Revista Mexicana de Astronomía y Astrofísica, **33**, 143–156 (1997)

BOLOMETRIC STELLAR DISTRIBUTION MODEL IN THE GALAXY AND ITS APPLICATION TO BAADE'S WINDOW

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Received 1997 April 1; accepted 1997 August 14

RESUMEN

Se desarrolla un modelo muy simple para la distribución de las estrellas en nuestra Galaxia. Este modelo es completamente arbitrario y se utiliza solamente como una descripción de la distribución estelar en la Galaxia. El modelo consiste de una componente plana (disco) con un perfil de densidad radial Gaussiano y con un decaimiento exponencial en la dirección perpendicular al plano galáctico; además presenta una componente esferoidal (bulbo) con un perfil de densidad radial Gaussiano. Este modelo se utiliza para predecir la función de luminosidad bolométrica observada (LF) para estrellas brillantes ($M_{\rm bol} \leq 0.0$).

El objetivo principal de este artículo es verificar, usándo nuestro modelo galáctico simplificado, si la población estelar de la Ventana de Baade (BW), puede estar conformada por una combinación de poblaciones de bulbo y de disco que se extiende hasta el centro galáctico. Se muestra que esta posibilidad es consistente con los datos, y que la LF observada puede ser separada en dos componentes Gaussianas que pueden corresponder consistentemente al disco y al bulbo.

ABSTRACT

We develop a very simple model for the distribution of the stellar component in our Galaxy. This model is completely arbitrary and is utilised only as a description of the stellar distribution in the Galaxy. The model consists of a flat (disc) component with a radial Gaussian density profile and an exponential density decay in the direction perpendicular to the galactic plane; moreover, it also presents a spheroidal (bulge) component with a radial Gaussian density profile. At this stage, the model is used for predicting the would-be-observed bolometric luminosity functions (LF) of bright ($M_{\rm bol} \leq 0.0$) stars.

The main purpose of this paper is to check, using our simplified galactic model, whether the stellar population in Baade's Window (BW), may consist of a combination of bulge and disc extending to the galactic centre populations. We show that this possibility is consistent with the data, and that the observed LF may be separated into two Gaussian components which may consistently correspond to the bulge and disc contributions.

Key words: **GALAXY – CENTRE — STARS – LATE–TYPE – LUMINOSITY FUNCTION – POPULATION II — STELLAR CONTENT**

1. INTRODUCTION

Several models for the stellar distribution of the stars in our Galaxy have been published (Elias 1978a,b; Bahcall & Soneira 1980; Jones et al. 1981; Bahcall 1986; Ruelas-Mayorga 1991a,b; Wainscoat et al. 1992 among others). They have been used for different purposes and have included a variety of different elements which contribute to the luminous output of our galactic system. Very complex models may give a highly accurate description of the Galaxy; however, we have to pay for this accuracy by having to select the values of some arbitrary parameters which, usually, are more plentiful the more complex the model is.

In this work we develop a very simple, almost analytical, model for the distribution of the stars in our Galaxy; we know it gives a good description of the Milky Way because we have reproduced very well a large number of stellar counts from the literature in different galactic directions and in several wavelengths (see Ruelas-Mayorga 1997). Moreover, the results given by this model are not significantly different from those obtained with other models (i.e., Bahcall & Soneira 1980; Jones et al. 1981; Ruelas-Mayorga 1991a,b).

Over the last few years we have conducted an extensive study of the stars in the low absorption window known as Baade's Window (BW), through which a stellar population presumably belonging to the galactic bulge may be observed. Our conclusions from these studies are interesting (see Ruelas-Mayorga & Teague 1992a,b; 1993) and they indicate that it is possible to divide the group of stars observed in BW consistently as partly belonging to a galactic component, which we like to call the 'true bulge', and partly belonging to an extension of the disc which reaches into the innermost sections of our Galaxy. This fact makes us face the interesting possibility of having stars belonging to two different galactic components, and sharing the same space around the galactic centre.

In a recent paper Ng et al. (1996) studied the stellar distribution in BW. They intended to reproduce the observed colour-magnitude [V vs. $(V-I)$] diagram for BW which they did quite successfully using seven different stellar populations; namely halo, bulge, old and metal poor disc, old disc, intermediate disc, young disc and the newly introduced bar population. The model presented in this paper is intended solely for a quantitative reproduction of the observed number-counts; therefore, it can be said that it encompasses, within the two proposed components, namely disc and bulge, all the detailed stellar populations considered by Ng et al. (1996). In fact, these authors are forced to invoke the bar population only from a detailed analysis of the stellar number distribution with respect to V-magnitude and $(V-I)$-colour simultaneously, because the distribution with respect to V-magnitude alone does not require the introduction of the bar population.

In this paper, using the simplified galactic model developed here, we shall check whether the suggestion of 'true disc' and 'true bulge' sharing the same space around the galactic centre is a viable possibility. In order to do this, we shall determine the bolometric luminosity function from our JHK photometric observations for a sample of stars in BW and other low absorption regions (see Ruelas-Mayorga & Teague 1992a, 1993). The Frogel & Whitford (1987) (hereinafter FW) transformations to bolometric magnitudes shall be used. In § 2 we develop our simplified galactic model and use it to predict the LF in the direction of BW. Section 3 describes the observations used for the derivation of the LF in BW; § 4 presents an analysis of these observations in the light of our galactic model, in § 5 our LF is compared with other LFs published in the literature and finally § 6 gives our summary and conclusions.

2. SIMPLIFIED GALACTIC MODEL

2.1. Numerical Simulations: The Disc

In order to predict the bolometric Luminosity Function in a particular direction in the Galaxy, we shall use a very simple, yet plausible model for the stellar distribution in the disc and bulge components of the Galaxy.

At this point we must remind the reader that the functional dependence chosen is completely arbitrary and that it may or may not have totally sound physical basis. This latter point will constitute the central topic of a future investigation. Therefore, we must think of this model purely as a convenient description of the stellar distribution in our Galaxy.

Let us assume that the number of stars along a radial unit-area cylinder on the galactic plane varies according to the following expression

$$D_{disc}(r)(\star/kpc^3) = n_{cd} exp\left[-\frac{r^2}{2\sigma^2}\right] \quad , \qquad (1)$$

where $n_{\rm cd}$ =central number of stars in the disc ($\star/{\rm kpc}^3$), r =distance from the centre on the galactic plane (kpc) and $\sigma_{\rm d}$ =radial scale length of the disc (kpc).

It will be this function and another function of the same character [eq. (4)] those used for modelling the density variations for the disc and the bulge.

In the vertical direction we shall assume an exponential density decay because there is a well known physical argument (hydrostatic equilibrium, see Rohlfs 1977) that implies the vertical density distribution to be proportional to $sech^2 Z/2Z_0$. This function is very well approximated by $4e^{-Z/Z_0}$ for $Z \geq Z_0$.

The density expression, including the vertical variation, becomes

$$N_{\rm disc}(r)(\star/kpc^3) = n_{\rm cd}\ \exp\left[-\frac{r^2}{2{\sigma_{\rm d}}^2}\right]\ \exp\left[-\frac{Z}{Z_0}\right] \quad , \qquad (2)$$

where now Z =height above or below the plane (kpc) and Z_0 =scale height for a specific stellar group (kpc).

It can be easily demonstrated, although a little laboriously, that equation (2) is transformed into a

Gaussian when viewed from the Sun at an arbitrary direction (l,b).

