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THE PRODUCTION AND DEPLETION RATES IN MODELS FOR THE CHEMICAL EVOLUTION OF THE GALAXY

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RESUMEN

Se calculan las tasas de producción y disminución de metales para usarse en la ecuación general de conservación de la masa, en modelos de evolución química de la Galaxia. Se muestra que la ecuación de conservación de los metales de Tinsley (1980) puede ser recuperada con la adopción de algunas hipótesis simplificadas, tales como ignorar los metales reservados en las remanentes. Con la ecuación derivada, mostramos que la masa total de un metal dado puede ser afectada por las hipótesis generalmente presentadas. Para la mayoría de los metales, encontramos que su masa total puede ser incrementada hasta un orden de magnitud relativa a la masa esperada de metales como ha sido presentada por Tinsley (1980).

ABSTRACT

The stellar production and depletion rates associated with a given metal are derived, to be used in the general equation for the mass conservation in models of the chemical evolution of the Galaxy. It is shown that the metal conservation equation of Tinsley (1980) can be recovered with the adoption of some simplifying hypotheses, such as neglecting the metals stored in the remnants. With the derived equation, we show that the total mass of a given metal can be affected by the hypotheses usually made. For the bulk of metals, we find that their total mass can be increased up to an order of magnitude relative to the expected mass of metals as given by Tinsley (1980).

Key words: **GALAXY - ABUNDANCES — GALAXY - EVOLUTION**

1. INTRODUCTION

In the framework of the one-zone model for the chemical evolution of the Galaxy with the sudden mass loss approximation (SMLA) and no infall or outflows, the mean metallicity Z_s of the stars ever formed can be obtained from the equation for the conservation of metals (cf., Tinsley 1980, equation 3.10)

$$Z_s M_s + Z M_g = \int_0^t \int_{m_{t'}}^{\infty} m p_{zm} \psi(t' - \tau_m) \phi(m) dt' dm \ , \tag{1}$$

where M_s and M_g are the star and gas masses, respectively; Z is the metal abundance in the gas or interstellar medium (ISM), and Tinsley (1980) assumed $Z(t=0)=0$ in equation (1); p_{zm} is the mass fraction of a star of mass m converted to metals and ejected; τ_m is the lifetime of a star of initial mass m; m_t is the turnoff mass, or the stellar mass with $\tau_m = t$; $\phi(m)$ is the initial mass function (IMF) assumed independent of time, and $\psi(t)$ is the star formation rate (SFR) at t. In this equation, $Z_s M_s$ is the mass of metals stored in stars, and $Z M_g$ is the corresponding mass stored in the gas, so that their sum is the mass of all metals ever ejected.

Equation (1) is only valid if we do not take into account the metals stored in the remnants. In this paper, we will derive more general expressions for the production and depletion rates, and compare their predictions with the simplified equation (1).

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2. DERIVATION OF THE PRODUCTION AND DEPLETION RATES

2.1. *The Mass Conservation Equation*

Let $M_s^i(t)$ and $M_g^i(t)$ be the mass of an element i stored at time t in the stars and in the ISM, respectively. We have for the total mass of element i

$$M^i(t) = M_s^i(t) + M_g^i(t) \; ; \tag{2}$$

(this equation is modified by the consideration of refuses, as discussed by Rocha-Pinto, Arany-Prado, & Maciel 1994). It should be noted that i may represent a particular metal, a radionuclide, or a family of elements, such as the metals.

A general mass conservation equation for element i can be written as

$$\frac{dM^i(t)}{dt} = P_s^i(t) + P_g^i(t) - D_s^i(t) - D_g^i(t) + G^i(t) - L^i(t) \; , \tag{3}$$

where P_s^i and P_g^i are the net conversion rates of any element to element i in stars and gas, respectively; D_s^i and D_g^i are the net conversion rates of element i to any other species for the same processes; G^i is the gain associated with infalling gas, and L^i the corresponding loss owing to gas outflow from the considered region. For simplicity we will refer to P and D as production and depletion rates, respectively.

