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EVALUATION BY NUMERICAL SIMULATIONS OF THE BLOCK ADJUSTMENT METHOD IN SMALL FIELD CCD ASTROMETRY

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RESUMEN

Se propone la determinación de posiciones astrométricas a partir de un mosaico de imágenes CCD directas con superposición parcial, tomadas con un telescopio reflector de gran distancia focal, utilizando la técnica de ajuste en bloque para la reducción. Por medio de simulaciones numéricas se evalúa la influencia de los errores introducidos por el método de medición en sí, los instrumentos, el centrado de las imágenes estelares y las posiciones de referencia. Finalmente se determina la precisión alcanzable en un campo cuadrado de 25′ de lado cubierto por dieciseis imágenes. Se encuentra que los errores introducidos por el método en sí y las posiciones de referencia, llevan a posiciones con precisiones del mismo orden que el catálogo de referencia empleado. Dado que estos errores no son significativos, si se emplea el ACT Reference Catalog, este método podría emplearse en la medición de posiciones de segunda época para la determinación de movimientos propios, utilizando placas de la Carte du Ciel como fuentes de posiciones de primera época. También se encuentra que los errores en las posiciones medidas están dominados por las aberraciones del sistema óptico y el error de centrado de las imágenes estelares.

ABSTRACT

The determination of astrometric positions from a mosaic of direct CCD frames with partial overlap taken with a long focus reflector telescope and the block adjustment technique for data reduction are proposed. The influence of errors introduced by: the measurement method itself, instruments, image centering, and reference positions is evaluated by means of numerical simulations. Finally the achievable accuracy in a 25′ square field covered with sixteen frames is determined. It is found that errors introduced by the method itself and the reference catalog lead to positions with an accuracy of the same order than the reference catalog. Since errors are not significant if the ACT Reference Catalog is the source of reference positions, this method could be used in a measurement of second epoch positions for the determination of proper motions using the Carte du Ciel plates as first epoch. It is also found that errors of the measured positions are dominated by aberrations of optical system and centering error of stellar images.

Key Words: **ASTROMETRY — METHODS: NUMERICAL**

1. INTRODUCTION

Some characteristics of CCD detectors like high quantum efficiency, wide dynamic range, linear response, dimensional stability, and the availability of digital images, ready to be processed with computers, make them suitable for astrometric programs (see for example Monet & Dahn 1983; Sinachopoulos & Seggewiss 1990; Geffert et al. 1994; Tinney, Reid, & Mould 1994), but the small field covered by a single frame can limit their applications. The block adjustment technique, originally developed for photographic astrometry (Stock 1981) would allow overcoming this limitation by making use of several frames taken with partial overlap. In order to obtain

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the position of every object in the field covered by the set of frames, they are reduced simultaneously constraining the coefficients to the condition of best fit to the reference catalog and to the neighboring frames.

Although the block adjustment technique has been successfully applied to CCD frames by other authors (Abad & García 1995) in our case it was necessary to have some previous estimations of its behavior under our particular conditions of observation before attempting to use it. Numerical simulations were chosen for this aim since in the last decades they have widely demonstrated their usefulness for the evaluation of processes.

The aim of this work is then to evaluate the feasibility of getting astrometric positions with a CCD detector attached to a long focus reflector telescope. It is intended to obtain the order of magnitude of errors in positions, and how is their dependence with different instrumental and observational effects. Since achievable accuracy in every astrometric measurement is ultimately limited by the accuracy of reference stars, errors were considered negligible when below that limit, that is about 30 mas for ACT Reference Catalog (Urban, Corbin, & Wycoff 1997), and limiting when above one tenth of arcsecond.

2. NUMERICAL SIMULATIONS

All the simulations carried out start with a set of positions α, δ that are projected on the plane of the frame to obtain their ideal rectangular coordinates, and these ideal coordinates are changed to obtain measured coordinates. Changes introduced consist in adding certain displacements ϵ_x, ϵ_y that may be random or systematic, depending on whether they represent centering errors or optical effects respectively. So it is generated a list of measured rectangular coordinates for every simulated frame, as if it was the output of the task CENTER in the package DAOPHOT of the program IRAF. These lists of measured x, y are then block adjusted to obtain the coefficients allowing to transform them to the measured celestial coordinates which are compared with the initial ones.

