Revista Mexicana de Astronomía y Astrofísica, **34**, 3–8 (1998)

ARE THERE SIGNIFICANT EQUATOR AND EQUINOX ERRORS IN THE FK5 SYSTEM?

R. L. Branham, Jr.

Centro Regional de Investigaciones Científicas y Tecnológicas, Argentina

and

J. G. Sanguín

Observatorio Astronómico "Felix Aguilar" and Yale Southern Observatory, Argentina

Received 1995 September 11; accepted 1997 June 9

RESUMEN

Se usaron 11739 observaciones en ascensión recta y 11656 en declinación, hechas entre 1847 y 1993, de 21 asteroides para estimar los errores del equinoccio y del ecuador del sistema fundamental FK5. Se corrigió la órbita de cada asteroide junto con la órbita de la Tierra, y los errores del equinoccio y del ecuador del sistema FK5 con sus variaciones seculares. No hay evidencia para un error del ecuador o una variación secular. Sin embargo, hay indicios de un error del orden de 0.18″ para la época 1970 del equinoccio con un *posible* movimiento de 0.11″–0.14″ por siglo. Hay evidencias externas que confirman por lo menos parte de estas conclusiones.

ABSTRACT

11739 observations in right ascension and 11656 in declination, made between 1847 and 1993, of 21 minor planets are used to estimate the equinox and equator errors of the FK5 fundamental system. Each minor planet's orbit was corrected along with corrections for the Earth's orbit and the FK5 equator, and equinox errors with their secular variations. No strong evidence exists for an equator correction or secular variation. Evidence does exist, however, for an equinox correction of the order of 0.18″ at the approximate epoch 1970 and for a *possible* motion of the equinox of about 0.11″–0.14″ per century. External evidence confirms at least part of these conclusions.

Key words: **ASTROMETRY — CATALOGS — MINOR PLANETS**

1. INTRODUCTION

This is the third paper attempting to find out if there are significant equator and equinox errors in the FK5 fundamental system (Fricke, Schwan, & Lederle 1988). The first paper, Paper I, used 413 photographic observations of 16 minor planets and arrived at inconclusive results (Branham & Sanguin 1992). Because of this unsatisfactory outcome, we added considerably more observations, 4528 photographic observations of 21 minor planets and reached firmer, but because of the nonnormality of the residuals, still not entirely convincing conclusions (Branham & Sanguin 1996), Paper II. In this paper the number of observations used is large, 11739 in right ascension (α) and 11656 in declination (δ), covers a long time span, nearly a century-and-a-half, and because 96% of the observations were used previously to successfully determine equator and equinox errors for the FK4 (Branham 1979), we feel that the conclusions presented in § 6 are relatively firm, although perhaps not completely unimpeachable.

Table 1 summarizes the observations by minor planet and Table 2 by type. There are three types: meridian, made with a mural circle, transit instru-

TABLE 1

THE OBSERVATIONAL DATA BY MINOR PLANET

Minor Planets	No. of Observ.	Mean Error	Time Span
6 (Hebe)	7098	1.822″	1847.516–1993.193
7 (Iris)	6809	1.825	1847.621–1989.346
8 (Flora)	3814	2.606	1847.801–1993.402
9 (Metis)	2758	2.532	1848.323–1993.632
15 (Eunomia)	2106	2.247	1851.626–1989.911
389 (Industria)	36	0.477	1975.452–1983.533
433 (Eros)	158	1.219	1965.509–1979.657
1224 (Fantasia)	36	0.718	1971.154–1985.218
1566 (Icarus)	58	0.447	1968.473–1987.659
1590 (Tsiolkovskaja)	26	0.914	1976.253–1987.893
1620 (Geographos)	22	1.464	1969.675–1983.135
1628 (Strobel)	22	0.781	1976.826–1980.557
1679 (Nevalinna)	36	0.813	1974.157–1978.016
1770 (Schlesinger)	22	0.729	1967.361–1979.640
1781 (Van Biesbroeck)	38	1.583	1976.638–1986.275
1791 (Patsayev)	30	0.665	1967.726–1971.543
1804 (Chebotarev)	94	0.638	1971.385–1989.175
1829 (Dawson)	52	1.638	1967.350–1981.528
1867 (Deiphobos)	72	0.865	1971.175–1984.309
1943 (Anteros)	58	3.082	1973.194–1985.149
2151 (Hannibal)	50	0.810	1975.127–1982.387