The stellar depletion rate D_s^i includes: (a) the depletion rate of the pre-existing mass of element i in stars, and (b) the depletion rate associated with the mass of i locked up in remnants.

2.2. *The Stellar Production Rate*

The production term $P_s^i(t)$ is a sum comprising all the stars that are contributing to species i at a given time t. These stars were born at a time $t'(m,t)$, which depends on the initial mass m and the time t, according to the SMLA. Since the lifetime of stars of mass m is τ_m, during the initial fraction f of their lives no production of element i occurs. For most metals, we have typically $f \sim 0.9$, which is essentially the fraction of the stellar lifetime spent on the main sequence. Therefore, in order to include the production of element i from all stars of mass m, we have to consider those stars that were born in the time range $t - \tau_m < t' < t - f\tau_m$. On the other hand, we must also sum all stellar masses from the minimum mass m_{pi} of stars that can produce species i to a maximum mass m_{max} of the most massive stars that can be formed.

Therefore, the production term can be written as

$$P_s^i(t) = \int_{m_{pi}}^{m_{max}} \int_{t-\tau_m}^{t-f\tau_m} m\theta_m^i(t,t')\psi(t')\phi(m)dt'dm \; , \tag{4}$$

where $m\theta_m^i(t,t')$ gives the net conversion rate of stellar mass to species i for a star of mass m born at t', and again ψ and ϕ are the star formation rate and initial mass function, respectively.

2.3. *The Stellar Depletion Rate*

A general equation for the stellar depletion rate D_s^i has the form

$$D_s^i(t) = \int_0^{m_{max}} \int_0^t m\beta_m^i(t,t')\psi(t')\phi(m)dt'dm + D_r^i(t) \; , \tag{5}$$

where $m\beta_m^i(t,t')$ is the net conversion rate of the pre-existing mass of element i to other species for a star of mass m born at t', and D_r^i is the depletion rate in the remnants. We have taken the minimum stellar mass as zero to account for depletion on the stellar residues (e.g., planets, comets, etc.) which are likely to be born with the (low mass) stars. This equation can be simplified if we consider only the metals, that is nuclides heavier than boron (excluding the radionuclides), for which we can write

$$D_s^i(t) = \int_{m_{di}}^{m_{max}} \int_{t-\tau_m}^{t-g\tau_m} m\beta_m^i(t,t')\psi(t')\phi(m)dt'dm + D_r^i(t) \; , \tag{6}$$

where m_{di} is the minimum stellar mass that depletes species i, and g is the initial fraction of τ_m during which no depletion occurs. For metals we can take $g \simeq f$.

3. DEFINITIONS: YIELD AND LOSS

Assuming the one-zone model and the SMLA, we will define the *yield* of species i as

$$y^i(t) = \frac{1}{[1-R(t)]\psi(t)} \int_{m_t}^{m_{max}} Q^i(m)\psi(t-\tau_m)\phi(m)dm \; ; \tag{7}$$

the *net yield* is defined as

$$y_n^i(t) = \frac{1}{[1-R(t)]\psi(t)} \int_{m_t}^{m_{max}} \Delta Q^i(m,t)\psi(t-\tau_m)\phi(m)dm \; ; \tag{8}$$

and we define the *destruction term* (or the *loss*) as

$$u^i(t) = \frac{1}{[1-R(t)]\psi(t)} \int_{m_t}^{m_{max}} Q_{ej}(m)\,\xi_{ej}^i(m)\psi(t-\tau_m)\phi(m)dm \; . \tag{9}$$

In these expressions, $Q^i(m)$ is the net mass converted to species i (*new* i) and ejected, for stars of mass m; ΔQ^i and ξ_{ej}^i are defined by equation (12) below; in the SMLA the returned fraction $R(t)$ can be written as

$$R(t) = \frac{1}{\psi(t)} \int_{m_t}^{m_u} Q_{ej}(m)\psi(t-\tau_m)\phi(m)dm \; , \tag{10}$$

and

$$Q_{ej}(m) = m - w_m \; , \tag{11}$$

is the ejected mass from a star of mass m, and w_m is the mass of the remnant.

