observations. The abnormal chemical composition of such stars can be explained by extensive mass loss —by stellar wind in massive stars or by mass exchange in a close binary system, (CBS)— and by mixing to the surface of the interior material which has undergone nuclear evolution. The former, most probably, leads the star to lose almost all of its hydrogen envelopes, so hydrogen is deficient not only in the atmosphere, but in all the star. The remnant will be a pure helium star (He-star). Probably in most cases, the formation of such stars is related with the evolution of binary systems. Moreover, He-stars of masses less than 10 $M_\odot$ can be formed only in CBS (Iben & Renzini 1983).

He-stars as intermediate products appear specially in the evolutionary scenarios of massive CBS. After the first mass exchange phase, the most evolved component (the primary) might be a neutron star (NS), a white dwarf (WD) or a black hole (BH); the secondary component (a massive main sequence star) may fill its Roche lobe and lose its hydrogen envelope in a common-envelope (CE) stage. In such a way, if the compact companion is not engulfed by the red giant's He-core, it may appear a very tight binary system composed by a He-star (its mass hardly exceeding 10 $M_\odot$) and a compact star. The subsequent evolution of such system depends upon the mass of the He-star. If its mass is less than some critical mass, M_{SN}, then the final product is a WD (Law & Ritter 1983; Iben & Tutukov 1985), and if it is greater, then a supernova (SN) explosion with the subsequent formation of a NS, is possible.

Another phenomenon that depends on the He-star mass is connected with the red giant (RG) phase. From Paczynski (1971) and Habets (1985) we can assume that He-stars in the mass interval of 0.85–2.7 $M_\odot$ become RGs. The outer layers of these stars expand extensively and they can reach their Roche lobe for almost all the possible orbital semi-major axes, so that the mass exchange phase is inevitable.

Although He-stars more massive than $\sim 2.7\ M_\odot$ do not become RGs, they can reach their Roche lobe, even if they are not very extended (after core-helium burning, the radii of the He-stars increases monotonically). When the system undergoes a CE stage, then this is the case. For these systems the separation between the components is very small due to their mutual approach during the CE stage. Therefore the Roche-lobe radius is also small. In this manner the He-star loses mass through the first inner Lagrangian point before the SN explosion. It is possible that a second CE stage arises and the components merge. If the components do not merge, then after the SN explosion the system may remain bound only if the ejected amount of mass is less than half of the total mass of the system

$$2\Delta M \leq M_{PSN} + M_2 ,$$

where

$$\Delta M = M_{PSN} - M_{SN} . \qquad (1)$$

The aim of this paper is the investigation of the mentioned He-star mass loss phase for all stars within the mass range of 2.0–6.0 $M_\odot$, assuming that such a phase occurs after the core-helium burning but before carbon ignition. We will name case BB such a mass loss phase.

In § 2 the evolutionary code, input physics, and mass loss algorithm, are described. In § 3.1 we present the evolutionary results for the 2.0, 2.2, 2.5, 2.9, 3.5, 4.0, and 6.0 $M_\odot$ He-stars (from the core-helium burning up to the radiative carbon ignition). In § 3.2 we present the evolutionary results for 2–6 $M_\odot$ He-stars which lose mass in a BB event. The relation between the initial He-star mass M_{He} and the mass M_r that remains after the mass loss phase, lifetimes, and other characteristics of such a phase, are given. In § 4 we discuss the evolutionary consequences of the case BB mass-loss event and, we analyze the possibility of binary radiopulsar formation, with special attention.

2. EVOLUTIONARY CODE AND INPUT PHYSICS

The evolutionary program used in our calculations, STEV, (Denissenkov 1990) is a program originally based on a scheme described by Paczynski (1970 *a*,*b*). We modified this program in order to follow the evolution of He-stars.

The central abundance (by mass) of ^{4}He from model to model was allowed to decrease by no more than 0.02. The time step was reduced for any faster decrease. The evolutionary models turned out to be very sensitive to this limiting value.

All the thermodynamic quantities and the opacity coefficients were used as in the original Paczynski code.

The reaction rates for the 3-α process were taken from Harris et al. (1983) (where a value of 0.1 was chosen for the factor 0.0–1.0), and for the C-α reaction from Fowler, Caughlan, & Zimmerman (1975). These reaction rates were corrected for electrostatic Coulomb screening using the factors by Graboske et al. (1973).

The neutrino loss rates given by Beaudet et al. (1967) were corrected for neutral current effects multiplying by the factors for the non-relativistic case given by Schinder et al. (1987).

The outer boundary conditions along the evolutionary sequence were calculated automatically. The mass in the envelope was taken to be 7–10%.

The initial He-star models were obtained using the SCH Paczynski program consecutively from X = 0.70 to X = 0.0 (where X is the hydrogen content). This program first integrates the stellar interior equations by a fitting technique and then improves the fit with the Henyey method. Table 1 gives the main parameters of ZAMS 2.0–6.0 $M_\odot$ He-stars.