TABLE 2

THE OBSERVATIONAL DATA BY TYPE OF OBSERVATION

Type	No. of Observ.	Mean Error	Time Span
Filar Micrometer (pre-1900)	5064	2.644″	1847.516–1900.000
Filar Micrometer (post-1900)	2767	1.964	1900.000–1954.763
Heliometer	44	2.066	1847.834–1851.741
Mural Circle	183	2.648	1847.535–1865.995
Photographic	9078	1.174	1914.121–1993.632
Ring Micrometer	1454	2.583	1847.661–1936.943
Transit Circle (pre-1900)	4270	2.608	1847.527–1900.000
Transit Circle (post-1900)	243	1.538	1900.000–1972.000
Transit Instrument	281	2.939	1847.535–1865.995
Vertical Circle	11	0.621	1899.700–1899.845
Total	23395	2.166	1847.516–1993.632

ment, or vertical circle; visual, made with an equatorial telescope and filar or ring micrometer or made with a heliometer; and photographic. Because some of the instruments observe in only one coordinate, the mural circle and the transit instrument only in α and the vertical circle only in δ, and because transit circle observations are frequently made only in one coordinate, the term "observation" refers to one coordinate only. Thus, a photographic observation will be counted as two, one in α and one in δ. The observational data base encompasses 23395 observations made from 1847 to 1993, a period of 147 years. There is little difference in quality between α and δ; with a cutoff of 8.5″ for an acceptable residual, the former have a mean error of unit weight, $\sigma(1)$, of $2.21''$ and the latter of $2.14''$.

Some question the utility of 19th century observations for studies of catalog equator and equinox errors. That they show greater scatter than their 20th century counterparts is undeniable (e.g., see Table 2. But various considerations militate against their arbitrary exclusion. The FK5 itself includes numerous 19th century, and even some 18th century catalogs. To investigate its possible equinox and equator errors one should incorporate the longest time base possible. A glance at Table 2 shows that the smallest mean error comes from vertical circle observations made in 1899, although there are few of them, whereas the minor planet in Table 1 with the largest mean error, Anteros, represents recent epoch photographic observations. One cannot state, therefore, that 20th century observations are necessarily better than 19th century ones, and we incorporated all of the observations from 1847 on.

2. TREATMENT OF THE OBSERVATIONS

The majority of the observations of planets 6–9 and 15 come from Branham (1979). These have been reduced to the FK4 system. Fricke (1982) used them in his study of the FK4 equator and equinox. The few remaining observations of these planets, photographic and transit circle, are on the FK4 or, in a few instances, on the FK5; the photographic observations on the FK4 were reduced to the FK5 by the procedure employed in Paper II. The other minor planets, 389 through 2151, have been reduced to the FK5 by the procedure mentioned in Paper I.

The visual and meridian observations of planets 6–9 and 15 required different treatment. To calculate an $(O-C)$ the ephemeris based on rectangular coordinates calculated for the equinox J2000 was reduced to the epoch of observation using the precepts in Kaplan et al. (1989) Then the $(O-C)$ was corrected for the difference FK4–FK5 by use of the tables of systematic differences given in the FK5 to calculate the differences for the epoch of observation.

In Branham (1979) nearly all of the visual and photographic observations had been reduced to the FK4 by re-reduction of the observations using reference star positions taken from the SAO or the AGK3, catalogs on the FK4. The reduction to the FK5 thus involves applying the systematic differences FK4–FK5. The meridian observations, however, could not be re-reduced this way. To go to the FK4 systematic differences, original catalog-GC, were used followed by GC-FK4; in a few instances the original catalog was the FK3 and one could go directly FK3–FK4. The reduction FK4–FK5 requires yet another application of systematic differences. Because accidental errors between two star catalog are generally larger than their systematic differences, one feels a little uneasy about applying a series of three systematic differences. The meridian observations, however, represent more than one fifth of the total and, although concentrated in the 19th century, cover a time span from 1847 to 1972. Given their large number, accidental errors should be minimized with respect to systematic errors, and we included all of the meridian observations.