The block adjustment of linear equations requires at least three stars in the overlapping zone of every frame, and at least three reference stars in the whole field to obtain a solution. In a real measurement there are many link equations between neighboring frames since CCD frames may contain many stars. In order to have as much links as possible, the simulated frames were arranged in a four-fold overlap center-edge arrangement.

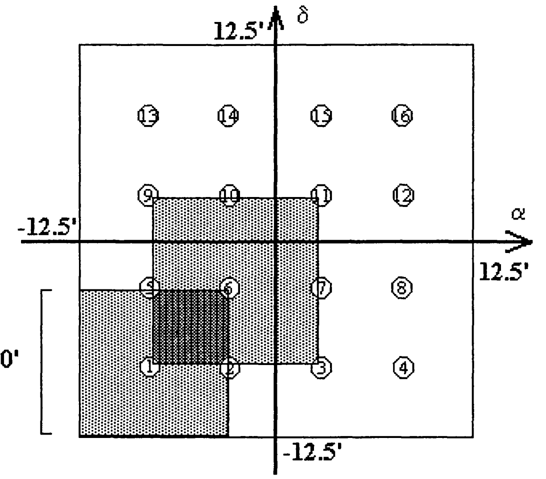

Fig. 1. Location of centers of the sixteen frames on the simulated field. Small shaded squares show the areas covered by frames number 1 and 6.

A first series of simulations intends to evaluate the influence of some field characteristics on the accuracy of measured positions: the number and relative positions of stars in the field, the quantity of reference stars and the accuracy of their positions. In case of a strong dependence on these characteristics, it could restrict the application of this technique to regions of the sky with the adequate characteristics.

The influence of instrumental effects such as: error in the determination of centroids of stellar images and displacements caused by the optical system was studied by means of other series of simulations. No simulations were carried out varying only the error in the location of the frames or in their alignment with the equatorial system, since previous simulations of measurement of photographic images (Bustos Fierro & Calderón 1997) showed that they do not produce any limitation to this measuring technique. Therefore, these two parameters of the simulations were kept constant, and their values were $\sigma_{\alpha_c,\delta_c} = 2''$ for the error in the location of the frames, and $\sigma_\theta = 1^\circ$ for their inclinations.

The errors in all analyzed quantities were generated as random numbers with gaussian distribution truncated at the 2σ level and added to those quantities. Such is the case of the error in centering stellar images and the error of reference positions.

2.1. *Simulated Field*

The high sensitivity of CCD detectors causes frames to be crowded of stars. Therefore, in order to make simulations as realistic as possible, they should

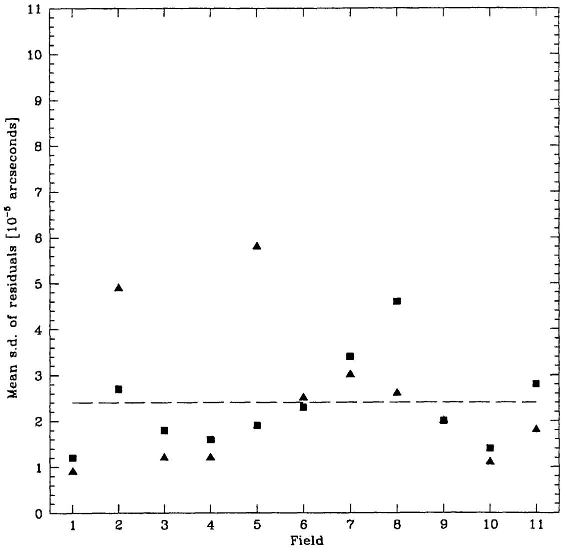

Fig. 2. Standard deviations of the residuals in simulations with error only in the location and alignment of the frames. Squares: $\sigma_{\alpha\cos\delta}$; triangles: σ_δ. The dashed line shows the average of dispersions in both coordinates of the eleven fields.

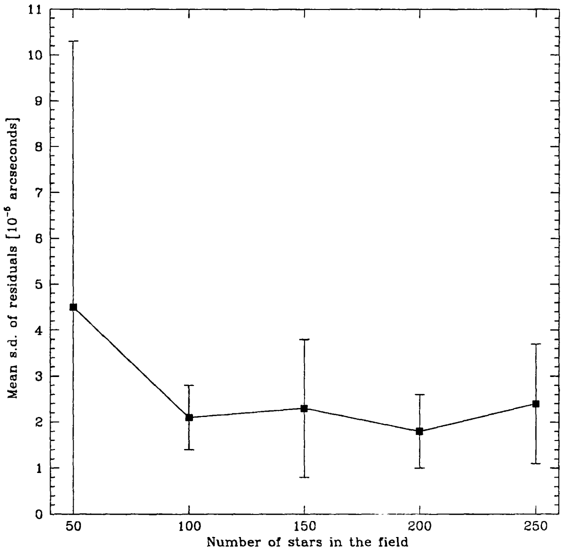

Fig. 3. Mean standard deviations of the residuals in sets of simulations with different number of stars in the field.

contain a big number of “stars”. In most of simulations shown here there are 250 stars in the whole field covered by the frames.