TABLE 1

PARAMETERS OF THE HOMOGENEOUS HELIUM STARS MODELS (X = 0.0, Z = 0.03)

$M/M_\odot$	$L/L_\odot$	log T_e	M_{bol}	log T_c	log ρ_c	M_c
2.0	3.370	4.850	−3.682	8.177	3.386	0.673
2.2	3.492	4.782	−3.991	8.187	3.337	0.756
2.5	3.660	4.893	−4.403	8.192	3.973	0.913
2.9	3.835	4.913	−4.848	8.201	3.206	1.139
3.5	4.060	4.945	−5.416	8.213	3.118	1.516
4.0	4.210	4.966	−5.796	8.221	3.060	1.831
6.0	4.711	5.050	−6.755	8.250	3.870	2.900

2.1. Determination of the Convective Core Boundary. Overshooting

The growth in mass of the helium burning convective core is a process that has been reported by several authors (e.g., Habets 1985; Iben 1986 and references therein). The core boundary propagates into the radiative helium-rich region around it by convective overshooting and fresh fuel appears in the core, increasing correspondingly by evolutionary time scale of the He-stars. Usually the condition to define the convective core boundary (CCB) is

$$\nabla_r^i = \nabla_a \ , \qquad (2)$$

(Schwarzschild 1958), and mixing is confined to the regions where $\nabla_r > \nabla_a$; ∇_r^i is the radiative gradient at the inner side of the convective core, ∇_a is the adiabatic gradient. This condition, however, is adequate only for the initial phase of central helium burning. When the helium content of the core decreases (and carbon and oxygen contents increase), a convectively unstable zone appears outside it (Castellani, Giannone, & Renzini 1971). As evolution progresses, the composition discontinuity increases and overshooting grows more effective. In order to maintain convective neutrality at the convective side of the boundary, this boundary is forced to enter into the radiative helium layer above it (Habets 1985). The "forced overshooting" of the boundary into the radiative layers may be performed by using for the definition of the CCB the condition $\nabla_r = \nabla_a$ k, where k < 1.0 (Massevitch et al. 1979).

This way the condition of convective neutrality shifts outward, and we obtain a more realistic boundary for the convective core in the course of evolution. In order to determine the values of the coefficient k, in every iteration we calculated the ratio ∇_r/∇_a at the mass point corresponding to the initial core boundary. We assume that this ratio is the value by which ∇_r should be multiplied in order to shift outward the condition of neutrality. This value depends on the core structure which in turn depends on its current helium content.

2.2. The Mass Loss Algorithm

In this paper we investigate binary systems consisting of a He-star which undergoes a SN-explosion and a compact object.

If the orbital separation between the components, and therefore its Roche-lobe radius R_R, is small enough, then the He-star loses mass before carbon ignition, when its radius has barely begun to increase. Most probably the mass loss rate will be such, that the companion will not accept all the matter lost by the He-star; that is why the mass transfer phase is non-conservative. In this case it is practically impossible to determine the behavior of the Roche lobe and of the mass loss rate with time. However it is possible to fix the final mass of the primary component if it is assumed that mass loss occurs until the star begins to shrink "catastrophically". In order to obtain such a remnant mass a mass loss algorithm described below was used.

A critical radius R_R (the Roche lobe of the helium component) was fixed. When the helium star reaches such a radius, the mass loss sets in; its rate is calculated for every model by the formulae

$$\dot{M}_{i+1} = \dot{M}_i \times \begin{cases} (R/R_R)^{\alpha}\ \beta, & \text{if } R < R_R \\ (R/R_R)^{\gamma}\ \delta, & \text{if } R > R_R \end{cases} \ , \qquad (3)$$

where α, β, γ, and δ are parameters and $\alpha \geq \gamma$, $\beta \geq 1.0$, $\delta \leq 1.0$. The values for α and γ in all the cases were in the 8.0 to 13.0 range. $\dot{M}_i$ was not allowed to differ from the value of $\dot{M}_{i-1}$ more than a factor of 3 to 4. This algorithm ensures that the stellar radius remains constant up to a few percent, although in some cases the radius differed from R_R by 35%.

Such a procedure has already been applied (e.g., Iben & Tutukov 1984; Iben 1986) and although it may appear to be somehow arbitrary it is motivated by the fact that during the course of evolution in the mass loss phase competing effects, that work in opposite direction, arise (Iben 1986). The stars have a self-regulating mechanism which prevents runaway mass transfer; they shrink in response to mass loss if it is high enough. Therefore, it is possible to adjust the mass loss rate, so that the model radius remains constant (Iben 1991).

3. THE RESULTS

3.1. Evolution of Single Helium Stars

The evolution of 2.0, 2.2, 2.5, 2.9, 3.5, 4.0, and 6.0 $M_\odot$ He-stars up to the radiative carbon burning, was calculated. A complete presentation and analysis of the evolutionary results was presented by Avila-Reese (1992).

3.1.1. Core-helium Burning Phase

In column (1) of Table 2 we present the initial convective core masses and their maximal extent during this phase for the mentioned stellar masses. Its duration is given in column (2).

In Figure 1 we show the evolutionary history of all the stars on the central density-central temperature plane, and in Figure 2 the evolutionary tracks in the H-R diagram.

In Figures 3 and 4 the evolution of the interior regions for the 2.0 and 3.5 $M_\odot$ stars are depicted (for the others stars the feature is similar).

TABLE 2

CHARACTERISTICS OF 2.0–6.0 $M_\odot$ HELIUM STARS DURING HELIUM BURNING

$M/M_\odot$	M_c-$M_{c(max)}/M_\odot$	t(10^6 yr)	X_C	X_O	E^a_{max}	(X_C)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
2.0	0.67–1.03	3.113	0.584	0.401	1.76	(0.34)
2.2	0.76–1.30	2.652	0.597	0.380	2.14	(0.26)
2.5	0.91–1.39	2.156	0.595	0.390	2.58	(0.30)
2.9	1.14–1.60	1.698	0.584	0.400	3.05	(0.37)
3.5	1.52–2.01	1.288	0.578	0.407	3.92	(0.45)
4.0	1.83–2.38	1.156	0.532	0.453	4.47	(0.20)
6.0	2.90–3.59	0.674	0.531	0.448	6.90	(0.32)

[a] The energy E_{max} is given in 10^5 erg g^{-1} s^{-1}.