3. EPHEMERIDES AND DIFFERENTIAL CORRECTIONS

The ephemerides were based on rectangular, equatorial coordinates and velocities for equinox J2000 and epoch of osculation JD2420000.5 (20 Aug. 1913) taken from DE200. We used our own numerical integration routine for the solar coordinates for the interval 1847–1993. The solar system is treated as an n-body problem, with the major planets integrated along with the minor planets. Some of the minor planets, such as Eros, pass close to the Earth, and thus the moon had to be carried as a separate body. This implies a small step-size, 0.25^d, and high order for the integrator. The integrator is 16th order Lagrangian predictor-corrector. For integrating the moon account is taken of tidal forces and terrestrial flattening. Given that there are 21 minor and 10 major planets, the solar system is treated as a 31 body problem. Starting coordinates for the integration were calculated from the Taylor series expansions that Le Guyader gives (1993). Because the initial coordinates and velocities were based on DE200, ephemerides referred to equinox J2000, the numerical integrations, and in particular the solar coordinates, are on that equinox. Had it been possible to use DE200 for the solar coordinates, it *may* have been preferable to do so, but the integrator actually employed assures dynamical consistency for both the minor planet and the solar coordinates. Furthermore, corrections to the Earth's orbit were included in the solution so that any error in the integrator will be largely eliminated.

After calculation of the $(O-C)$'s each minor planet's orbital elements were differentially corrected by use of Brouwer & Clemence's Set III equations (1961). Because none of the minor planets suffers strong perturbations, these corrections, based on 2-body approximations, are entirely adequate and far less computationally expensive than numerically integrated partial derivatives. For each minor planet there are 6 unknowns, for a total of 126 unknowns for the 21 minor planets. The Earth's orbit was also corrected by use of Brouwer & Clemence's Set VI, which includes the equinox correction, ΔE, an additicn of 5 more unknowns. The equator correction, ΔD, adds one more unknown. Because of the long time span we decided to include partial derivatives for the secular variations of the equator and equinox corrections, $\Delta\dot{E}$ and $\Delta\dot{D}$. There is, therefore, a total of 134 unknowns. The epoch of osculation was changed to JD2439581.5 (1 April 1967) to fall in the time span covered by observations of nearly all of the minor planets; 83.4% of the observations occur before this date and 16.6% after. For the secular variations 1 Jan. 1900 was used as the origin. This gives a baseline of 53 years and 48.2% of the observations before the origin and 93 years and 51.8% after.

The matrix of the equations of condition is sparse: in a row of 134 entries only the 6 unknowns for a given planet's orbital elements and the 8 unknowns common to all of the observations are nonnull. The matrix, therefore, is only 10.4% dense. When the equations of condition are accumulated into an upper triangular system, the resulting matrix becomes doubly-bordered, block diagonal and 16.4% dense. To take advantage of this sparsity solutions were calculated by a procedure (Branham 1992) developed for just this purpose.

Before a linear system is solved, one should see how well, or ill, conditioned it is. This can be done by using the singular value decomposition (SVD) and calculating the ratio of the largest to the smallest singular value, one definition of the condition number of a linear system (Branham 1990). We employed Branham's unpublished modification of the SVD that calculates only the singular values, not the left and right singular vectors, and needs space only for an upper Hessenberg matrix, not the full data matrix. The condition number of the unscaled data matrix was 1.2×10^{7}. The singular values show that the five minor planets with the longest observing histories, 6–9 and 15, make the strongest contribution to the solution. The weakest contribution comes from the secular variations $\Delta\dot{E}$ and $\Delta\dot{D}$.

With a condition number of 1.2×10^{7} for the data matrix and over 23 000 equations of condition, it is a nice question whether one should calculate the solution by normal equations or orthogonal transformations; the former suffer from greater sensitivity to the condition number, the latter from greater accumulation of round-off or chopping errors (Branham 1990). We finally decided to use the fast Givens transformation, although normal equations would have served equally well, especially should one scale the data matrix or accumulate the equations of condition with extended precision. (All calculations were performed in double-precision, machine epsilon of 1.39×10^{-17}).