Simulated field consisted of an approximately 25′ square area centered at $\alpha = 0^h$, $\delta = 0°$ in order to simplify calculations. This centering choice does not introduce limitations neither in the method nor in the numerical simulations, since the purpose was to analyze differences between true and measured coordinates, regardless of their values. The results may be applied to any declination by considering errors in right ascension as $\sigma_{\alpha\cos\delta}$.

Simulated frames (Figure 1) consisted of an approximately 10′ square area. In order to cover the field in a center-edge arrangement, sixteen frames are necessary. They were arranged in four rows 50% overlapped, each row made of four frames also 50% overlapped. As a simplification, the CCD was considered to have 1000 × 1000 pixels.

The stars, each one identified with a number n, were positioned within the field by generating their coordinates α and δ as random numbers, both with a flat distribution. In simulations with n_r reference stars, stars with $1 \leq n \leq n_r$ were chosen for this purpose.

3. RESULTS

3.1. *Relative Positions of the Stars in the Field*

This series consists of eleven simulations. All the simulated fields contain 250 stars but their positions are different on each field, which was achieved by initializing the random numbers generator with a different seed before positioning the stars. The first three stars were chosen to be the reference ones.

The standard deviations of differences measured-true coordinates are below 0.10 mas, being the average 0.024 mas (Figure 2). All these errors are well below the error of reference positions nowadays available for this kind of applications.

All following results are based on these eleven fields, that means that the random numbers generator was initialized with the same seeds as in this section before locating the stars in the field, causing the same distributions of stars and reference stars, but changing the seeds before the generation of random errors.

For every analized effect, each value plotted corresponds to the mean standard deviation of residuals in both coordinates in a series of eleven fields, thus resulting the average of twenty-two data, eleven for each coordinate, that are presented with the error bar indicating the standard deviation of this average. Both coordinates were treated together without any differentiation, since no asymmetries between them were introduced in the simulations.

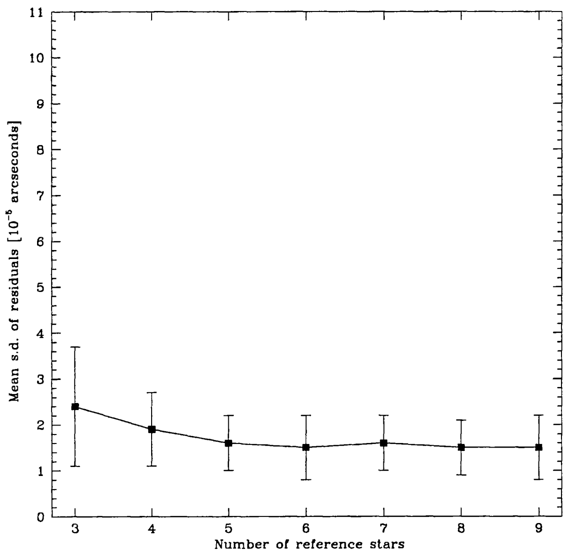

Fig. 4. Mean standard deviations of the residuals in sets of simulations with different number of reference stars in the field.

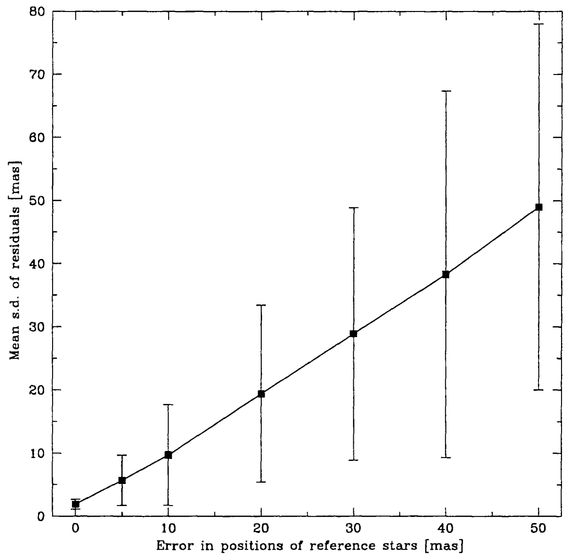

Fig. 5. Mean standard deviations of the residuals in sets of simulations with different accuracy of the reference positions.