Equation (2) becomes

$$N_{\rm disc}(s,l,b) = n_{\odot} \exp\left[\frac{B^2}{8A\sigma_{\rm d}^2}\right] \exp\left[-\frac{\left(s-\frac{B}{2A}\right)^2}{2\left(\frac{\sigma_{\rm d}}{\sqrt{A}}\right)^2}\right] \quad ; \tag{3}$$

where $n_{\odot}$ = Number of stars in the solar neighbourhood ($\star$/kpc^3), s = distance from the Sun (kpc), l = galactic longitude, b = galactic latitude, $R_{\odot}$ = distance from the Sun to the galactic centre (kpc), $A = cos^2 b$, $B = 2R_{\odot} cosl cosb - 2\frac{\sigma_d{}^2}{Z_0} sinb$.

Equation (3) represents a Gaussian with a maximum value of

$$n_{\rm max} = n_{\odot} \exp\left[\frac{B^2}{8A\sigma_{\rm d}{}^2}\right] \quad ,$$

which occurs at a distance from the Sun

$$s_{\rm max} = \frac{B}{2A} = R_{\odot} cosl secb - \frac{\sigma_{\rm d}{}^2}{Z_0} tanb secb \quad .$$

Note that $s_{\rm max}$ depends on Z_0; therefore, the maximum may occur at different distances from the Sun for different spectral-type stellar groups. The standard deviation for the disc distribution is equal to

$$\sigma(b) = \frac{\sigma_{\rm d}}{\sqrt{A}} = \frac{\sigma_{\rm d}}{cosb}$$

2.2. Numerical Simulations: The Bulge

In this paper we consider the bulge as the central part of the overall spheroidal component of the Galaxy; this implies that the halo would also present a Gaussian density profile. It has been found that the halo density profile is proportional to $\rho^{-3.5}$ (Freeman 1996 and references therein). It is not difficult to mimic this dependence, within observational errors, with a Gaussian function with properly chosen parameters. Moreover, Ng et al. (1996) mention the fact that both the halo and the bulge present a radial density profile proportional to $\rho^{-3.5}$. They also establish that for them, bulge in BW means metal-rich stars and halo means metal-poor stars. In the light of these assertions, the name spheroid for the bulge in this paper might have been a far better choice.

We shall model the bulge as a radially symmetric component for which the number of stars along a line of sight from the centre, obeys the following expression

$$N_{\rm bulge}(r)(\star/kpc^3) = n_{\rm cb} \exp\left[-\frac{r^2}{2\sigma_{\rm b}^2}\right] \quad , \tag{4}$$

where $n_{\rm cb}$ = central number of stars in the bulge ($\star$/kpc^3), r = distance from the centre (kpc) and $\sigma_{\rm b}$ = radial scale length of the bulge (kpc).

As seen from the Sun, equation (4) is also a Gaussian as follows

$$n(s,l,b) = n_{\rm max} \exp\left[-\frac{(s - R_{\odot} cosl cosb)^2}{2\sigma_{\rm b}^2}\right] \quad , \tag{5}$$

with

$$n_{\rm max} = n_{\odot} \exp\left[\frac{R_{\odot}^2 cos^2 l cos^2 b}{2\sigma_{\rm b}^2}\right] \quad , \tag{6}$$

which occurs at a distance from the Sun

$$s_{\rm max} = R_{\odot} cosl cosb$$

where s is the distance from the Sun in kpc and the other variables have their usual meaning.

Table 1 shows in column 1, the Spectral Type of the stars used (only those with important contribution to the K-band), column 2 is the logarithm of their solar neighbourhood density in stars/kpc^3, column 3 shows their scale height in kpc above the galactic plane, column 4 gives their absolute bolo-

TABLE 1

PARAMETERS FOR THE STARS OBSERVED IN THE IR THAT CONTRIBUTE IN AN IMPORTANT FORM TO THE BOLOMETRIC LUMINOSITY FUNCTION

Sp. Type	$\log(\rho_{\odot})$ ($\star$ kpc^{-3}) ($\pm\sqrt{N}$)	Z_0 (kpc) (±0.01)	$M_{\rm bol}$ (±0.12)
M4-5 V	7.35	0.30	9.09
M2-3 V	7.25	0.30	8.11
M0-1 V	7.15	0.30	7.57
K4-5 V	7.05	0.3	6.73
K0-1 III	5.59	0.20	1.34
K2-3 III	5.23	0.20	0.67
K4-5 III	4.28	0.30	–0.73
M0 III	3.48	0.30	–1.42
M1 III	3.13	0.30	–1.58
M2 III	3.13	0.30	–1.92
M3 III	3.13	0.30	–2.32
M4 III	3.00	0.30	–3.30
M5 III	3.00	0.30	–4.08
M6 III	2.45	0.30	–4.84
M7 III	2.09	0.30	–5.74
M8+ III	1.65	0.30	–6.56
K-M2 I-II	1.49	0.05	–7.06
M3-4 I-II	1.10	0.05	–8.18

metric magnitudes, derived using data from Ruelas-Mayorga (1991a) and equations (11) and (12).

If there were no absorption present, our galactic model in any arbitrary direction (l, b) would consist of two sets of 18 Gaussians each one, which would represent respectively the disc and bulge components.

As a matter of convenience, we shall require that eq. (1) be normalised in such a way as to produce the value for the solar neighbourhood density for a particular spectral type, when integrated over a volume of 1 kpc^3 centered on the Sun. This, of course, implies the introduction of a normalisation factor (F_{Ndisc}) such that

$$\int_{-0.5\ kpc}^{+0.5\ kpc} F_{Ndisc} n_{cd} exp\left[-\frac{r^2}{2\sigma_d^2}\right] = n_{\odot disc} \ , \qquad (7a)$$

where $n_{\odot disc}$= density of disc stars in the solar neighbourhood ($\star$/kpc^3).

The expression for the bulge (eq. 4) is normalised the same way; that is

$$\int_{-0.5\ kpc}^{+0.5\ kpc} F_{Nbulge} n_{cd} exp\left[-\frac{r^2}{2\sigma_b^2}\right] = n_{\odot bulge} \ , \qquad (7b)$$

where $n_{\odot bulge}$= density of bulge stars in the solar neighbourhood ($\star$/kpc^3).

This model is used to predict the LF of the Galaxy, provided the stars follow Gaussian distributions such as the ones proposed earlier for the disc and the bulge (eqs. 3 and 5).

Figure 1 shows the model-predicted LF for BW with a simulated completeness limit of $M_{\rm bol} \sim -3.0$. The triangles represent the disc contribution, the circles the bulge contribution and the diamonds the total would-be-observed LF. The horizontal uncertainty reflect the error in the determination of the bolometric correction ±0.12 (see eqs. 11 and 12) and the vertical uncertainty is due to Poisson statistics $(N \pm \sqrt{N})$. It is interesting to note that even though the theoretical stellar density distributions are symmetrical, the generated LF is not, particularly for the disc. Figure 2 presents the points of the theoretically generated LF (diamonds) and the two best fitting Gaussian functions (dashed-lines). The agreement between these functions and the model disc and bulge contributions shown in Fig. 1 is remarkable except for the fact that the model disc contribution appears to be slightly asymmetrical towards the bright end. This effect, of course, is not shown by the fitted Gaussians.