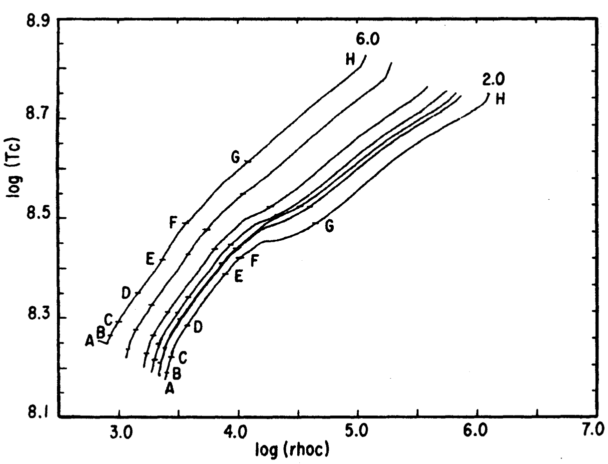

Fig. 1. The central density, temperature evolution of the 2.0–6.0 $M_\odot$ helium stars.

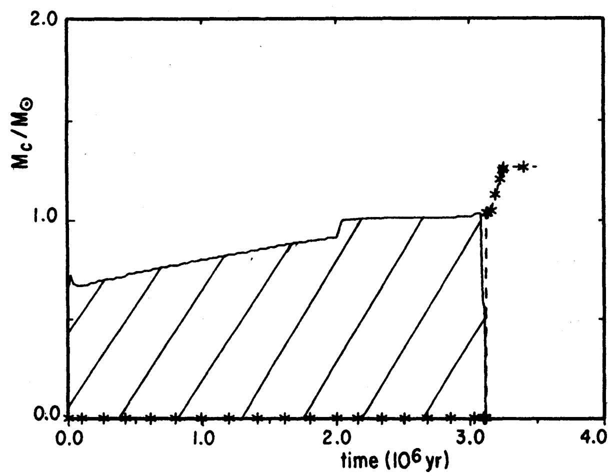

Fig. 3. The interior evolution of the 2.0 $M_\odot$ helium star. Hatched regions are convective. The asterisks indicate the region of maximum energy generation.

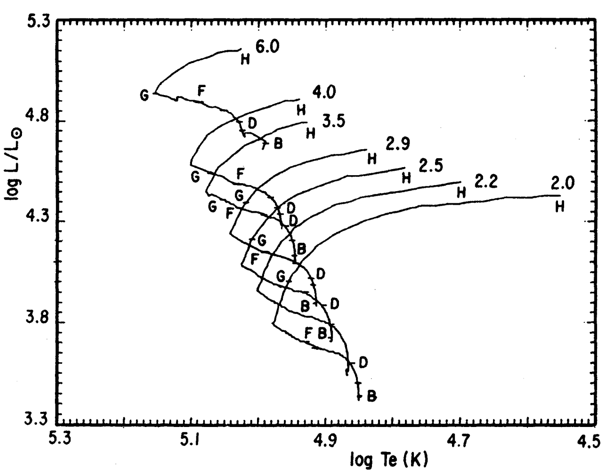

Fig. 2. The evolutionary tracks of the 2.0–6.0 $M_\odot$ helium stars in the H-R diagram.

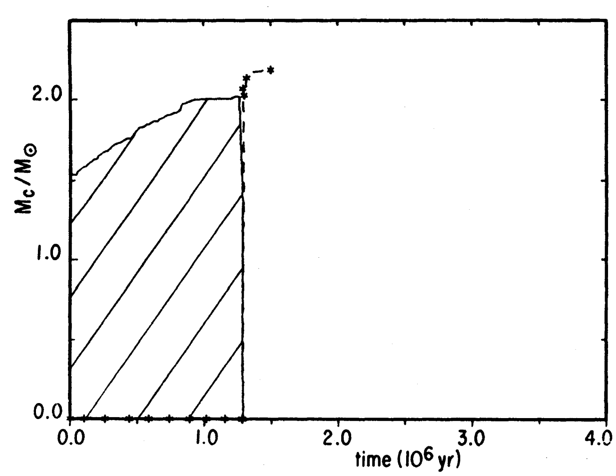

Fig. 4. Same as Figure 3 for the 3.5 $M_\odot$ helium star.