3.2. *Number of Stars in the Field*

In the same way mentioned above, four new sets of eleven fields with 50, 100, 150, and 200 stars were generated initializing the random numbers generator with the same seeds as in the previous section for every field. The case with 250 stars was taken from the previous section. Three reference stars were used as above.

Figure 3 shows that the standard deviations of residuals keep still below 0.10 mas and show no marked dependence with the quantity of stars. Only in fields with 50 stars the dispersions are higher, but since there are very few stars per frame —an average of 8— it is not expected to happen in a real measurement because real CCD frames will be much more crowded.

3.3. *Quantity of Reference Stars*

The minimum condition to solve the block adjustment equations is to have three reference stars in the whole field and three link stars on every frame. The former condition impose no limitation to measurements with CCD detectors, since due to their high sensitivity, there will usually be many link stars per frame, even with small overlap. On the contrary, the quantity of reference stars impose a lower limit to the size of the field.

The same set of eleven fields in § 3.1 was block-adjusted adding one reference star each time up to nine, the resulting mean standard deviations of residuals beeing shown in Figure 4. It can be seen that they are still negligible when compared with errors of available reference positions, showing a slight trend of residuals to improve with increasing number of reference stars, although accuracy may be significantly worse with only three reference stars.

The following simulations contain four reference stars, that is the average for a 25′ × 25′ field in ACT Reference Catalog.

3.4. *Error in Reference Positions*

Once the four reference stars were chosen, reference positions introduced in the block adjustment were their true coordinates plus certain displacements $\epsilon_\alpha, \epsilon_\delta$:

$$\begin{aligned} \alpha_r &= \alpha + \epsilon_\alpha \\ \delta_r &= \delta + \epsilon_\delta \end{aligned}$$

ϵ_α and ϵ_δ were generated as random numbers with gaussian distribution truncated at 2σ level, having null average and r.m.s. deviation equal to σ_r, the error in positions of reference stars.

The value of σ_r was a constant during every single set of simulations but varied from 0 to 40 mas in different ones, so considering most of ACT stars. One set of eleven simulations was performed with every different value of σ_r, the mean standard deviations of the resulting residuals being shown in Figure 5.

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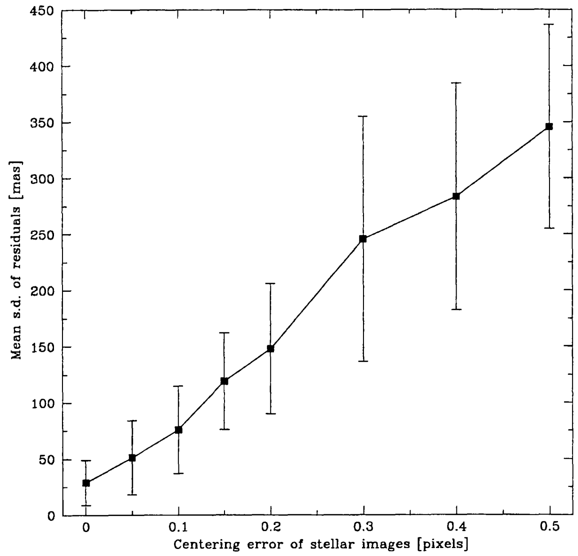

Fig. 6. Mean standard deviations of the residuals in sets of simulations with different accuracy of the centroids of stellar images.

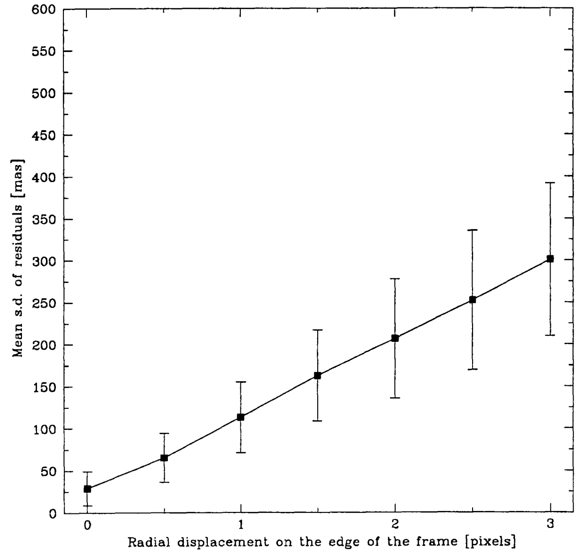

Fig. 7. Mean standard deviations of the residuals in sets of simulations with radial distortions modeled by $\Delta r(r) = ar^2$.