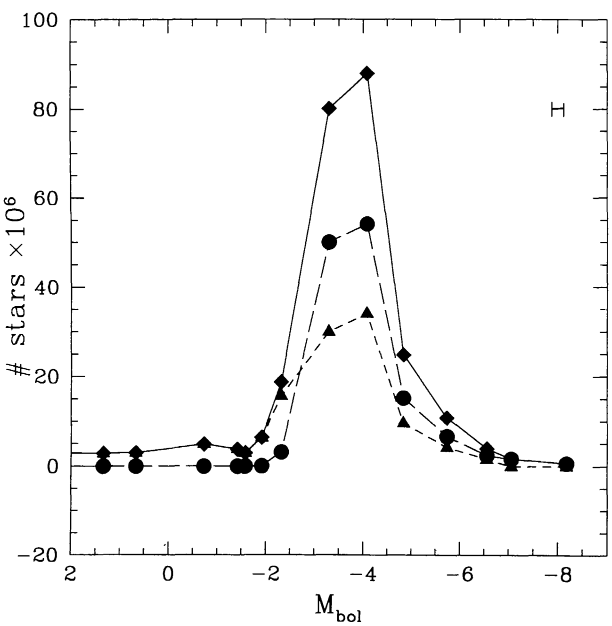

Fig. 1. Model-predicted LF for BW (completeness limit $M_{\rm bol} \sim -3.0$). The triangles represent the disc and the circles the bulge contributions. The diamonds represent the total contribution which is fitted by Gaussians (see Fig. 2). Note the close agreement between the Gaussians in Fig. 2 and the disc and bulge components in this figure. The error bar at top right is the corresponding error in the determination of the BC_K. Vertical uncertainties are typically smaller than the symbols.

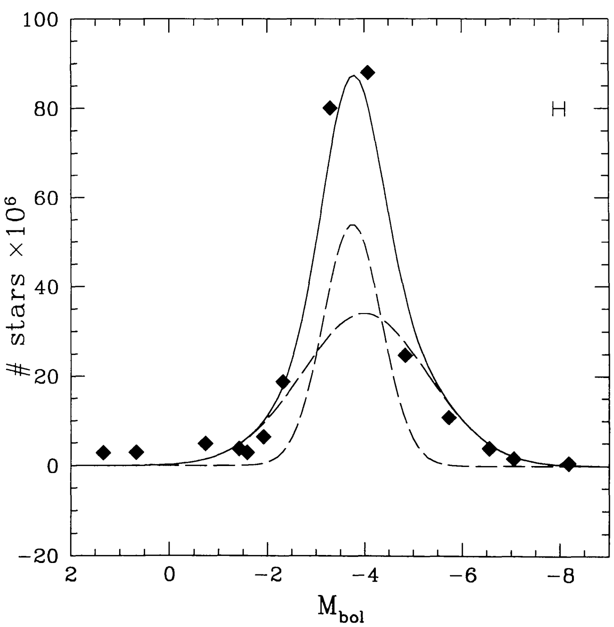

Fig. 2. The diamonds represent the LF derived using the model discussed in § 2 of this paper. The dashed-lines represent the Gaussians which we use empirically to fit the disc (low) and bulge (tall) contributions to this stellar sample. The solid line gives the combined effect of the disc and bulge populations. The error bar at top right is the corresponding error in the determination of the BC_K. Vertical uncertainties are typically smaller than the symbols.

The purpose of this exercise is to show that a plausible observed LF may be very adequately represented by two Gaussians, fitted exclusively according to statistical requirements, and that the characteristic parameters of these curves may be consistently associated with the physical extent for the disc and bulge. We have generated LFs with other stellar distribution models; their general appearance is exactly similar to the functions depicted in Fig. 1, and the characteristic parameters of the fitted Gaussians reproduce, in and adequate way, the physical extents for the disc and bulge components used in the different generating models.

Our theoretically produced LF is now empirically fitted by two Gaussians with the following mathematical expressions

$$N_{\rm bulge}(M_{\rm bol}) = 54 \times 10^6 \exp\left[-\frac{(M_{\rm bol}+3.75)^2}{2(0.6)^2}\right] , \qquad (8)$$

and

$$N_{\rm disc}(M_{\rm bol}) = 34\times 10^6 \exp\left[-\frac{(M_{\rm bol}+4.0)^2}{2(1.3)^2}\right] . \qquad (9)$$

We have assumed each spectral type to be distributed around the galactic centre with particular characteristic distances, σ_d and σ_b, for the disc and bulge components respectively. This distribution is mathematically represented by a function $\rho(r)$, where, for a particular direction (l, b), the maximum always occurs within a small interval around the same value for r, independently of the spectral type in question. As we observe and produce the number-counts versus magnitude function, what we are really doing is transforming the function $\rho(r)$ for each spectral type to a function $\rho(M_{bol})$ through the use of the relation $m - M = 5 \log r - 5$. However, the functions $\rho(M_{bol})$ for each spectral type do not attain their maxima at the same value of M_{bol} and since the observed function is the combination of the $\rho(M_{bol})$ for each individual spectral type, the resulting function presents a larger width than that corresponding to the spatial distribution. We ran several numerical experiments and established that the width of the observed magnitude distribution (Δm_{obs}) is approximately $2.6 \times (\Delta m)$, where Δm is the magnitude-width for a single spectral type. If we now take the equation for the distance modulus $m - M = 5 \log r - 5 = \frac{5}{\mu} ln\ r - 5$, where $\mu = ln\ 10 = 2.30$. Taking the differential of this equation we get $\Delta m = \frac{5}{\mu}\frac{\Delta r}{r}$ from which $\Delta r = \frac{r\mu}{5}\Delta m$, but $\Delta m \sim \frac{\Delta m_{obs}}{2.6}$, therefore

$$\Delta r \sim \left(\frac{r}{5}\right) \Delta m_{obs} . \qquad (10)$$

From the standard deviation of functions (8) and (9), and using relationship (10), we derive standard deviations for the physical distribution of the stars in the disc and bulge components whose values are: $\sigma_{\rm d} = 2.28$ kpc and for the bulge $\sigma_{\rm b} = 1.05$ kpc; these values are in perfect agreement with the characteristic radial scale lengths for the disc and the bulge, used in the model for generating the LF which would be observed. These values are: for the disc $\sigma_{\rm d} = 2.0$ kpc and for the bulge $\sigma_{\rm b} = 1.0$ kpc. These numerical simulations and checks and those presented in Ruelas-Mayorga (1997) have permitted us to see that the simplified model which we have used for representing the Galaxy is an adequate one, and that the results obtained from the data, using this model, are trustworthy as claimed in Ruelas-Mayorga & Noriega-Mendoza (1995).

3. THE OBSERVATIONS

The observations were collected with the Infrared Photometer attached to the 1.9-m telescope at the Mount Stromlo and Siding Spring Observatories. They were obtained scanning our detector at sidereal rate over four low absorption areas, two of which are contained within the BW region (BW, R2), and two others very near this area. Subsequently, we performed detailed JHK photometric observations of subsamples of the stars found in the scans. The photometric values may be consulted in Table 2 of Ruelas-Mayorga & Teague (1992a), Table 1 of Ruelas-Mayorga & Teague (1992b), and in Tables 2a, 2b and 2c of Ruelas-Mayorga & Teague (1993). For further details see the papers cited above.

Table 2, column 1 gives the names of the areas studied, columns 2 and 3 give approximate galactic coordinates l and b respectively, column 4 lists their surface area in square minutes of arc, columns 5 and 6 show an area factor and a factor derived from the de Vaucouleurs $r^{\frac{1}{4}}$ law respectively; used to normalise the number of stars arbitrarily to the BW area. The normalisation is achieved by multiplying the appropriate number of stars by the factors in columns 5 and 6 of Table 2. The evidence shows the $r^{\frac{1}{4}}$ law to be valid for the bulges of other spiral galaxies, if we assume for them, as we have for our own Galaxy, that the disc extends all the way to their centres, then the normalisation factor applied to our Galaxy, due to the $r^{\frac{1}{4}}$ law is fully justified because it obviously applies to the light distribution of disc and bulge populations mixed in the surroundings of other galactic centres.