TABLE 3

THE EVOLUTIONARY TIMES[a], t, AND THE MAXIMUM NET ENERGY RATES[b],E, IN THE HELIUM STARS CORRESPONDING TO THE CENTRAL HELIUM ABUNDANCES Y_C (BY MASS)[c]

Yc		0.97	0.70	0.40	0.10	0.01	0.00	...	...
$M/M_\odot$		A	B	C	D	E	F	G	H
2.0	t	0.00	0.762	1.708	2.764	3.107	3.113	3.166	3.256
	E	1.40	1.63	1.71	1.02	0.53	0.01	...	...
2.2	t	0.00	0.032	1.458	2.377	2.647	2.652	2.689	2.752
	E	1.66	2.01	2.06	1.89	1.30	0.27	...	...
2.5	t	0.00	0.557	1.207	1.929	2.151	2.156	2.183	2.237
	E	2.10	2.32	2.45	2.29	1.38	0.32	...	...
2.9	t	0.00	0.430	0.986	1.519	1.669	1.698	1.723	1.758
	E	2.67	2.79	3.00	2.71	2.40	0.36	...	...
3.5	t	0.00	0.344	0.768	1.139	1.284	1.288	1.293	1.329
	E	3.35	3.61	3.83	3.49	2.01	0.52	...	...
4.0	t	0.00	0.302	0.645	0.994	1.154	1.156	1.162	1.190
	E	3.88	4.23	4.37	4.07	2.89	0.60	...	...
6.0	t	0.00	0.189	0.385	0.588	0.672	0.674	0.682	0.699
	E	7.00	6.42	6.75	6.39	4.00	1.18	...	...

[a] In millions of years. [b] In 10^5 erg g^{-1} s^{-1}. [c] These points are depicted in Figures 1 and 2.

Table 3 gives the corresponding evolutionary times, central helium abundances, and maximum net energy rates during the core-helium burning phase corresponding to the points depicted in Figures 1 and 2.

The growth in mass of the cores in the course of evolution of the 2.0, 2.2, 2.5, 2.9, 3.5,4.0, and 6.0 $M_\odot$ helium stars is 57%, 71%, 53%, 40%, 32%, 30%, and 24% respectively. Such a behavior of the growth of the core as a function of stellar mass is more regular than that obtained by Habets (1985, 1986 *a,b*) (we did not carry out a special treatment of semi convection). In columns (3) and (4) of Table 2 we give the carbon and oxygen abundances by mass (X_C and X_O) at the helium exhaustion in the core.

Our results are close to a linear relation between stellar mass and X_C as was proposed by Arnett (1972). These results also show that a maximum central mass fraction of ^{12}C is attained for 2.2–2.5 $M_\odot$ helium stars. Figure 5 depicts the time variation of the central abundances by mass of ^4He, ^{12}C and ^{16}O during core-helium burning for a 3.5 $M_\odot$ (the behavior is similar for the other He-stars).

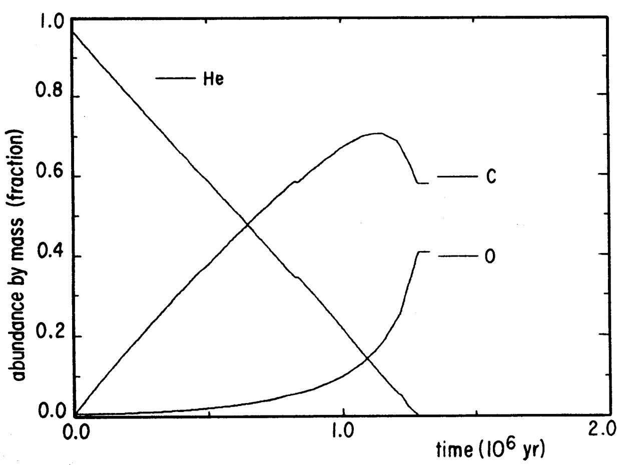

Fig. 5. The time variation of the central abundance by mass of ^4He, ^{12}C, ^{16}O during convective core-helium burning in the 3.5 $M_\odot$ helium star.

The values of X_C obtained by Habets are lower than those derived by us. In order to find the source of such differences we made test calculations for a 2.0 $M_\odot$ star. If the $^{12}C(\alpha, \gamma)^{16}O$ reaction rates are increased, the resulting X_C becomes smaller. The 3α reaction rates used by us are smaller than those used by Habets. Test calculations for a 2.0 $M_\odot$ star have shown that if we decrease the reaction rates, then the value of X_C increases. Test calculations have shown also that X_C depends on the efficiency of convective overshooting in the course of the evolution. Such a fact could be also a reason for the differences between Habets' t_{evol} and our values.

Concerning the final mass concentration of oxygen, X_O, we obtained lower values than Savonije & Takens (1976) for a 2.0 and 4.0 $M_\odot$ stars and than Habets for all the helium stars investigated here. Our models did not present large semi convective zones at the end of helium burning as was reported in the models of the mentioned authors. This is probably the reason for the low values of X_O in our models (see Savonije & Takens 1976).

In the interior of the star the maximum net energy production E_{net} takes place in the center (see Figs. 3 and 4). The maxima of central E_{net} during the core-helium burning phase occur when the helium abundance is in the range 0.20–0.45 for all the stars investigated in this work (see column 5 in Table 2). After this maximum, the central E_{net} decreases quickly.

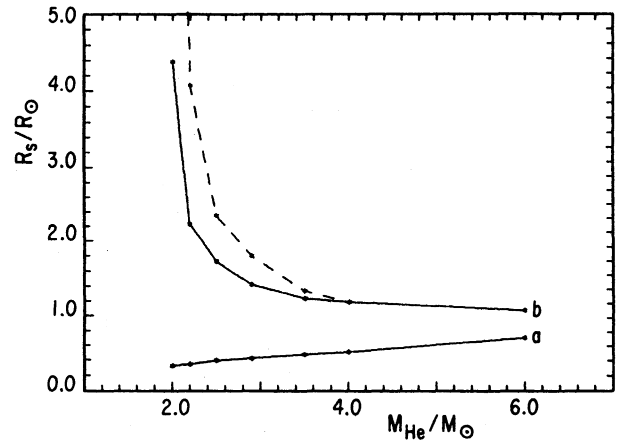

Fig. 6. *(a)* The radii of the He-stars with masses M_{He} at the onset of the core He-burning and *(b)* at the onset of radiative C-ignition. The dotted curve are the radii at the onset of convective carbon burning (Habets 1985).