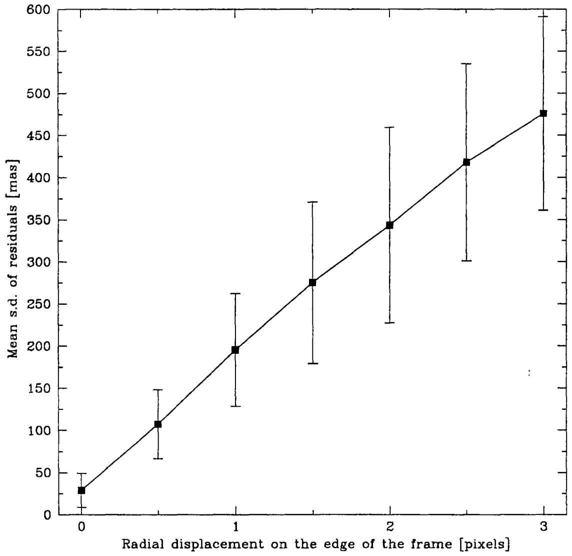

Fig. 8. Mean standard deviations of the residuals in sets of simulations with radial distortions modeled by $\Delta r(r) = ar^3$.

As expected, the accuracy of measured positions are approximately directly proportional to the accuracy of reference positions, with the proportionality constant very close to 1. It is also noticeable that the r.m.s. deviation of the average residual of the eleven fields increases almost linearly with the error of reference positions.

In the following simulations, the error in the positions of the reference stars was set in $\sigma_r = 30$ mas; that is, the median for stars in ACT Reference Catalog.

3.5. *Error in the Centroids of Stellar Images*

When centering stellar images, many factors contribute to the error in measured position, such as the centering algorithm itself, f.w.h.m. of stellar images and star's magnitude and color. Since stellar images were not actually centered in these simulations, magnitudes and colors were not simulated, but only positions; these effects could not be explicitly and individually taken into account. However, their overall consequence is the increasing of the global centering error due to the fact that stars with different magnitudes and colors are expected to be randomly distributed in the field.

Another way to interpret the results is that they are valid for stars in the range of magnitudes and colors such that their centering errors amount to the value introduced in the simulations.

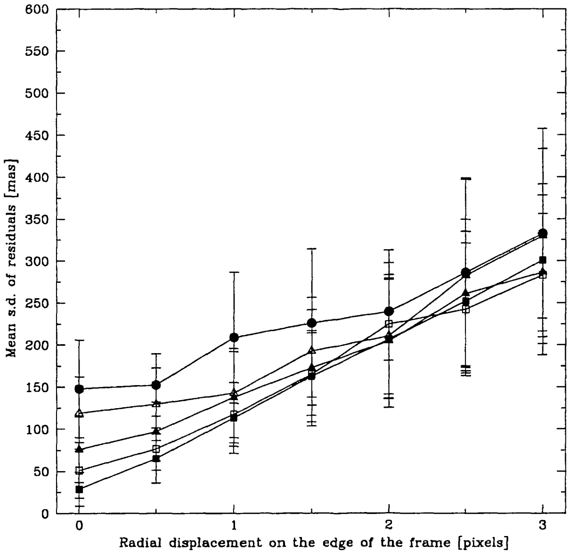

Fig. 9. Mean standard deviations of the residuals in sets of simulations with radial distortions modeled by $\Delta r(r) = ar^2$ and centering error. Each curve corresponds to a different value of the centering error $\sigma_{x,y}$ as follows: full squares: $\sigma_{x,y} = 0$; open squares: $\sigma_{x,y} = 0.05$ pixels; full triangles: $\sigma_{x,y} = 0.10$ pixels; open triangles: $\sigma_{x,y} = 0.15$ pixels; full circle: $\sigma_{x,y} = 0.20$ pixels.