We use the Bolometric Corrections to the K magnitude $(BC_{\rm K})$ as a function of $J - K$ colour given by FW properly transformed to the Anglo Australian Observatory (AAO) IR system in which our K magnitudes are given (see McGregor & Hyland 1981; Elias et al. 1983; Ruelas-Mayorga & Teague 1992a). The mathematical expressions are obtained from a

TABLE 2

OBSERVED REGIONS USED FOR THE DERIVATION OF THE LF OF THE STARS IN THE BULGE

Region	l (°) (±12″)	b (°) (±12″)	A (sq. arcmin) (±0.1)	A Factor	$r^{\frac{1}{4}}$ Factor
BW	0.0	−4.0	201.0	1.00	1.000
R1	0.0	−3.5	139.0	1.45	0.931
R2	0.0	−4.0	56.3	3.57	1.000
R3	0.0	−4.5	185.0	1.09	1.064

graphical fit to Figure 1b of FW. An uncertainty of $\sim \pm 0.12$ mag for $BC_{\rm K}$ is read directly from this graph. The relations, valid only for our data, are as follows

$$BC_{\rm K}(\pm 0.12) = 1.61(J-K)_{\rm AAOOBS} + 0.64 \ , \quad \text{for } (J-K)_{\rm AAOOBS} \leq 1.55 \ ; \tag{11}$$

$$BC_{\rm K}(\pm 0.12) = 0.38(J-K)_{\rm AAOOBS} + 2.55 \ , \quad \text{for } (J-K)_{\rm AAOOBS} \geq 1.55 \ . \tag{12}$$

Due to the different photometric systems used, the FW and our values for the observed and dereddened magnitudes and colours are obviously different; however, the value of the bolometric magnitude for a given star must be the same whether it is obtained using the FW observed quantitites or ours. In the appendix we show the calculation of the BC_K using eqs. (11) and (12) for some assumed observed values in the AAO system and for the corresponding values in the FW system. At this point, we would like to stress the fact that all the observational data (stellar counts and K-magnitudes) have been arbitrarily normalised to the BW area.

The calculations presented in the appendix assure us that the analytical expressions applied to our observed quantities in the AAO system do give the same bolometric corrections as those obtained from the FW calibration.

Therefore, the absolute bolometric magnitude may be calculated as follows

$$M_{\rm bol} = K_{\rm OBS} - A_{\rm K} + BC_{\rm K} - 5 log \ r + 5 \ . \tag{13}$$

where $M_{\rm bol}$ = absolute bolometric magnitude, $K_{\rm OBS}$=observed K magnitude in the AAO system, $A_{\rm K}$ = absorption in K, $A_{\rm K}(BW)$ = 0.24 mag, $A_{\rm K}(R1)$ = 0.38 mag, $A_{\rm K}(R2)$ = 0.39 mag, $A_{\rm K}(R3) = 0.32$ mag, $BC_{\rm K}$ = Bolometric correction obtained from eqs. (11) and (12) and r = distance in parsecs.

The values for $A_{\rm K}(BW)$, $A_{\rm K}(R1)$, $A_{\rm K}(R2)$, and $A_{\rm K}(R3)$ were taken from Ruelas-Mayorga & Teague (1992a [Fig. 6]; 1993).

Since the observations that we have were taken over four different areas, we would like to correct the number counts as though all of them had been obtained from the same region. We choose this region arbitrarily to be that named BW and correct the counts by an Area factor (see Table 2). It is known that the light distribution of bulges of spiral galaxies obey the $r^{\frac{1}{4}}$ law. In order to have a completely homogeneous sample referred to region BW, we shall also correct the counts by an $r^{\frac{1}{4}}$ factor (see Table 2). This supposes that the characteristics of the stellar populations within the four observed areas are the same. The magnitudes and colours for the stars in each region are also corrected so that the entire stellar sample appears to have $A_{\rm K} = 0.24$ mag and $E(J-K) = 0.27$, which are the values we found for BW. Both these corrections assure that our combined stellar sample has the appropriate characteristics as if it had all been drawn from the BW region. We may now proceed to test our assumptions with this homogeneous sample and the stellar distribution model proposed in this paper.

The number of stars in each window as a function of bolometric magnitude is given in Table 3. Column 1 gives the value $m_{\rm bol} - A_{\rm bol}$, column 2 gives the absolute bolometric magnitude ($M_{\rm bol} = m_{\rm bol} - A_{\rm bol} - 14.7$), where 14.7 is the adopted value for the distance modulus to the galactic centre (8.75 kpc), which corresponds to an average of somewhat recent determinations of the value of this parameter (Graham 1979). Newer values for this parameter have been found by measurements of the proper motions of several H_2O-masers and give a value 7.6 ± 1.6 kpc $DM = 14.4 \pm 0.5$ (Reid et al. 1987; Gwinn et al.

TABLE 3

OBSERVED (O) AND CORRECTED (C) NUMBERS OF SOURCES AS A FUNCTION OF APPARENT AND ABSOLUTE BOLOMETRIC MAGNITUDE FOR THE 4 REGIONS OBSERVED[a]

		BW	*R*1		*R*2		*R*3		*T*	
$m_{\rm bol} - A_{\rm bol}$	$M_{\rm bol}$	$O = C$	O	C	O	C	O	C	O	C
(1)	(2)	(3)	(4)		(5)		(6)		(7)	
14.75	+0.04	2	0	0	0	0	0	0	2	2
14.25	−0.46	2	0	0	0	0	0	0	2	2
13.75	−0.96	5	0	0	0	0	0	0	5	5
13.25	−1.46	10	0	0	0	0	0	0	10	10
12.75	−1.96	12	1	1	4	14	0	0	17	27
12.25	−2.46	23	8	11	8	29	3	3	42	66
11.75	−2.96	49	8	11	7	25	3	3	67	88
11.25	−3.46	43	4	5	2	7	8	9	57	64
10.75	−3.96	17	2	3	2	7	1	1	22	28
10.25	−4.46	11	0	0	1	4	0	0	12	15
9.75	−4.96	1	1	1	1	4	1	1	4	7
9.25	−5.46	4	2	3	0	0	0	0	6	7
8.75	−5.96	0	0	0	0	0	1	1	1	1
8.25	−6.46	2	0	0	0	0	0	0	2	2
7.75	−6.96	0	0	0	0	0	1	1	1	1
7.25	−7.46	0	0	0	0	0	0	0	0	0
6.75	−7.96	2	0	0	0	0	0	0	2	2
6.25	−8.46	0	0	0	0	0	0	0	0	0
T	...	183	26	35	25	90	18	19	252	327

[a] The corresponding uncertainties (not shown) obey the Poisson statistics.

1989) for the distance to the galactic centre (for a more recent account of the latest determinations of this parameter see Reid 1989). If our adopted DM were in error by this much (0.31), which seems quite unlikely, our values for $M_{\rm bol}$ would be shifted towards fainter values by $\sim$ 0.3 mag. In view of the difficulty of establishing with certainty the distance to the galactic centre and the success obtained with the value of $DM = 14.7$ in applications of several galactic models for the stellar distribution along the galactic plane (see Ruelas-Mayorga 1991a,b), we shall adopt it and $E(J-K) = 0.27$ for our normalising region (BW) (see Ruelas-Mayorga & Teague 1992a for further details). A very recent determination of the distance to the Galactic centre by means of infrared photometry of RR Lyrae variables (Carney et al. 1995), gives a value of $R_\odot = 8.3$ kpc which implies $DM = 14.6$ which is only 0.1 units smaller than our value.

Columns 3 to 6 of Table 3 show the observed and corrected number of sources for the regions BW, R1, R2 and R3, respectively; finally column 7 gives the observed and corrected total number of sources per magnitude bin. At the bottom ot the table the total number of sources in each area is given.