3.1.2. Evolution from Core-Helium Exhaustion to Carbon Ignition

After helium exhaustion in the core, helium burns in shells around the C-O core. The stellar radius, which at the end of helium-burning in the core attained a minimum value, increases rapidly.

The maximum of the net energy generation rate, ε_{max}, in the helium shell-burning phase is reached at shell temperatures varying from 1.75 to 2.3×10^8 K.

The evolutionary calculations were terminated at C-ignition. As was obtained in previous works, only the 2.0 $M_\odot$ star moves far into the degenerate region of the ρ-T diagram before carbon is ignited (Fig. 1).

The time spent between the ending of core-helium burning and carbon ignition for the 2.0 $M_\odot$ star was 1.33×10^5 yr. This time decreases for the more massive helium stars to 2.42×10^4 yr in the case of the 6.0 $M_\odot$ star.

3.2. Evolution of Binary Helium Stars

Our attention is fixed in CBS with helium components which, after the case of a BB mass loss event, evolve to a NS.

The lower mass limits for a helium star to form NS have been determined by Law & Ritter (1983) by using calculations of Law (1982) and Delgado & Thomas (1981), and by Habets (1985). These limits are 3.2 (±0.1) $M_\odot$ and 2.20 (±0.05) $M_\odot$, respectively.

The criterion used by these authors is that the

TABLE 4

REMNANT AND CORE MASSES AFTER A CASE BB MASS-LOSS PHASE, THE DURATION OF SUCH A PHASE AND THE AVERAGE MASS LOSS RATE, FOR $R_R = 0.6\ R_\odot$

$M_{He}/M_\odot$ (1)	$M_r/M_\odot$ (2)	$M_{CO}/M_\odot$ (3)	t(10^3 yr) (4)	$\dot{M}(M_\odot$/yr) (5)
2.0	1.40±0.03	1.22±0.02	54.62±1.01	1.1 10^{-5}
2.5	1.73±0.04	1.52±0.02	32.08±0.97	2.4 10^{-5}
2.9	2.01±0.03	1.73±0.01	27.39±0.69	3.2 10^{-5}
3.5	2.58±0.06	2.13±0.04	17.27±0.87	5.3 10^{-5}
4.0	2.93±0.01	2.43±0.03	17.95±0.93	6.0 10^{-5}
6.0	4.70±0.02	3.70±0.05	5.85±0.31	2.2 10^{-4}

TABLE 5

REMNANT AND CORE MASSES AFTER A CASE BB MASS-LOSS PHASE, THE DURATION OF SUCH A PHASE AND THE AVERAGE MASS LOSS RATES, FOR $R_R = 0.7\ R_\odot$

$M_{He}/M_\odot$ (1)	$M_r/M_\odot$ (2)	$M_{CC}/M_\odot$ (3)	t(10^3 yr) (4)	$\dot{M}(M_\odot$/yr) (5)
2.0	1.45±0.02	1.20±0.02	35.12±1.10	1.6 10^{-5}
2.5	1.82±0.04	1.53±0.03	16.34±1.55	4.2 10^{-5}
2.9	2.07±0.03	1.68±0.03	20.27±0.80	4.1 10^{-5}
3.5	2.65±0.01	2.07±0.01	10.89±0.05	7.8 10^{-5}
4.0	3.08±0.02	2.40±0.02	10.32±0.90	8.9 10^{-5}
6.0	4.78±0.03	3.33±0.05	5.52±0.72	2.2 10^{-5}

core mass of the He-star grows larger than the Chandrasekhar limiting mass M_{Ch} in the course of evolution.

Although our calculations were performed only to the onset of carbon ignition, we can affirm that the mentioned lower mass limit does not exceed 2.2 $M_\odot$ because the core mass achieved by such a star already exceeds M_{Ch}.

The mentioned mass limits were also obtained by Arnett (1978) (>3.0 $M_\odot$), Nomoto (1984) (2.5–2.8 $M_\odot$) and Hillebrandt et al. (1984) (2.0–2.5 $M_\odot$).

After core-helium burning the He-stars increase in radii. Figure 6 depicts the dependence on stellar masses of the initial radii of the main-sequence He-stars and the radii attained just before carbon ignition, according to our calculations. In order for the mass exchange phase to set in, the Roche lobe must be equal or smaller than such radii for the component masses. Such a phase would be non-conservative; for this, we have used the mass loss algorithm described above (§ 2.2). The values of 0.6 and 0.7 $R_\odot$ were taken as the critical radii R_R (Roche-lobe radius); if the radius of the star exceeds R_R, then it begins to lose mass until the equilibrium stellar model sets in with a very reduced radius (where $R_s < R_R$ by factors smaller than 1/2). Naturally the mass loss calculated by (3) depends on the values of the coefficients α, β, γ, and δ on the initial mass loss rate adopted. All efforts were made in order to obtain a complete interval of final masses. Fortunately, as shown in our calculations, such an interval turned out to be very small (column 2 of Tables 4 and 5). It confirms the validity of the described mass loss algorithm.