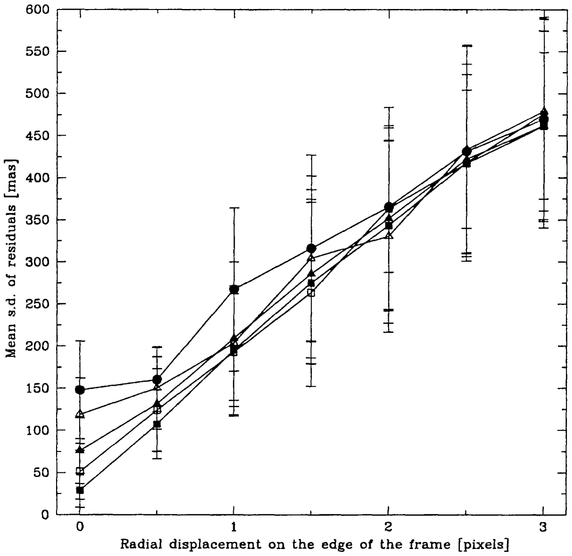

Fig. 10. Mean standard deviations of the residuals in sets of simulations with radial distortions modeled by $\Delta r(r) = ar^3$ and centering error. Each curve corresponds to a different value of the centering error $\sigma_{x,y}$ as follows: full squares: $\sigma_{x,y} = 0$; open squares: $\sigma_{x,y} = 0.05$ pixels; full triangles: $\sigma_{x,y} = 0.10$ pixels; open triangles: $\sigma_{x,y} = 0.15$ pixels; full circle: $\sigma_{x,y} = 0.20$ pixels.

Figure 6 shows the mean standard deviations of residuals found in sets of simulations, where the positioning of the stellar images on the frames have centering errors from 0 to approximately 12 μm (one half of the size of a pixel). The accuracy of centroids determined with the task CENTER in the package DAOPHOT of the program IRAF is typically between 0.1 and 0.2 pixels.

In the range studied residuals show a trend to increase linearly with the centering error, with the proportionality constant approximately equal to 660 mas/pixel, close to the scale factor of these simulations that is 620 mas/pixel, plus an additive constant approximately equal to 20 mas.

3.6. *Displacement Introduced by the Optical System*

Displacements in the radial direction introduced by the optical system were simulated by means of functions depending on the radial coordinate on the frame, that were added to the "measured" coordinates of the stars. Two groups of simulations were carried out, each one with a particular dependence of displacements Δr with the radial coordinate r:

- Group 1: $\Delta r(r) = ar^2$,
- Group 2: $\Delta r(r) = ar^3$.

In every simulation the constant a is calculated given the condition that Δr reaches a pre-determined amount on the edge of the frame ($r_e = 500$ pixels).

Figures 7 and 8 show the mean dispersions of residuals found in every group of simulations. In both cases, it can be seen that the mean s.d. of residuals and its dispersion increase almost linearly, but faster when the cubic model was introduced.

3.7. *Combination of Centering Error and Distortions*

Considering that, as seen in the last two series of simulations, instrumental effects can raise errors considerably, and therefore they can limit the achievable accuracy, they were both combined into a single series of simulations. Once again, two groups of simulations were carried out with the same dependence of displacements Δr with the radial coordinate r as in the previous section.

Figures 9 and 10 show the mean dispersions of residuals found in every group of simulations with their corresponding error bars. It is still noticeable the trend of residuals to be higher, and to have bigger

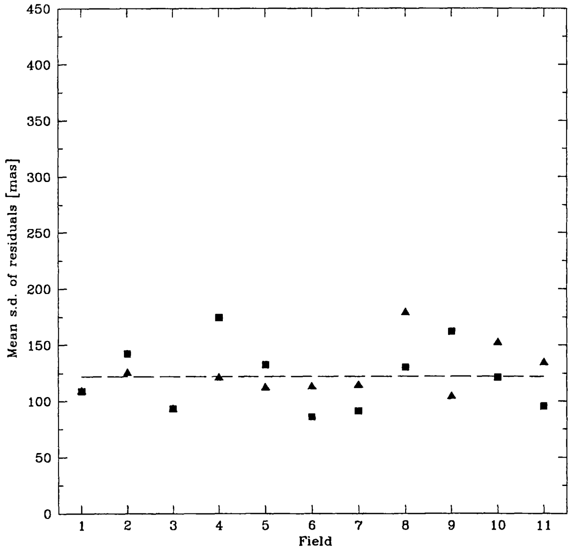

Fig. 11. Standard deviations of the residuals in simulations with parameters similar to a real observation. Squares: $\sigma_{\alpha\cos\delta}$; triangles: σ_δ. The dashed line shows the average of dispersions in both coordinates of the eleven fields.

dispersions with the cubic model for optical distortions than with the quadratic one. It is also observed that with increasing distortions the dependence of residuals on the centering error becomes weaker.