4. ANALYSIS

Figure 3 presents both the Differential Luminosity Function (DLF) (triangles) [Number of sources within an arbitrary area with absolute bolometric magnitude in the interval $M-\frac{1}{2}, M+\frac{1}{2}$] and the Cumulative Luminosity Function CLF (circles) [Number of sources within an arbitrary area down to absolute bolometric magnitude equal to M]. The dia-

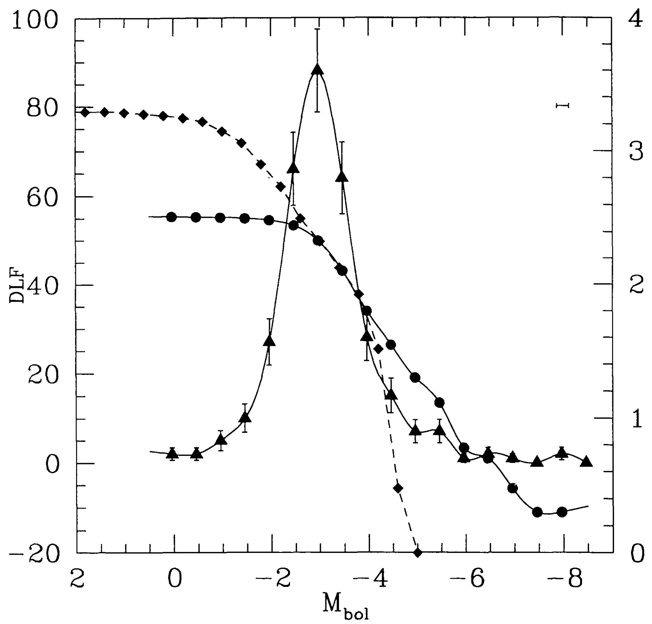

Fig. 3. On the left axis we present the observed differential luminosity function (DFL) (triangles) and the logarithm of the cumulative luminosity function (right axis) for both our data (circles) and for those of Frogel & Whitford (1987) (diamonds). The error bar at top right is the corresponding error in the determination of the BC_K. Vertical uncertainties are obtained from Poisson statistics and will be so obtained for all the figures in this paper.

monds illustrate the Frogel & Whitford (1987) BW CLF (FWCLF) displaced and reddened appropriately as if their stars underwent the same absorption and reddening as our stars do. As noted before, we shall assume a distance modulus and a reddening to the galactic bulge in BW of $(m-M)_0 = 14.7$ and $E(J-K) = 0.27$ respectively (see Ruelas-Mayorga & Teague 1992a).

A comparison between the triangles in Fig. 3 (observed points) and the diamonds in Fig. 1 (model generated points) is interesting since the maxima of each graph coincide within an interval of ± 0.5 mag. This fact further supports the idea that the simplified galactic model developed in this paper does represent appropriately the distribution of stars in our galaxy.

It is prudent at this point to say that both stellar samples are located within the same approximate region on the sky, and that FW's group and ours assume these stars to be in the innermost sections of the Galaxy in the surroundings (± 2 kpc) of the galactic centre. We, therefore, would expect both samples to belong to the same stellar population and to have the same physical characteristics.

It is important to note that our CLF shows the following characteristics: (1) a strong decrease around $M_{\rm bol} = -3$ which corresponds to M4-M5 III stars, (2) both stellar samples have similar slopes in the interval $-4.0 \leq M_{\rm bol} \leq -2.5$.

An analysis of our data indicates that our stellar sample is complete down only to $M_{\rm bol} = -3.0$, at which point it starts to decrease in a more or less systematic form. We shall assume that the FW's DLF presents a similar behaviour; therefore, they must achieve completeness in the interval $-1.4 \leq M_{bol} \leq -1.5$.

Figure 4 shows our DLF (triangles) as well as that from Frogel & Whitford (1987) (circles) after the latter has been divided by 4. This scaling has little importance and is intended solely to produce graphs of comparable dimensions. From this figure we clearly see that the maxima in the LFs do not coincide but appear shifted by ~ 1.5 mag. An error in the value of $A_{\rm K}$ would produce an effect like this one; however, its magnitude (~ 1.5 mag) needs to be so large that it seems unlikely to us. Differences in the value of the distance modulus (DM) can also produce such a shift; we adopt DM = 14.7 while FW use DM = 14.2, these values allow only for a 0.5 shift.

In Ruelas-Mayorga & Teague (1992a) we noted that FW obtain their sample for IR study from the Blanco et al. (1978, 1984) 'I' plates of BW and that not all the IR-detected sources which we studied appeared on their survey. Frogel & Whitford (1987)

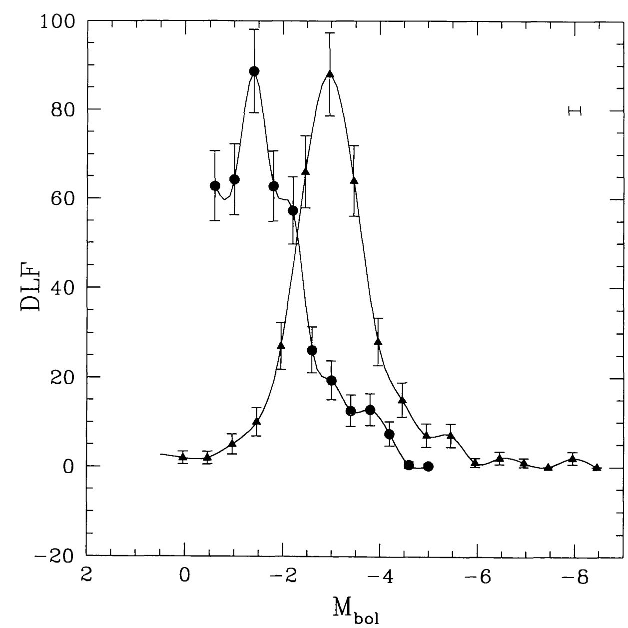

Fig. 4. Differential luminosity functions for our data (triangles) and for FW data (circles). Note the difference in $M_{\rm bol}$ for the maxima. This difference may be interpreted mainly as produced by a difference in depth between our survey and that of FW. The error bar at top right is the corresponding error in the determination of the BC_K.

seem to study a slightly less red population than we do. From our mean $(J-K)_0 \sim 1.33$ we conclude that most of our stars are M5-M6 giants. For FW, the average $(J-K)_0 \sim 1.06$ implies a less red stellar population, and since most of the stars are giants, this also implies a fainter spectral type in the K filter.

As mentioned above, our stellar sample is complete down only to $M_{\rm bol} \sim -3.0$. If FW's sample is complete to a deeper $M_{\rm bol}$ limit, it is clear that the maximum of their LF will appear at fainter magnitudes. We, however, emphasise the paucity of sources in the FW LF in the interval $-5.0 \leq M_{\rm bol} \leq -8.0$ which certainly may not be explained away by invoking a difference in depth between our sample and theirs. Their LF is clearly missing stars at bright bolometric magnitudes. We are, therefore, inclined to interpret this lack of coincidence between maxima, mainly as due to the difference in depth of both surveys, and partly as produced by the different stellar populations which we study.

As it has been suggested by previous results of ours, the stellar population in BW may be constituted by circumnuclear disc and bulge components. In what follows, we shall show that this interpretation is not inconsistent with the data. We want to stress the fact that this paper does not intend to prove the presence of circumnuclear disc and bulge populations in BW from the data, rather it only aims at showing that this possibility is consistent with the data.

As the simplified galactic model developed in § 2 shows, the observed DLF may be adjusted by two Gaussian functions: (1) one function is wide and low. It presumably represents the disc contribution; (2) the other function is narrow and high. We claim that it represents the bulge.