4. THE SOLUTIONS

Various solutions were calculated, but the final ones on which we base our conclusions are shown in Table 3. Pierce's criterion (Branham 1990) indicated that 6.3% of the observations should be eliminated. This corresponds to a cutoff of 8.5″ and leaves 21 829 equations of condition. This often used criterion assumes that the residuals arise from a normal distribution. Statistics on the residuals, however, show that they are not well represented by a normal distribution. The Q factor, 2.56 for a normal distribution, measures the weight in the tails of a distribution and is 3.83 for our residuals. They are also leptokurtic because the normal distribution's kurtosis of 3.00 becomes 6.07. The coefficient of skewness, 0 for the normal distribution, is here 0.065. In short, the residuals are more narrowly peaked, heavier tailed, and somewhat skewed compared with residuals from a normal distribution. Figure 1 shows graphically the behavior of the residuals.

Granted that the residuals are nonnormal, does this imply that they are nonrandom as well? This

TABLE 3

THE SOLUTIONS[a]

Unknowns	6.3% Trim	6.3% Trim, No $\Delta\dot{E}$, $\Delta\dot{D}$	Cauchy
Mean Error	2.08″	2.08″	0.563″
$\Delta\epsilon$	−0.061″ ± 0.006″	0.001″ ± 0.007″	−0.029″ ± 0.000″
Δl_0	−0.449 ± 0.006	−0.482 ± 0.003	−0.694 ± 0.002
$\Delta\omega$	−0.119 ± 0.027	0.117 ± 0.031	0.085 ± 0.001
Δe	0.000 ± 0.000	0.000 ± 0.000	0.000 ± 0.000
ΔE	−0.135 ± 0.033	0.116 ± 0.031	0.081 ± 0.003
$\Delta\dot{E}$	0.587 ± 0.005	...	0.856 ± 0.002
ΔD	−0.051 ± 0.011	−0.079 ± 0.023	−0.004 ± 0.005
$\Delta\dot{D}$	0.056 ± 0.023	...	0.049 ± 0.009

[a] From a 6.3% trim of the 23395 observations, a 6.3% trim suppressing equator and equinox motion, and for an assumed Cauchy distribution.

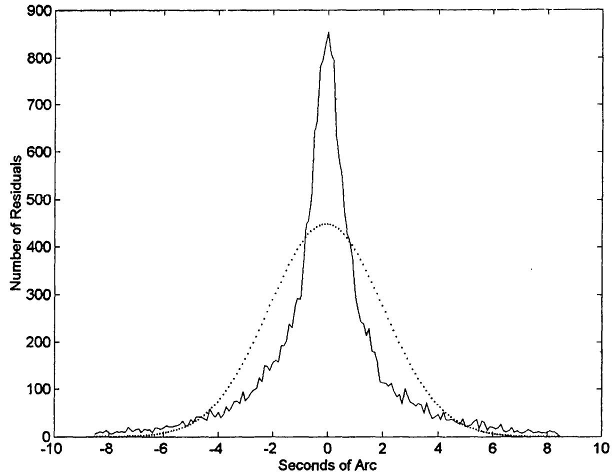

Fig. 1. Behavior of the Post-Fit Residuals. The Solid line represents the data and dotted line, the normal distribution.

can be checked by a runs test of alternation of sign. With 23395 residuals there are 11337 runs whereas we expect 11667 runs with a standard deviation of 76. Thus, the actual runs, although somewhat fewer than expected from genuine randomness, are nevertheless well represented by the assumption of randomness.

But also given the nonnormality of the residuals, it is important to calculate a solution independent of this assumption, the basis for the least squares criterion. The L1 distribution has Q factor of 3.31 and kurtosis of 6.00, closer to the actual values for our distribution. Unfortunately, to calculate an L1 solution one needs the complete data matrix of size 23 395×134. This is too large to fit into the physical, or even virtual memory, of the computer used. But it is possible to calculate an *approximate* L1 solution. The L1 distribution is heavier tailed and more peaked than the normal. The Cauchy distribution represents this behavior and was approximated by calculating weights, based on the size of the residual, that agree with a theoretical Cauchy distribution.