The corresponding core masses are given in column (3), and the time intervals between the onset of mass transfer and the beginning of "catastrophic" shrinking, in column (4). Column (5) gives the average mass loss rates.

The remnant stars are compact objects formed by a massive C-O core and a small helium envelope. These stars at the onset of carbon ignition might be homogeneous C-O stars starting their carbon-main sequence phase.

Mass loss does not influence strongly the internal structure of the core. Thus, the onset of carbon ignition occurs almost under the same conditions

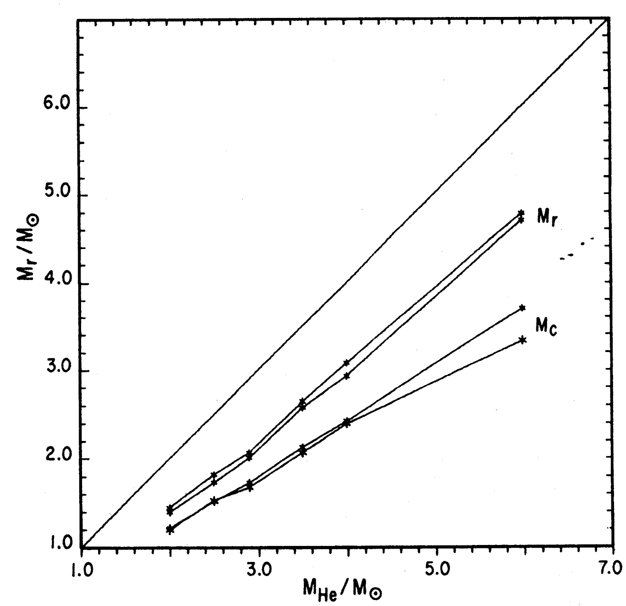

Fig. 7. The remnant star and core masses after the BB mass transfer in the 2.0–6.0 $M_\odot$ helium stars for R = 0.6 $R_\odot$ (lower curve) and R = 0.7 $R_\odot$ (upper curve).

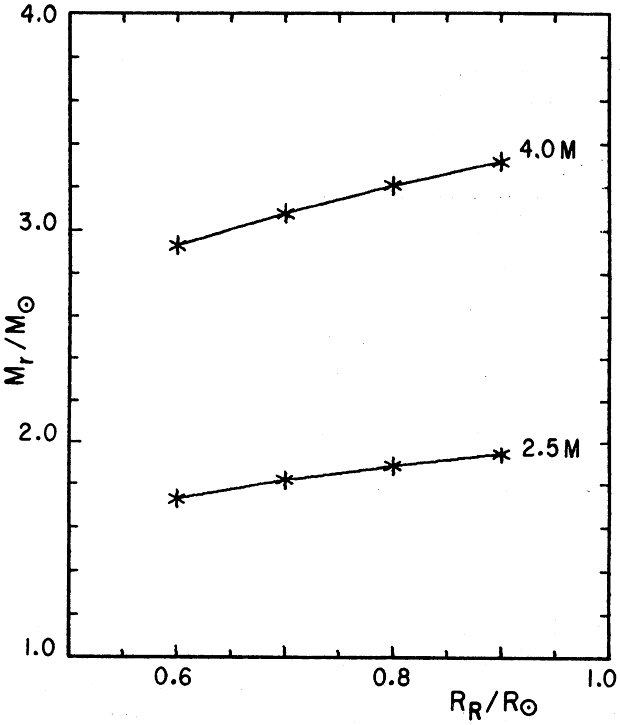

Fig. 8. Dependence on R_R of the remnant masses (after a BB event) for the 2.5 and 4.0 $M_\odot$ stars.

for stars that have lost mass and for those that have not lost any mass.

The helium-shell burning phase shortens by a factor of 2–3 and the core masses become slightly reduced due to mass loss.

Figure 7 shows the relation between the initial He-star mass M_{He} and the final mass M_r for all the masses investigated in this work for the two critical radii, R_{cr}, 0.6 and 0.7 $R_\odot$; the figure is based on the data of Tables 4 and 5.

In the case of 0.6 $R_\odot$ we found an approximation formula for such a relation:

$$M_r \approx 0.89 \ M_{He}^{0.96} - 0.40 \quad . \qquad (4)$$

If one increases the critical radius, R_R, after the mass loss phase the remnant mass proves to be slightly larger for a given initial mass. We found the final masses of the 2.5 and 4.0 $M_\odot$ helium star for 0.6, 0.7, 0.8, and 0.9 $R_\odot$. Figure 8 presents the obtained relations between the remnant mass M_r and the critical radius R_R. These relations can be approximated by $M_r \sim (R_R/0.6)^{0.2}$. Now considering this proportionality in (4) we obtain

$$M_r \approx (0.89 \ M_{He}^{0.96} - 0.40) \ (R_R/0.6)^{0.2} \quad . \qquad (5)$$

We assume that this formula is right for values of R_R less than 2.2 $R_\odot$. It is possible that M_r is no longer dependent on R_R for even greater values.