Assuming for ΔQ^i the same definition as in Yokoi, Takahashi, & Arnould (1983), *the net difference between the mass of element i in the ejecta ("old" and "new") and that in the corresponding material at birth*, it can be written as

$$\Delta Q^i(m,t) = Q^i(m) - Q_{ej}(m)X^i(t-\tau_m)\xi_{ej}^i(m) \; , \tag{12}$$

where X^i is the abundance of species i in the ISM, $X^i = M_g^i/M_g$, and the *destruction function* ξ_{ej}^i is the fraction of the original mass of i, present in the ejecta, which has been destroyed during stellar evolution (see Appendix § A1). The second term on the right of equation (12) is the total mass of original i, present in the ejecta, which has been destroyed due to both convection and local depletion during stellar evolution. ξ_{ej}^i has the same meaning as ξ in Malaney, Mathews, & Dearborn (1989) (Appendix, § A2). The total destruction function, ξ_{tot}^i, obeys the relation

$$mX^i(t-\tau_m)\xi_{tot}^i(m) = \left[Q_{ej}(m)\xi_{ej}^i(m) + w_m\xi_w^i(m)\right]X^i(t-\tau_m) \; , \tag{13}$$

where ξ_w^i refers to the destroyed i stored in the remnant. Note that the above equation gives the total net mass of *original* i depleted in a star during its life, that is, the total net mass of i converted to other species in a star born at $(t-\tau_m)$.

An equation analogous to equation (12) for the new i in the remnant is

$$\Delta N^i(m,t) = N^i(m) - w_m X^i(t-\tau_m)\xi_w^i(m) \; , \tag{14}$$

where N^i is the net mass converted to especies i and locked up in the remnant.

The use of y^i or y_n^i defined above depends on the knowledge of how much the original i survives stellar evolution. In general, this is not known and the net yield is used. However, the r-process radionuclides are produced only in the final stages of very massive stars, so the old i can be described through the decay law. As discussed by Yokoi et al. (1983), for long lived radionuclides y^i is the r-process yield itself (cf., Appendix, equation A8). It is worth noting that both yields above are equal when $u^i = 0$, due to $\xi_{ej}^i = 0$ in equation (9), that is, there was no stellar destruction of i in the ejected mass. Hence by equation (12), $\Delta Q^i(m) = Q^i(m)$. These last equalities are valid for the bulk of metals and the reason is that during stellar evolution generally one metal transmutes into another one, resulting that only the "new" neutrons and protons contribute to Q^z and the destruction of metals does not need to be considered. So for the bulk of metals we have $y^z = y_n^z$ and

$$\Delta Q^z(m) = Q^z(m) = mp_{zm} \ . \tag{15}$$

The last equality stands for the fact that mp_{zm} in equation (1) has the same meaning as Q^z. Analogously, we consider $\xi^z_w = 0$ in equation (14), so $\Delta N^z(m) = N^z(m)$. Moreover, in a first approximation, we can consider that the only stars which end their lives as metal rich remnants have masses in the range $1 \lesssim m \lesssim m_{up}$ $(M_\odot)$, where m_{up} is the maximum mass of a star that produces a white dwarf. Therefore, we can write for the metals

$$\Delta N^z(m) = N^z(m) \approx w_m \quad \text{where} \quad 1 \lesssim m \lesssim m_{up}(M_\odot) \ . \tag{16}$$

4. A SIMPLIFIED EQUATION FOR THE MASS CONSERVATION OF METALS

In order to derive the approximate equation (1), we will make some simplifying assumptions:

(i) Assuming the time range in equation (4) $(1-f)\tau_m << t$, the SFR can be considered as nearly constant. Therefore, we can write $\psi[t'(m,t)] \approx \psi(t-\tau_m)$ and equation (4) becomes

$$P^i_s(t) = \int_{m_{pi}}^{m_{max}} \left[\int_{t-\tau_m}^{t-f\tau_m} m\theta^i_m(t,t')dt'\right] \psi(t-\tau_m)\phi(m)dm \ . \tag{17}$$