Because the secular variations of the equator and equinox corrections are the most weakly determined quantities judged by their singular values, a solution was calculated with these quantities suppressed.

5. DISCUSSION

The various solutions in Table 3 all show that the corrections to the Earth's coordinates are small. The integrator used, therefore, seems to have been adequate. This conclusion is reinforced when one realizes that the small corrections ensue from both the least squares and the L1, or better stated approximate L1, criteria and thus appear to be independent of the criterion used.

Let us refer to the second column of Table 3 as solution 1, the third column as solution 2, and the last column as solution 3. Both solution 1 and solution 3 indicate a high equinox motion, surprising given that to eliminate the FK4 equinox motion was one of the goals of the FK5. This implies that solutions 1 and 3 are probably unacceptable, but confirming evidence is needed. Part of the confirming evidence is internal: $\Delta\dot{E}$ and $\Delta\dot{D}$ are the weakest contributors to the solution according to the SVD, and their values may be largely fictitious regardless of their formal mean errors.

If we indeed suppress these variables and look at the least squares solutions for ΔE and ΔD from Paper I, Paper II, and this paper we find: $0.181'' \pm 0.195''$ and $-0.092'' \pm 0.060''$ from 826 observations with median date 1975; 0.191 ± 0.086 and 0.027 ± 0.016 from 9036 observations for 1966 and 0.116 ± 0.031 and -0.079 ± 0.023 from 23395 observations for 1913, respectively.

We see here evidence for a motion of the equinox. If we use Paper I, the motion would be $(0.181'' - 0.116'')/(19.75\text{–}19.13) = 0.104''$ per century; from Paper II, the corresponding value is $(0.191''- 0.116'')/(19.66\text{–}19.13) = 0.142''$ per century. Because Paper II uses nearly eleven times more observations, we take the latter equinox motion as more likely. A standard comparison of means test (Wonnacott & Wonnacott 1972) shows that this conclusion is valid at well over the 99% level. But the comparison of means test also assumes the normality of the residuals, a dubious assumption for the residuals here. To bolster this indication of a possible equinox motion we calculated a solution for just the 20th century observations, but with the secular variations suppressed. This solution gives $\Delta E = 0.171'' \pm 0.010''$ and $\Delta D = 0.040'' \pm 0.015''$ at the midpoint of the observations, JD2437589 (17 Oct. 1961). This equinox agrees well with those calculated from Papers I and II, which also used only 20th century observations (but exclusively photographic ones): 16 minor planets were used in Paper I and 21 in Paper II. It would imply an equinox motion of $(0.171''\text{–}0.116'')/(19.61\text{–}19.13) = 0.115''$ per century. The time spans among just the 20th century solutions are too short, 1961, 1966, 1975 to calculate reliable equinox motions. That the 20th century observations yield concordant equinox corrections perceptibly higher than the combined 19th and 20th century observations suggests that there is indeed a small but real equinox motion. A solution for just the 19th century observations could not be calculated because the observing history of minor planets 6–9 and 15 in the 19th century is hardly uniform. All of these minor planets were heavily observed after their discoveries followed by long periods of sparse observing. As a consequence, half of the 19th century observations fall before JD2399911 (20 Aug. 1858) and include numerous observations made with mural circle and transit instruments, which have higher mean errors than other types of instrument. The ensuing linear system for just these observations is so poorly conditioned that no useful solution results.