4. DISCUSSION

We have investigated numerically He-stars in the mass range 2–6 $M_\odot$ which lose mass by Roche-lobe filling in a binary system, when they expand after the core-helium burning, but before carbon ignition (we called such a mass loss event, case BB). The companion might be a compact object, most probably a NS. If we fix the mass of the companion, then we can find the maximal orbit semi major axis of the system, for which the He-stars could fill their Roche lobe before the core carbon burning phase. A good approximation for the dependence of the He-star radius, R_{He}, on its mass, M_{He}, is given by the formula (from Fig. 6)

$$R_{He} \approx (M_{He} - 1.45)^{-2} + 1.00 \quad . \qquad (6)$$

Commonly one uses the following approximate formula for the Roche-lobe radius

$$R_R \approx 0.52 \ [M_1/(M_1 + M_2)]^{0.44} \ a \quad . \qquad (7)$$

From the equality $R_{He} = R_R$ for a given M_1 and M_2 and using (6) and (7), the values of a are derived easily (M_1 in this case is the He-star mass M_{He}, M_2 is the mass of the companion, a is the semi major axis). In Figure 9 the dependence of M_{He} of maximal

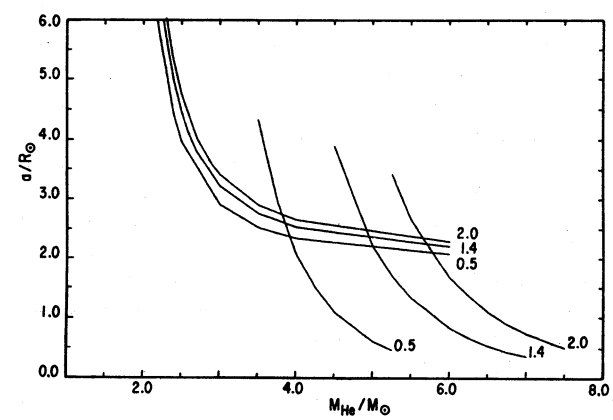

Fig. 9. The maximal orbital semi-major axes of a BS consisting of a He star and of a compact object with mass M (0.5, 1.4, 2.0 $M_\odot$), for which the He components with masses M_{He} will lose mass in a BB event (the curves at the top) and the maximal M_{He} for which the system will stay bound after the ML-phase and the SN explosion (the curves at the right).

a for different values of M_2 are given (the upper curves). In such a way only systems with semi-major axes less than $\sim$ 2-10 $R_\odot$ (for He-components in the mass range 2–6 $M_\odot$) might undergo a BB event. The systems could have attained such a separation during the CE stage; namely during this phase the He-core was "undressed". For wider systems the He-components could reach their Roche lobe only during or after the core-carbon burning.

The mass loss phase has a duration of the order of 10^4 years and the average mass loss rate is 10^{-5} $M_\odot$ yr^{-1}. Therefore it may form a super critical accretion disk around the accretor, if it is a NS or a BH. Such a system could be an SS 433 like object. It is also probable that the system will evolve to a second CE phase. If the interaction energy of the components is low enough then the components can merge.

After the mass loss phase the remnants of He-stars more massive than 2.2 $M_\odot$ —compact objects consisting of a C-O core at the onset of its carbon burning, and a helium envelope— may undergo a SN explosion. This SN might be of Type Ib/Ic. Low mass He-stars of 3–4 $M_\odot$ according to Shigeyama (1990) and Uomoto 1986), and 3–6 $M_\odot$ according Nomoto et al. (1990), seem to be the most relevant progenitors of a Type Ib SN. Tutukov, Yungelson, & Iben (1992) found a theoretical frequency of SNIb with He-stars in CBS as precursors, of 0.0087 yr^{-1}. Such an estimate is also in fair agreement with the observed frequency. The formation frequency of He-stars in the mass range 2–6 $M_\odot$ (15–30% of all massive stars) found by Podsiadlowski (1992) is consistent with the estimates for the frequency of Type Ib/Ic SN without requiring isolated W-R stars as additional contributors.

The explosion of the more massive component causes the disintegration of the system if the condition (1) is not fulfilled (for systems with circular orbits). The orbit of the systems that survive the explosion, become eccentric with $e > 0$. It is possible to calculate such eccentricity for systems with initial circular orbits, with the formula