In equation (17) the bracketed term gives the total net mass converted to species i in a star with mass m, during its lifetime. This term includes the ejected mass $Q^i(m)$ and the mass $N^i(m)$ that is locked up in the stellar remnant [cf., equations (12) and (14)], so that we have

$$P^i_s(t) = \int_{m_t}^{m_{max}} \left[Q^i(m) + N^i(m)\right] \psi(t-\tau_m)\phi(m)dm \ . \tag{18}$$

The lower mass limit has been changed to the turnoff mass, as the sum is concerned with stars that eject at t.

(ii) We have seen that the stellar depletion term can be simplified for metals as in equation (6). Applying to this equation a similar procedure as to (17) and (18) we can write [see equation (13) and Appendix § 3]

$$D^i_s(t) = \int_{m_t}^{m_{max}} \left[mX^i(t-\tau_m)\xi^i_{tot}(m)\right] \psi(t-\tau_m)\phi(m)dm + D^i_r(t) \ . \tag{19}$$

(iii) Except for some of the light elements (^{6}Li, ^{9}Be, ^{10}B, ^{11}B) which are produced by spallation of interstellar atoms by galactic cosmic rays (see for example Audouze 1986), and for the radionuclides, no production or depletion occurs in the ISM, so that P^i_g and D^i_g are generally negligible.

(iv) Recent models of the chemical evolution of the Galaxy have emphasized the importance of infall onto the galactic disk (Tosi 1990; Chiappini & Maciel 1994; Chiappini, Matteucci, & Gratton 1997). However, the chemical abundance of the infalling gas is not well determined, and can be negligible for several species found in the ISM. Moreover, the simple model for the chemical evolution of the Galaxy does not take into account gas infall and outflows on the disk (Tinsley 1980; Searle & Sargent 1972), so that it is illustrative to consider the evolution of the galactic disk with no infall or outflow, which renders $G^i = L^i = 0$ in equation (3).

(v) In view of the approximations above, equation (3) becomes

$$\frac{dM^i(t)}{dt} = P^i_s(t) - D^i_s(t) \ , \tag{20}$$

so that the net production and depletion terms include the stellar contribution only. Using equations (12), (13), (14), (18) and (19) we have from the above equation

$$\frac{dM^i(t)}{dt} = \int_{m_t}^{m_{max}} \left[\Delta Q^i(m,t) + \Delta N^i(m,t)\right] \psi(t-\tau_m)\phi(m)dm - D^i_r(t) \, . \tag{21}$$

Assuming further the *rather unlikely* hypothesis that the depletion rate in the remnants is essentially given by the rate at which the net new mass of species i is locked up [see equations (13), (14) and (19)], we have

$$D^i_r(t) = \int_{m_t}^{m_{max}} \Delta N^i(m,t)\psi(t-\tau_m)\phi(m)dm \, . \tag{22}$$

Equations (21) and (22) lead to

$$\frac{dM^i(t)}{dt} = \int_{m_t}^{m_{max}} \Delta Q^i(m,t)\psi(t-\tau_m)\phi(m)dm\,. \tag{23}$$

The above equation can be integrated over time and for the bulk of metals we have

$$M^z(t) = \int_0^t \int_{m_{t'}}^{m_{max}} \Delta Q^z(m)\psi(t'-\tau_m)\phi(m)dm\; dt'\,, \tag{24}$$

which is again equation (1) in Tinsley's (1980) form, since our net mass ΔQ^z is the same as the product mp_{zm} defined by Tinsley (1980) [see equation (15)].