Both functions may be represented mathematically as follows

$$N(M_{\rm bol}) = A \exp\left[-\frac{(M_{\rm bol} - M_{{\rm bol}_0})^2}{2\sigma^2}\right] \quad , \qquad (14)$$

where for the disc case: A = 34, $M_{{\rm bol}_0} = -2.96$, and $\sigma = 1.16$; for the bulge case: A = 54, $M_{{\rm bol}_0} = -2.96$ and $\sigma = 0.53$, where the ratio $B/D \sim 1.58$ is the appropriate value found by Ruelas-Mayorga & Teague (1992a) down to $K \sim +9.0$.

Figure 5 shows the observed DLF (triangles) and the two components into which we have separated it: the disc (dashed) and bulge (solid). The dotted line represents the sum of the disc and bulge components. Note the agreement with the observed DLF.

From the above mathematical expression we may obtain several pieces of information. The bolometric magnitude at which the maximum is present is ~ -3.0 mag which corresponds to M3-M4 III stars, slightly earlier in spectral type than expected from the results in Ruelas-Mayorga & Teague (1992a).

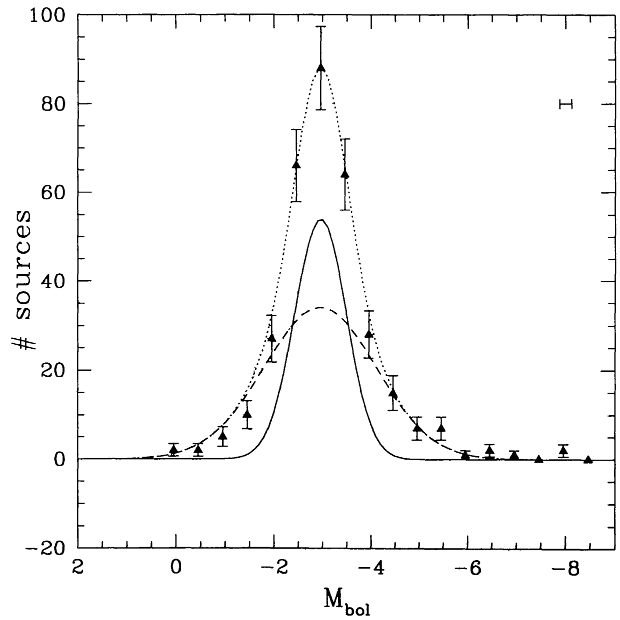

Fig. 5. Observed differential luminosity function (DLF) for our data (triangles). It may be decomposed into two Gaussians which are associated with the bulge (solid line) and the disc (dashed line). The dotted line represents the sum of the two components. Note how well it reproduces the observed DLF. The error bar at top right is the corresponding error in the determination of the BC_K.

The standard deviation of the Gaussian representations used to model the DLF may, as indicated in § 2, give us an indication as to the size of the two galactic components which they represent; we may, therefore, say that the total radius of the bulge $\sim 3\sigma \sim 2.8$ kpc, and that the total radius of the disc $\sim 3\sigma \sim 6.1$ kpc.

The presence of the disc population within BW, apart from the obvious projected one, is not a consequence of the particular model used to fit the observations. It is rather an assumption which is later proved to be consistent with the data, independently of the galactic model in use. In this paper it is shown that this assumption is consistent with the data through the use of the simplified galactic model presented here; however, in Ruelas-Mayorga & Teague (1992a) this assumption is also proved to be consistent with the data using a different stellar distribution model. We propose that this consistency will be present independently of the model utilised to fit the data.

In order to study whether there really is a circumnuclear disc-like component in BW, we take the LF in the solar neighbourhood for M III stars from Ruelas-

Mayorga (1991a) and calculate the total M III star distribution in the solar vicinity. Figure 6*a* illustrates this function. The tall solid line represents the total function whereas all the other lines show the distribution of individual M spectral types. As later spectral types are reached, a shift towards brighter $M_{\rm bol}$ is noted (see Figure 6*b*).

If we integrate the distribution of M III stars in the solar neighbourhood along a pyramid in the direction of BW, with a base area equal to the sum of the areas of the 4 observed regions (581.3 square arcmin) and whose height is 8.75 kpc, the number of sources observed with an absolute bolometric magnitude $M_{\rm bol}$ is

$$N(M_{\rm bol}) = K\rho_{\odot}\theta_{\rm a}\theta_{\rm b} \quad ; \qquad (15)$$

where K is a constant which needs to be determined by calculations and, whose value varies from region to region (K = 405 for BW and R2, K = 494 for R1 and K = 330 for R3), $\rho_{\odot}$ represents the density in the solar neighbourhood and $\theta_{\rm a}\theta_{\rm b}$ represents the area of the region in square radians. When this is applied to our study we obtain the following equation

$$N(M_{\rm bol}) = 2 \times 10^{-2} \times \rho_{\odot}(M_{\rm bol}) \qquad (16)$$

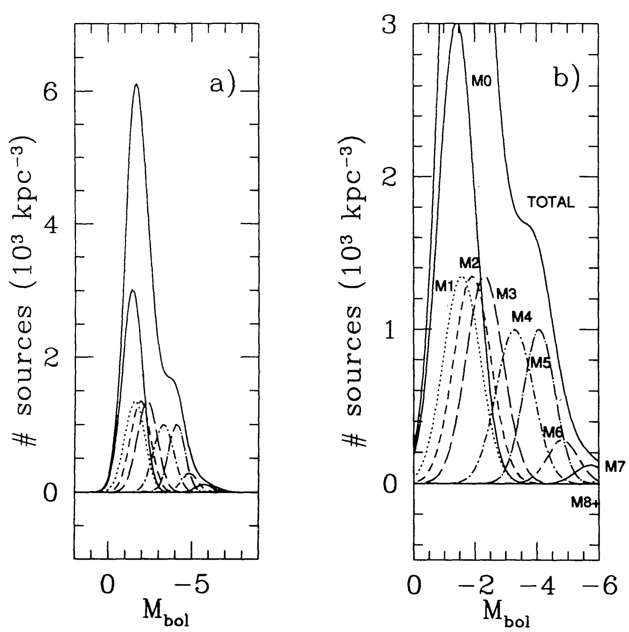

Fig. 6*a*). Local distribution for the M III stars (tall-solid line). All the other lines represent the individual contributions of spectral types from M0 to M8 III. *b*) This figure is an enlargement of the central section of Fig. 6*a* so that it is possible to see the individual contributions of each M-spectral type to the stellar density. As later spectral types are reached, a shift towards brighter $M_{\rm bol}$ is noted.

The integrated and projected distribution has the same width as that for the solar neighbourhood and with a maximum of observed sources equal to 120 centred at $M_{\rm bol} \sim -1.5$ which agrees with the maximum of the FWLF. Our LF is centred at $M_{\rm bol} \sim -3.0$ which rather agrees with the distribution produced by the combination of M3 to M8 III stars as seen in Figure 7. The width of the theoretical distribution ($\sim$ 2.5) and that of the observed one ($\sim$ 2.0) agree reasonably well. The maximum number of sources seen, at $M_{\rm bol} = -3.0$, on the bright component of the theoretical distribution for M III stars is $\sim$ 1600 kpc^{-3}. This figure, when projected towards BW, predicts a maximum observed number of

$$\sim 1600 \times 2 \times 10^{-2} = 32 \text{ sources} \quad , \qquad (17)$$

which agrees very well with the maximum observed number (34) of sources for the wide component of our LF.