$$e \approx (M_1 - M_{NS})/(M_2 + M_{NS}) \ . \qquad (8)$$

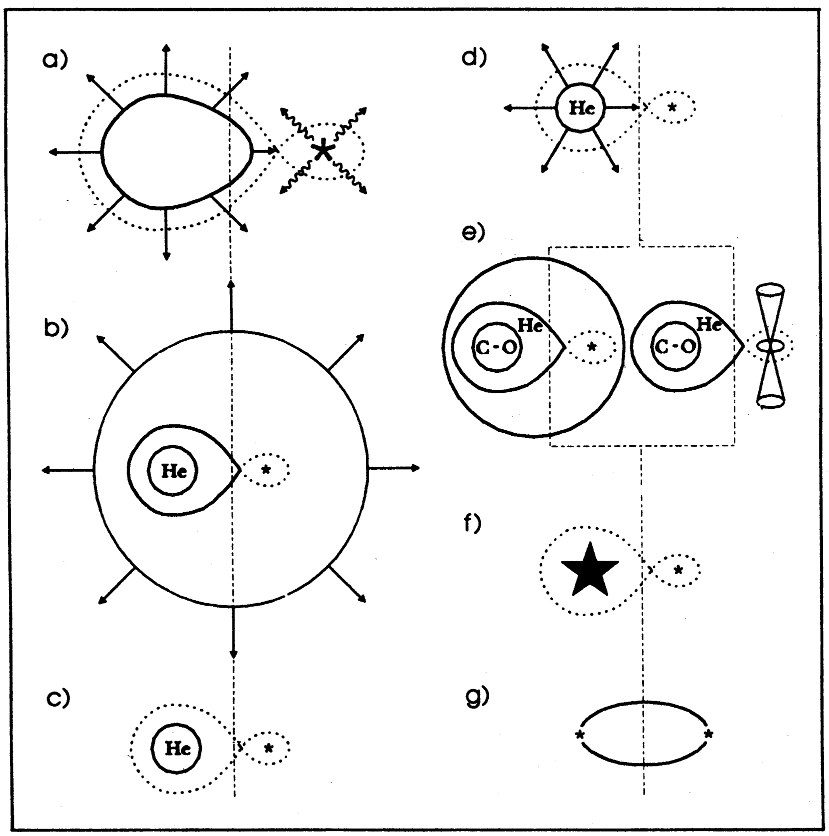

Fig. 10. The origin of He-stars in massive CBS and subsequent evolution. *(a)* O, B star + compact object. *(b)* First ML phase; common envelope; orbital shrinkage. *(c)* He-star + compact object. *(d)* Core He-exhaution. The star begins to expand. *(e)* Second ML phase; common envelope or S 433-like object. *(f)* Supernova explosion (SNIb). *(g)* Binary radiopulsar with $e > 0$.

known quantities: $M_{NS}, M_2, a_{ff}, e.$

$$\begin{cases} M_{NS}, M_2, e \\ a_{ff} \end{cases}$$

$$\Downarrow$$

$$\begin{cases} M_{PSN} = e\,(M_2 + M_{NS}) + M_{NS} \equiv M_r \\ M_{He} = [(M_r/(R_R/0.6)^{0.2} + 0.4)/0.89]^{1/0.96} \\ a_f = a_{ff}\,(M_2 - M_r + 2M_{NS})/(M_2 + M_{NS}) \end{cases}$$

$$\Downarrow$$

$$\begin{cases} a = (M_{He}^2\, a_f \alpha_{CE})/(M_r\, M_2) \\ R_R = 0.52\, a[M_{He}/(M_{He} + M_2)]^{0.44} \end{cases}$$

Fig. 11. Flow diagram of expressions used to determine a and M_{He}.

From equation (1) (or from equation (8) for $e = 1$) the maximal possible pre-supernova mass for which the system does not disintegrate can be found. For instance, if we assume that the second component is a NS and that the mass of both neutron stars is $M_{NS} \simeq 1.4\ M_\odot$, then such a maximal mass is $M_{PSN} \simeq 4.2\ M_\odot$. Thus, systems with He-pre-supernovas less massive than 4.2 $M_\odot$ and with a NS companion, remain bound after the SN explosion (in the case of a spherically symmetric explosion). Similar situations, but with other maximal masses are obtained if the second component is a WD or a BH. All these systems might be progenitors of binary radio pulsars.

The pre-supernova mass in our case is the mass of the remnant star, that underwent the case BB mass loss phase. According to our calculations such a mass is connected with the He-star mass for a given Roche-lobe radius by the formula (5). The Roche-lobe radius depends on the semi major axis a [eq. (7)].

In the M_{He}-a plane, the maximum He-star masses, for which a given system after the case BB mass loss phase and the SN explosion, remain bound, are determined in Figure 9 (the curves at the right of the graph). Thus, with the help of this graph we can find which systems with He-components probably give rise to a binary radiopulsar.

The formation, evolution, and final stage (binary radiopulsar formation) of the 2–6 $M_\odot$ He-stars in CBS with separations $\leq 10\ R_\odot$ is schematically presented in the evolutionary scenario depicted in Figure 10.

If the orbital parameters (e and a_{ff}) and the masses of the components of a binary radiopulsar are known then it is possible to find the initial He-star mass and orbital separation (M_{He} and a). Figure 11 shows the sequence of equations which are necessary to use in order to derive M_{He} and a. In these formulae there are two free parameters: the efficiency of transformation of orbital energy into energy of dispersal α_{CE}, and the Roche-lobe radius. However, using a complementary formula for a (equation 7), it is possible to reduce these parameters to one. Applying the sequence of equations in Figure 11, for instance, to the binary radiopulsar 1913 + 16 ($e = 0.62$, $a_{ff} = 2.8\ R_\odot$) we obtain that $M_r = 3.14\ M_\odot$ and $a_f = 1.06\ R_\odot$, and for $\alpha_{CE} = 0.7$ then $M_{He} = 4.02\ M_\odot$, $a = 1.66\ R_\odot$, and $R_R = 0.76\ R_\odot$. According to the approximate formula $M_{He} \approx 0.1\ M_{MS}^{1.44}$ the progenitor of a He-star of 4.0 $M_\odot$ would be a MS-star of $\sim 13\ M_\odot$.

We selected $\alpha_{CE} = 0.7$ because such a value is in agreement with the values derived from well-studied binaries (Iben & Tutukov 1989; Yungelson & Tutukov 1993), and because if $\alpha_{CE} < 1$ then the CE could be ejected (Iben & Tutukov 1985).

According to Figure 9 He-stars more massive than $\sim$6 $M_\odot$ might fill their Roche lobe only after core carbon-ignition. A study of the corresponding mass loss phase is necessary. These He-stars could explode as SNIc. Due to the large amount of matter expelled in the explosion, the remnant system (a possible binary radiopulsar) would have a larger eccentricity and orbital separation than the binary radiopulsars formed by the above studied scenario.

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Vladimir A. Avila Reese: Observatorio Astronómico Nacional, c. La Madrid 710, Tarija, Bolivia.