(vi) Under the same assumptions *(i)* to *(iv)* above, we can integrate equation (21) in the following mass ranges: a) $m_t < m \leq m_{up}$ and b) $m_{up} < m < m_{max}$. We also assume that in the range (a) ΔQ^i is negligible compared to the production due to stars in the second range, and that the rate of depletion in remnants D_r^i is null; in range (b) we assume that ΔN^i equals the rate of depletion in the compact remnants left from the supernova. So, we have from equation (21)

$$\frac{dM^i(t)}{dt} = \int_{m_t}^{m_{up}} \Delta N^i(m,t)\psi(t-\tau_m)\phi(m)dm + \int_{m_{up}}^{m_{max}} \Delta Q^i(m,t)\psi(t-\tau_m)\phi(m)dm\,. \tag{25}$$

From equations (23) and (25), we see that Tinsley's equation (24), does not consider the rate at which "new mass" is stored in the remnants [the first integral in equation (25)].

5. EFFECT ON THE TOTAL MASS OF METALS

In view of the previous discussion, the general conservation equation (3) should be used in order to consider all possible production/destruction mechanisms, since the calculation of the production and depletion rates by equations (4) and (6) may produce important discrepancies upon the total mass of element i, if assumption *(v)* [equation (22)] is released. Therefore, it is interesting to investigate the differences between the results produced by equation (23) *and* those produced by equation (25), which accounts for the more realistic approximation *(vi)*.

Let $M_T^i(t)$ be the total mass of element i obtained by equation (23), and $M^i(t)$ the corresponding quantity derived from equation (25). We can then write

$$\frac{M^i(t)}{M_T^i(t)} = 1 + \delta^i(t)\,, \tag{26}$$

where, under assumption *(vi)* [$\Delta Q^i \approx 0$ for masses below m_{up}], we have introduced for each metal a parameter defined by

$$\delta^i(t) \equiv \frac{\int_0^t \int_{m_{t'}}^{m_{up}} \Delta N^i(m,t')\psi(t'-\tau_m)\phi(m)dmdt'}{\int_0^t \int_{m_{t'}}^{m_{max}} \Delta Q^i(m,t')\psi(t'-\tau_m)\phi(m)dmdt'}\,. \tag{27}$$

Of course, if we do not consider the rate of new mass stored in remnants we have $\delta^i = 0$ and $M^i(t) = M_T^i(t)$. Instead, let us consider as an example the case of the bulk of metals. Besides equations (15) and (16), let us consider, for the sake of simplicity, the instantaneous recycling approximation (IRA). In this case and for a given turnoff mass $m_t < m_{up}$, we rewrite equation (27) as

$$\delta^z = \frac{\int_{m_t}^{m_{up}} \Delta N^z(m)\phi(m)dm}{\int_{m_t}^{m_{max}} \Delta Q^z(m)\phi(m)dm}\,. \tag{28}$$

Equations (15), (16), (A3), (A4) [see Appendix] and (28) lead to

$$\delta^z = \frac{\int_{m_t}^{m_{up}} w_m\phi(m)dm}{(1-R)y^z}\,. \tag{29}$$

We evalutate δ^z employing the grid of models by Maeder (1992, Tables 8 and 9, and 1993, the revised Table

TABLE 1

SOME ESTIMATES OF THE δ^Z PARAMETER

Case ($Z = 0.001$)	y^z	R	δ^z
Scalo 1986			
B	0.01022	0.4187	30.804
x =1.35			
C	0.0383	0.6219	12.103
D	0.0222	0.5882	19.171

7). We take the values of y^z and R from the cases which best fit the *observed* net yields of metals and oxygen, respectively, $0.01 < y^z < 0.036$ and $0.008 < y_n^o < 0.013$; and the ratio $\Delta Y/\Delta Z$ of the relative helium to metal enrichment, $3 < \Delta Y/\Delta Z < 6$ (see Figures 13 to 15 of Maeder 1992, and also Chiappini & Maciel 1994). We can reasonably reproduce the mass of white dwarfs (in solar units) according to Weidemann (1987, Table 3 —which has been used by Maeder) by the following equation

$$\begin{aligned} w_m &= 0.050\, m + 0.500 \qquad 1 \leq m < 6 \\ &\simeq 0.144\, m \qquad\qquad\quad 6 \leq m < m_{up} . \end{aligned} \tag{30}$$