The low and wide distribution seen in Fig. 5 (dashed), which we claim represents the disc contribution, may be reproduced very well from the combined effects of the disc M3-M8+ stars as seen in Fig. 7 (dashed, low). This contribution is centred at $M_{\rm bol} \sim -3.0$ and has a width of $\sim$ 2.5 mag in good agreement with the fitted Gaussian claimed to represent the disc. The disc also contributes a more important and narrower part to the distribution (see Fig. 7, long dashes) but since it appears

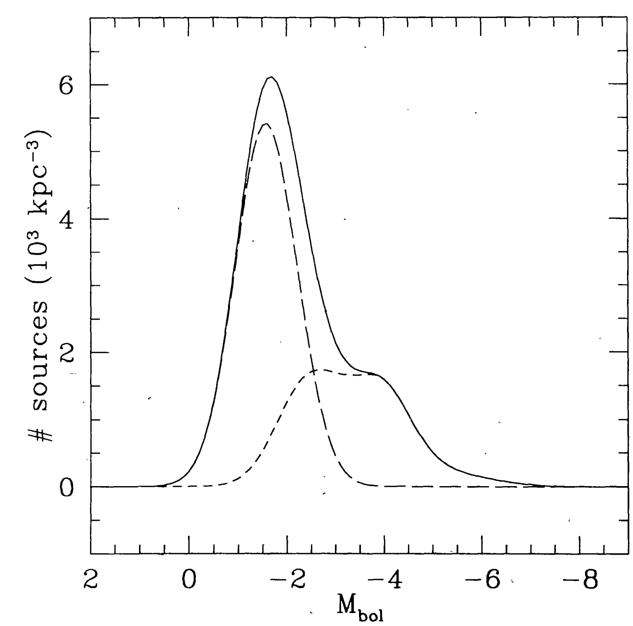

Fig. 7. Local distribution for the M III stars (tall-solid line). The dashed lines represent the contribution of M0, M1 and M2 stars (tall) and M3 to M8+ stars (low).

at fainter magnitudes, its presence cannot be seen in our observed data. If the FW data are indeed deeper than ours, they could have important contributions from disc M0-M2 giant stars in their reported stellar counts.

Our suggesting that the low and wide component of the observed LF may represent an extension of the disc into the innermost sections of the Galaxy receives an important support from this calculation. This might clearly mean that there is in our sample of stars in the four clear windows towards the galactic centre, a subsample that can consistently be interpreted as made by members of the disc population and as having physical characteristics (brightness and maybe mass, age, and metallicity) similar to those of the stars in the solar neighbourhood.

Figure 8 illustrates on the same graph and with no scaling factors the DLF from Frogel & Whitford (1987) (circles) and our DLF (triangles). A comparison between Figs. 7 and 8 is rather interesting since both graphs appear to present the same general character, being the ratio of their maxima equal to ~ 4. This suggests that an important part of the brighter section of the FWDLF is disc-produced; whereas the fainter ($M_{\rm bol} \geq 0.0$) section of the FWDLF corresponds mainly to the 'true bulge' contribution. The tall long-dashed line in Fig. 7 may correspond partly to the FWDLF, and the low short-dashed line may —almost completely— correspond to our DLF.

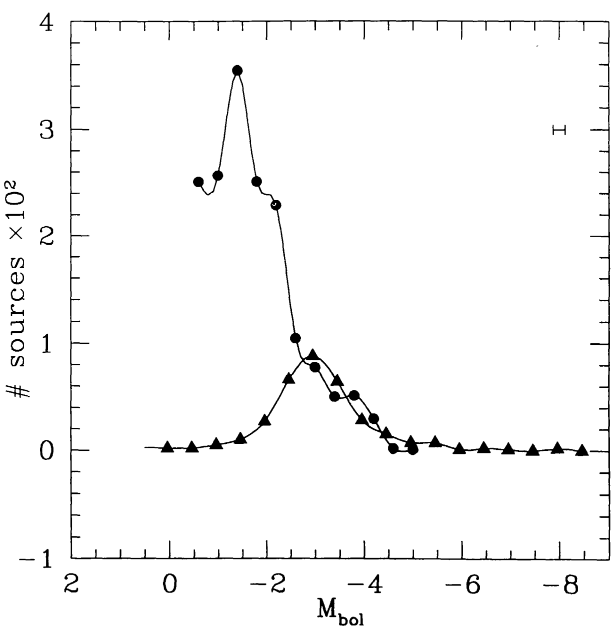

Fig. 8. This figure shows the differential luminosity functions for the FW data (circles) and for our own data (triangles), at their real measured sizes. Compare this figure with Fig. 7 and note how similar in character both figures are. The error bar at top right represents the error in the determination of the BC_K. Vertical uncertainties are typically smaller than the symbols.

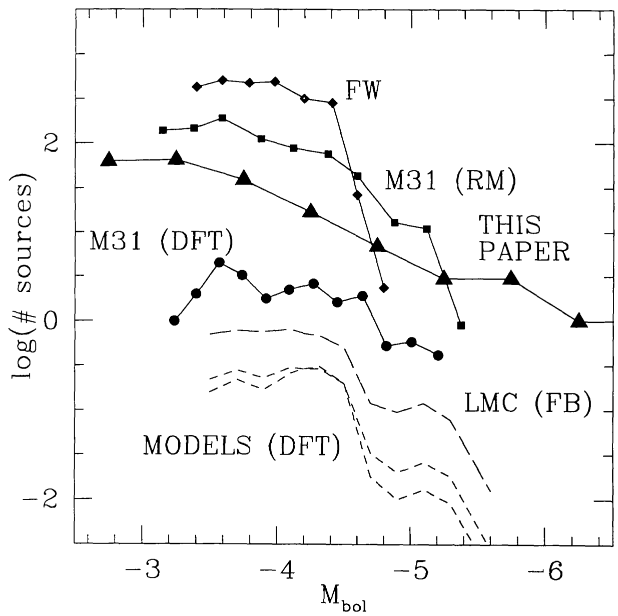

Fig. 9. Plot of LFs of several stellar populations; the triangles represent the LF derived in this paper, the diamonds picture the galactic bulge LF found by FW (shifted by +1.2), the dashed line (long dashes) is the LF for the LMC Bar West (Frogel & Blanco 1990) (shifted by −1.6), the squares and circles depict the M31 bulge LF derived by Rich & Mould (1991) (shifted by +0.6) and Davies, Frogel, & Terndrup (1991) (shifted by −1.0) respectively. The short-dashed-lines represent models (shifted by −2.0) Davies et al. (1991).

5. COMPARISON WITH OTHER LUMINOSITY FUNCTIONS

In Figure 9 we show a plot of LFs for several stellar populations, they have been shifted vertically to avoid confusion; the triangles picture the LF derived in this paper, the diamonds represent the galactic bulge LF found by FW, the long-dashed line is the LF for the LMC Bar West (Frogel & Blanco 1990), the squares and circles depict the M31 bulge LF derived by Rich & Mould (1991) and Davies, Frogel, & Terndrup (1991) respectively. Finally the short-dashed lines represent different models for bulge stellar populations (see Davies et al. 1991); the LF for bulge stars published recently by Tiede, Frogel, & Terndrup (1995) is not included in this plot, since it agrees well with the formerly published FW LF and since this paper does not extend to K-magnitudes fainter than $K \sim +11.0$. It is clear that a vertical shift would put our LF in close agreement with all the other LFs, except with that for the galactic bulge found by FW. The sharp cut-off at $M_{\rm bol} \sim -4.2$ found by FW is not present in our data. It is comforting to see that our derived LF for BW is quite similar to the LFs for the bulge of M31 reported by

Rich & Mould (1991) and by Davies et al. (1991). This fact does give support to the idea that the stellar population in BW may be used as a template population for spheroidal populations in general, and in particular for the bulges of other spirals.