Completely independent evidence for the equinox correction and a possible equinox motion comes from two different sources. G. Carrasco (personal communication) finds, when comparing 9800 observations in α and 5150 in δ made with the Cerro Calán (Santiago, Chile) transit circle with the FK5 that "there still exists a systematic difference in right ascension, especially in the zone between $-50°$ and the south pole". From one of his graphs one sees that $\Delta\alpha \cos\delta$ increases from near null at $-50°$ to over $0.2''$ near the south pole, a positive correction just as we find. Miyamoto & Sôma (1993) use over 24 000 proper motions on the FK5 system to determine some parameters of galactic kinematics. One of these is $\Delta\dot{E}+\Delta\lambda$, where $\Delta\lambda$ is a correction to the planetary precession. The latter should be small, and most of the value found, $-0.12'' \pm 0.03''$, may be attributed, therefore, to an equinox motion. Because of the difference of sign compared with our value this hardly seems confirming evidence. But Miyamoto & Sôma also correct their observations for the luni-solar precession, Δp, for which they use a value $-0.30''$ per century. We do not include a correction for this unknown because in theory it should be null, although in practice, as Miyamoto & Sôma point out, some evidence for a correction of the order of the value they use exists. But more importantly a correction Δp would be, with our observations, extremely weakly determined, as an SVD indicates; to include such an unknown would be parasitic to the solution. The important point is that Miyamoto & Sôma correct their observations for Δp, we do not. But both Δp and $\Delta\dot{E}$ behave similarly, although not identically: they will both systematically affect the right ascensions. If we take their behavior as *approximately* the same, then we should *add* $0.30''$ to Miyamoto & Sôma's equinox motion to make it comparable with ours; this would give for their equinox motion a value $0.18'' \pm 0.03''$, roughly equivalent to ours.

It is more difficult to arrive at trustworthy conclusions regarding the equator and its secular variation. The 1913 value is negative, the 1961 and 1966 values positive. This may imply, and is confirmed by a comparison of means test, a small motion of the equator. But the 1975 value is negative, although based on far fewer, but higher quality ($\sigma(1) = 0.412''$), observations. Carrasco (personal communication) found some, but hardly overwhelming, evidence for an equator error in the FK5. It is perhaps best to adopt a cautious policy, especially given the nonnormality of our residuals, and state that little conclusive evidence for an equator correction or secular variation exists.

6. CONCLUSIONS

Based on 23 395 observations of 21 minor planets made over a 147 year time span, no strong evidence exists for an equator correction or secular variation in the FK5 system. Evidence, both internal and external, does exist; however, for a correction to the equinox of the order of $0.18''$ at approximate epoch 1970 and for a possible motion of the equinox of the order of $0.11''$–$0.14''$ per century.

REFERENCES

Branham, R. L. Jr. 1979, Astron. Papers Amer. Ephem., 21, Part 3

__________. 1990, Scientific Data Analysis (New York: Springer), Sec. 8.1, pp. 110-115, pp. 79-80, Sec. 5.5, Sec. 6.4

__________. 1992, AJ, 104, 1658

Branham, R. L. Jr., & Sanguin, J. G. 1992, AJ, 103, 2099

__________. 1996, in III International Workshop on Positional Astronomy and Celestial Mechanics, The FK5 Equator and Equinox, eds. A. López García, E. J. Yagudina, M. J. Martínez Uso, & A. Cordero (Valencia: Univ. of Valencia Press), 429

Brouwer, D., & Clemence, G. M. 1961, Methods of Celestial Mechanics (New York: Academic Press), 241

Fricke, W. 1982, A&A, 107, L13

Fricke, W., Schwan, H., & Lederle, T. 1988, Veröffentlichungen Astron. Rechen Institut Heidelberg, Nr. 22

Kaplan, G. H., Hughes, J. A., Seidelmann, P. K., Smith, C. A., & Yallop, B. D. 1989, AJ, 97, 1197

Le Guyader, C. 1993, A&A, 272, 687

Miyamoto, M., & Sôma, M. 1993, AJ, 105, 691

Wonnacott, T. H., & Wonnacott, R. V. 1972, Introductory Statisitcs, 2nd ed. (New York: Wiley), Sec. 8.1

Richard L. Branham Jr.: Centro Regional de Investigaciones Científicas y Tecnológicas (CRICYT), calle Dr. Adrián Ruiz Leal s/n, Parque General San Martín, 5500 Mendoza, Argentina (rlb@lanet.losandes.com.ar).

Juan G. Sanguín: Observatorio Astronómico "Félix Aguilar" and Yale Southern Observatory, Avda. Benavidez 8175 (O), 5407 Rivadavia, San Juan, Argentina (leoncito@unsjfa.edu.ar).