We take $m_{up} \simeq 8\ M_\odot$ and the detailed IMF tabulated by Scalo (1986, Table VII), with a galactic age $T_o = 15$ Gyr, with lower and upper mass limits equal to 0.1 and 120 $M_\odot$ (see also Maeder 1993); we also take the IMF as a power law $\phi(m) = am^{-(1+x)}$, where $a = [\int_1^{120} m^{-x}\, dm]^{-1}$, with $x = 1.35$. Our Table 1 shows the values from Maeder (1992 and 1993) and the corresponding ones for δ^z. When we consider the fitting mentioned above, cases B and C can be considered as the limiting ones. So, we can establish the range $12 < \delta^z < 31$. Taking these values and equation (26), the relation between the total mass of metals in the Galaxy based on our general equations and that on Tinsley (1980, equation 3.10) is in the range $13 \leq M^z(t)/M_T^z(t) \leq 32$.

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APPENDIX

SOME REMARKS ON THE CONCEPTS OF YIELD AND LOSS

In what follows we show the self-consistency of our basic concepts of yield and loss, and also compare our definitions with those in other works.

A1. RELATION TO TINSLEY'S FORMALISM

Using the parameters defined in § 3, we can write the "old" mass of i (surviving astration) and present in the ejecta as $Q_{ej}(m)X^i(t-\tau_m)[1-\xi^i_{ej}(m)]$; the "new" one as $Q^i(m)$ and the *corresponding material at birth* (the original one) as $Q_{ej}(m)X^i(t-\tau_m)$. So, from the definition of ΔQ^i we obtain equation (12).

With the terms above, we can also confirm a problem in Tinsley's formalism which has already been noted by Maeder (1992, equations 1 to 6). We write the total ejection rate of species i from stars as a sum of the old plus new mass of i ejected by stars

$$E^i(t) = \int_{m_t}^{m_{max}} \{Q_{ej}(m)X^i(t-\tau_m)[1-\xi^i_{ej}(m)] + Q^i(m)\}\psi(t-\tau_m)\phi(m)dm \ , \tag{A1a}$$

or, using equation (12)

$$E^i(t) = \int_{m_t}^{m_{max}} \{Q_{ej}(m)X^i(t-\tau_m) + \Delta Q^i(m,t)\}\psi(t-\tau_m)\phi(m)dm \ . \tag{A1b}$$

The corresponding rate in Tinsley (1980, equation 3.9) is

$$E_z(t) = \int_{m_t}^{m_u} [(m - w - mp_{zm})Z(t - \tau_m) + mp_{zm}]\psi(t - \tau_m)\phi(m)dm \ , \tag{A2a}$$

which can be written in our formalism [see equations (11) and (15)] as

$$E_T^z(t) = \int_{m_t}^{m_{max}} \left\{ [Q_{ej}(m) - \Delta Q^z(m,t)] X^z(t - \tau_m) + \Delta Q^z(m,t) \right\} \psi(t - \tau_m)\phi(m)dm \ . \tag{A2b}$$

Comparing (A1b) with (A2b) we see that mp_{zm} $(= \Delta Q^z)$ must be removed from the first term in the integral of (A2a).

A2. RELATION TO OTHER WORKS

Assuming the instantaneous recycling approximation (IRA), we simplify equation (7) to (12) as

$$y^i = \frac{1}{(1-R)} \int_{m_t}^{m_{max}} Q^i(m)\phi(m)dm \ ; \tag{A3}$$

$$y_n^i(t) = \frac{1}{(1-R)} \int_{m_t}^{m_{max}} \Delta Q^i(m,t)\phi(m)dm \ ; \tag{A4}$$

$$u^i = \frac{1}{(1-R)} \int_{m_t}^{m_{max}} Q_{ej}(m)\, \xi_{ej}^i(m)\phi(m)dm \tag{A5}$$