In Rich et al. (1989), Rich & Mould (1991) and Davies et al. (1991) a difference between the M31 bulge LF and that given by FW for the galactic bulge is reported. These papers are unable to account for the presence of the precipitous drop at $M_{\rm bol} \sim -4.2$, which FW have interpreted as representing the end of a stars's evolution on the asymptotic giant branch (AGB) in an old, metal rich population. In order to explain away the lack of precipitous drop the idea of dealing with an atypical stellar population, either in the galactic bulge or in the M31 bulge, is put forward. We, however, propose a different solution, and it is that the presence of the sharp drop in the FW galactic bulge LF is an artifact due to the incompleteness of their stellar sample. As noted in Ruelas-Mayorga & Teague (1992a), a section of our K-scanned region in BW overlaps the FW region and, although every single one of their sources is well identified with our K-selected sources, there are many of our K-selected sources not matched by stars in their region. In that paper we concluded that "to draw conclusions as to the IR characteristics of the stellar population in BW based on the Blanco, McCarthy, & Blanco (1984) stars is clearly erroneous because many other important sources in the IR are being discriminated against." It simply seems that very bright IR sources were not included in the FW sample. Recently DePoy et al. (1993) have conducted a two dimensional IR study of BW. One of their conclusions is that no apparent lack of detection of stars is noticed between this IR study and the Blanco et al. (1984) and Blanco (1986) 'I' plates from which FW obtained the stars for their 1987 paper. Unfortunately, the very region in which we did find the discrepancy could not be observed in the above mentioned two dimensional IR paper, leaving the discrepancy pointed out by Ruelas-Mayorga & Teague (1992a) still outstanding.

The observed LF presented in this paper agrees perfectly with that published by Ng et al. (1996) (see their Fig. 7*c*) in the interval where it is complete ($M_{bol} \leq -3.0$). Ng's function does present a noticeable drop in the interval $-5.0 \leq M_{bol} \leq -4.0$ which suggests that the precipitous drop reported by FW at $M_{bol} \sim -4.2$ might be an effect present within a very narrow interval of bolometric-magnitude.

6. SUMMARY AND CONCLUSIONS

i) A simplified model for the bolometric stellar distribution in our Galaxy is obtained.

ii) From this model we show that the DLF for bright magnitude stars in any direction in the Galaxy may be fitted reasonably well by the combination of two Gaussians. They represent the disc and the bulge contributions respectively.

iii) We obtain, for a sample of stars in four clear windows towards the galactic centre, the bolometric LF aided with our JHK photometry and the $BC_{\rm K}$ calibration published by Frogel & Whitford (1987).

iv) The LF towards the galactic centre may be decomposed into 2 components. As suggested by previous studies and our simplified galactic model, each may be consistently associated with the contribution to this stellar sample of the disc and of the bulge respectively.

v) The bulge contribution turns out to be narrow with respect to $M_{\rm bol}$, with a characteristic radius of ~ 0.9 kpc.

vi) The disc contribution is wide with respect to $M_{\rm bol}$ and has a characteristic radius of ~ 2.0 kpc.

vii) The wide component may be perfectly reproduced when the LF for M3-M8+ giant stars in the solar neighbourhood is projected over the BW area and integrated over a distance equal to $R_\odot$ (8.75 kpc). This suggests that there must be a component of the stellar population in the regions surrounding the galactic centre with characteristics very similar to those of the stars in the disc of our Galaxy at the solar neighbourhood. This component is what we have claimed to be the circumnuclear disc contribution to the stellar population in BW (see Ruelas-Mayorga & Teague 1992a,b; 1993a).

viii) Our LF and that of Frogel & Whitford are centred at $M_{\rm bol} \sim -3.0$ and ~ -1.5 respectively. This has been interpreted mainly as due to the difference in the K-depth of our surveys, and partly due to observations of slightly different stellar populations in the same galactic component due to the way they have been selected.

ix) An important section of the Frogel & Whitford LF may be interpreted as disc-produced.

x) Comparison of the LF derived in this paper with other LFs published in the literature reveals that the stellar population in BW may be used as a template population for spheroidal systems, and in particular, for the bulges of spiral galaxies.

The authors are grateful to A. R. Hyland, T. J. Jones, R. J. Wainscoat, and M. M. Casali for discussions and help during the data collection phase of this paper. They also thank P. Pişmiş and A. Arellano for their comments and criticism which improved greatly the presentation of this work as well as C. Chiosi and S. Torres-Peimbert for several suggestions offered. The suggestions given by an anonymous referee are thanked. Both CONACyT (Consejo Nacional de Ciencia y Tecnología) and DGAPA (Dirección General de Asuntos del Personal Académico) have provided financial support to the authors on different occasions. ARM would like to express his deepest thanks to the memory of Dr. A. Ruelas S. from whom he learnt that work and dedication are the only means to achieve success.

APPENDIX

We show that the FW relation between BC_K and $J-K$ produces the same value for the BC_K for the K-magnitude of a star measured in the FW system, as the value obtained with eqs. (11) and (12) for the K-magnitude of the same star measured in the AAO system.

In order to show that this is so let us calculate two cases:

Let us take $(J-K)_{AAOOBS} = 1.11$, where applying eq. (11) we get

$$BC_K = 1.61 \times 1.11 + 0.64 = 2.43 \quad .$$

Now let us deredden this value

$$(J-K)_0 = (J-K)_{obs} - 0.27 \quad ,$$

therefore we obtain

$$[(J-K)_0]_{AAO} = 1.11 - 0.27 = 0.84 \quad .$$

Using the transformation equation in Elias et al. (1983) we get

$$[(J-K)_0]_{CIT} = 0.897(J-K)_{AAO} + 0.006 \quad ,$$

therefore we derive

$$[(J-K)_0]_{CIT} = 0.897 \times 0.84 + 0.006 = 0.76 \quad .$$

For Fig. 1*b* of FW we may represent the variation of the BC_K with $[(J-K)_0]_{CIT}$ as follows

$$BC_K = 2[(J-K)_0]_{CIT} + 0.9 \quad \text{for } [(J-K)_0]_{CIT} \leq 1.5 \quad ;$$

substitution in this equation gives

$$BC_K = 2 \times 0.76 + 0.9 = 2.42 \quad .$$

This value is the same as the value obtained from our eq. (11), upon the consideration of the errors attached to this calculation.

In order now to apply eq. (12), let us take $(J-K)_{AAOOBS} = 1.70$. After its application we get

$$BC_K = 0.38 \times 1.70 + 2.55 = 3.20 \quad .$$

Now dereddening this value we obtain

$$(J-K)_0 = (J-K)_{obs} - 0.27 \quad ,$$

which transformed to the AAO system gives

$$[(J-K)_0]_{AAO} = 1.70 - 0.27 = 1.43$$

Now, transforming this value using the equation in Elias et al. (1983)

$$[(J-K)_0]_{CIT} = 0.897(J-K)_{AAO} + 0.006 \quad ,$$

we get

$$[(J-K)_0]_{CIT} = 0.897 \times 1.43 + 0.006 = 1.29 \quad .$$

For Fig. 1*b* of FW we may represent the variation of the BC_K with $[(J-K)_0]_{CIT}$ as follows

$$BC_K = 0.5[(J-K)_0]_{CIT} + 2.55 \quad \text{for} \quad [(J-K)_0]_{CIT} \geq 1.5 \quad ;$$

substitution in this equation gives

$$BC_K = 0.5 \times 1.29 + 2.55 = 3.20 \quad ,$$

which is the same value obtained with our eq. (12).

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