3.8. *Simulations with Parameters from a Real Observation*

Observations in Bustos Fierro & Calderón (2000) consist of 16 frames arranged in the same way as in former simulations. These observations were performed with a CCD TEK1024, that is a square CCD with 1024 × 1024 square pixels 24 μm sided, without binning. The CCD was attached to the 2.15 m reflector telescope in CASLEO (Complejo Astronómico El Leoncito, San Juan, Argentina) with focal reducer, which gives on the frames a nominal scale factor of $0.033''/\mu$m or $0.792''$/pixel. The f.w.h.m. of stellar images was $2.4''$ (3 pixels).

The frames cover an area approximately square $25'$ sided that contains seven ACT stars, all of them with accuracy in their positions close to 30 mas given in ACT Reference Catalog.

In order to reproduce the conditions of these observations, some parameters are already fixed but some others are arbitrary, such as the error in centroids of stars and the model and amount of optical deformations. In addition, according to previous simulations, they are limiting factors to the achievable accuracy, since it has been shown that if their values were "too high" measured positions would result with very poor quality and consequently useless for astrometric purposes. On the other hand, the utilization of focal reducer makes feasible the presence of optical deformations, but there is no information about this aspect of the camera used.

This series of ten simulations was carried out with the following parameters:

- Number of reference stars: 7.
- Error in reference positions: 30 mas.
- Error in centroids of stars: 0.15 pixels.
- Model function for optical deformations: $\Delta r(r) = ar^2$.
- Displacement of images at the edge of the frame: 1 pixel.

The fields are different in every simulation, that means that the random numbers generator was initialized with a different seed in each one. Their results in Figure 11 show that with moderate optical distortion, the expected accuracy is mostly between $0.1''$ and $0.2''$, being the average $0.12''$.

4. CONCLUSIONS

Simulations without any kind of instrumental errors show that the method itself introduces errors in the measured positions that are always smaller than 0.1 mas. Furthermore, these negligible errors show no strong correlation neither with the density of stars nor the quantity of reference stars. It is important to notice that these results reflect the accumulation of roundoff and truncation errors in the calculations; therefore, they may be highly dependent of the software and hardware utilized.

In the case of simulations with errors in the reference positions, the achievable accuracy shows an approximately linear correlation with it. According to these results ACT Reference Catalog would introduce errors smaller than $0.05''$.

Also the error of the stellar centroids show a linear correlation with the error of the measured positions that can be approximately expressed by

$$\sigma = 0.66''/\text{pixel} \cdot \sigma_{x,y} + 0.02'',$$

where σ is the error in the measured positions and $\sigma_{x,y}$ is the error of the stellar centroids in fraction of a pixel. The routines commonly used in IRAF lead to centroids with accuracy typically around 0.15 pixels;

therefore, positions could be measured with accuracy around 0.12″ if optical distortions are not important.

Simulations with errors in the centroids and optical distortions show an approximately linear dependence of the error in the measured positions with the amount of the distortion on the edge of the frame, with the proportionality constant dependent on the model function used for the optical distortions. In cases with no severe distortion, the error of measured positions increases with the error of centroids of stellar images, but when distortions are bigger than 2 pixels on the edge of the frame, the error of measured positions seems to be independent of the error of centroids, at least in the range of values studied here. This is more evident in the case of distortions modeled with a cubic function of the radial coordinate.

It has been shown in artificial fields that the block adjustment technique applied to direct CCD frames could provide positions with astrometric quality. The accuracy of those positions would be limited mainly by distortions introduced by the optical system, and in second place by centering error of stellar images. With distortions not exceeding 1 pixel on the edge of the frame, the smallest achievable errors in measured positions would be between 0.10″ and 0.20″.

The combination of positions obtained from today observations with accuracy of 0.20″ and from Carte du Ciel plates with accuracy of 0.3″ (according to numerical simulations in Bustos Fierro & Calderón 1997) would allow the determination of proper motions of stars up to $B = 14$ mag with accuracy between 4 and 5 mas/year, with only one pair of epoches.

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