The returned fraction is written, as usual as

$$R = \int_{m_t}^{m_u} Q_{ej}(m)\phi(m)dm \ . \tag{A6}$$

Equation (12) is rewritten as

$$\Delta Q^i(m,t) = Q^i(m) - Q_{ej}(m)X^i(t)\xi_{ej}^i(m) \ . \tag{A7}$$

Note that equations (A3), (A5) and (A6) equally hold if a constant SFR is used in equations (7), (9) and (10). The above equations (for which the IRA is assumed) lead to

$$y_n^i(t) = y^i - u^i \ X^i(t) \, , \tag{A8}$$

that is, we can either consider a net yield or we can consider destruction detached from production whenever possible (e.g., in the evolution of the r-process cosmochronometers).

The "decay rate" due to astration, λ_d, defined by Malaney et al. (1989, equation 10) can be written with our definitions as $\lambda_d^i = (1/M_g) \int \xi_{ej}^i Q_{ej} \psi \phi dm$, which with equation (A5) gives $\lambda_d^i = (1/M_g)(1-R)\psi u^i$. So, if we assume the linear dependence of the SFR on the gas mass $(1-R)\psi = \omega M_g$, where ω is a constant (Clayton 1985), we will have the direct relation $\lambda_d^i = \omega u^i$.

It is worth noting that the works of Yokoi et al. (1983, equation 12), Malaney et al. (1989, equation 7) and Brown (1992, equation 3.8), which analyze the evolution of elements known to be affected by astration, employ parameters with the same meaning, respectively, χ^a, $1 - \xi$ and A_i. Perhaps, this similarity has been masked by the lack of a mathematical relation such as (A7).

A3. CONSISTENCY OF EQUATIONS (6) AND (19)

In order to prove the consistency between equations (6) and (19), for metals, we define $F_m^i(t)$ as the mass of i in the star of mass m at the time t and $\eta_m^i(t,t')$ as the fraction of the *original mass i* (at t') which has been converted into another species, in the time range from t' to t of the stellar evolution. We have

$$F_m^i(t')\ \eta_m^i(t,t') = F_m^i(t') - F_m^i(t) \, . \tag{A9}$$

From the above definitions and that in § 2.3, we write

$$m\beta^i_m(t,t') = F^i_m(t')\,\frac{d\eta^i_m(t,t')}{dt} = -\frac{dF^i_m(t)}{dt}\,, \tag{A10}$$

or

$$\int_{t-\tau_m}^{t-g\tau_m} m\beta^i_m(t,t')\,dt' = -\int_{t-\tau_m}^{t-g\tau_m} \frac{dF^i_m(t)}{dt}\,dt' = -\int_{t-\tau_m+g\tau_m}^{t} dF^i_m(t')\,. \tag{A11}$$

The transformation $t = t' + g\tau_m$ is applied on the last equality.

We have assumed that a star of mass m has no depletion of i between its birth at $(t-\tau_m)$ up to the time $(t-\tau_m+g\tau_m)$. So we have $F^i_m(t-\tau_m+g\tau_m) = F^i_m(t-\tau_m)$. Therefore, from equations (A9) and (A11) we can write

$$\int_{t-\tau_m}^{t-g\tau_m} m\beta^i_m(t,t')\,dt' = F^i_m(t-\tau_m) - F^i_m(t) = F^i_m(t-\tau_m)\,\eta^i_m(t,t-\tau_m)\,. \tag{A12}$$

Finally, the assumption of a uniform distribution of metals in the ISM and the definitions on the Appendix and in § 3, lead to $F^i_m(t-\tau_m) = m\,X^i(t-\tau_m)$ and $\eta^i_m(t,t-\tau_m) = \xi^i_{tot}(m)$, so we can write from equation (A12)

$$\int_{t-\tau_m}^{t-g\tau_m} m\beta^i_m(t,t')\,dt' = m\,X^i(t-\tau_m)\,\xi^i_{tot}(m)\,. \tag{A13}$$

Since the same assumption (i) of § 4 is used, equation (19) can be obtained from (6) through equation (A13).

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