

## QUANTITATIVE STELLAR SPECTRAL CLASSIFICATION. II. EARLY TYPE STARS

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*Received 2001 September 18; accepted 2002 May 9*

### RESUMEN

Se ha extendido a las estrellas del tipo espectral B el método desarrollado por Stock y Stock (1999) para estrellas del tipo espectral A-K, con el cual es posible derivar magnitudes absolutas y colores intrínsecos a partir de los anchos equivalentes de las líneas de absorción de los espectros estelares. Espectros de estrellas tipo B para las cuales el catálogo de Hipparcos proporciona paralajes con un error menor al 20%, fueron observados con el reflector de 1 m del CIDA equipado con espectrógrafo Richardson y un detector CCD Thompson 576×384. Utilizando una rejilla de 600 líneas/mm se obtuvo una dispersión en el primer orden de 1.753 Å/pixel. Para cubrir el rango espectral comprendido entre 3850 Å y 5750 Å fue necesario utilizar la rejilla en dos posiciones distintas, teniendo un solapamiento en la región entre 4800 Å y 4900 Å. Fueron observadas un total de 116 estrellas, pero no todas en las dos posiciones de la rejilla. Se identificaron un total de 12 líneas de absorción en los espectros y se midieron sus anchos equivalentes. Estos fueron relacionados con las magnitudes absolutas derivadas del catálogo Hipparcos y con los colores intrínsecos (deducidos de los tipos espectrales MK), por medio de polinomios de primero y segundo orden y combinaciones de dos o tres líneas como variables independientes. Las mejores soluciones fueron obtenidas con polinomios de tres líneas, reproduciendo las magnitudes absolutas con un residuo promedio de 0.40 magnitudes, y los colores intrínsecos con un residuo promedio de 0.016 magnitudes.

### ABSTRACT

The method developed by Stock & Stock (1999) for stars of spectral types A to K to derive absolute magnitudes and intrinsic colors from the equivalent widths of absorption lines in stellar spectra is extended to B-type stars. Spectra of this type of stars for which the Hipparcos catalogue gives parallaxes with an error of less than 20% were observed with the CIDA one-meter reflector equipped with a Richardson spectrograph with a Thompson 576×384 CCD detector. The dispersion is 1.753 Å/pixel using a 600 lines/mm grating in the first order. In order to cover the spectral range 3850 Å to 5750 Å the grating had to be used in two different positions, with an overlap in the region from 4800 Å to 4900 Å. A total of 116 stars was observed, but not all with both grating positions. A total of 12 measurable absorption lines were identified in the spectra and their equivalent widths were measured. These were related to the absolute magnitudes derived from the Hipparcos catalogue and to the intrinsic colors (deduced from the MK spectral types) using linear and second order polynomials and two or three lines as independent variables. The best solutions were obtained with polynomials of three lines, reproducing the absolute magnitudes with an average residual of about 0.40 magnitudes and the intrinsic colors with an average residual of 0.016 magnitudes.

*Key Words:* STARS: FUNDAMENTAL PARAMETERS

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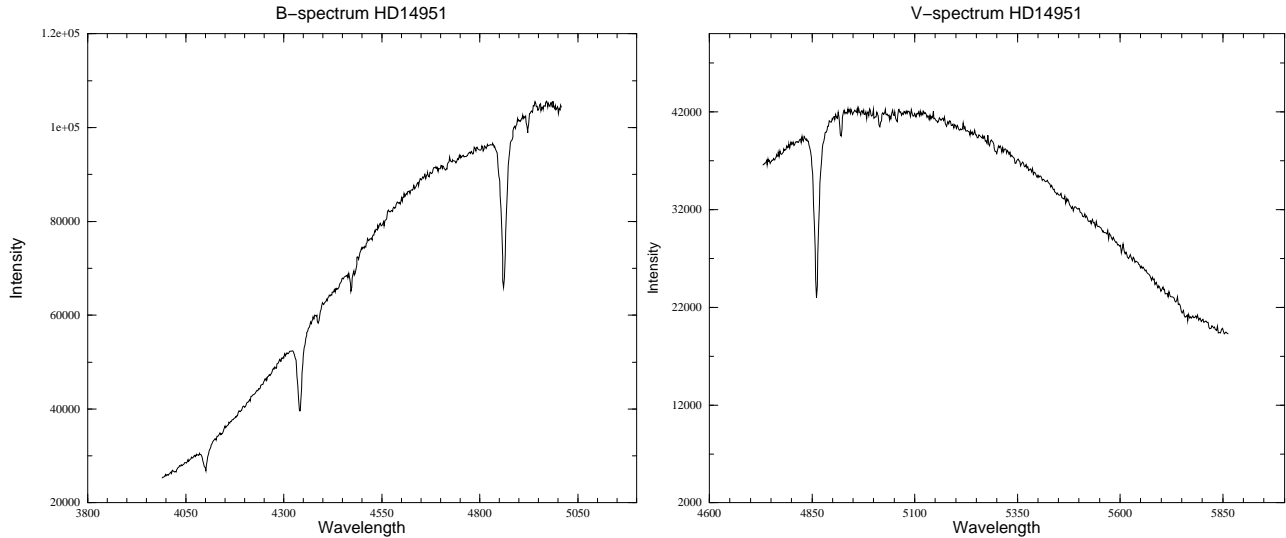


Fig. 1. Observed  $B$  and  $V$  band spectra of HD14951.

## 1. INTRODUCTION

In a previous work, two of the authors (Stock & Stock 1999) published a method for the derivation of stellar physical parameters such as the absolute magnitude, an intrinsic color, and a metallicity index from the equivalent widths or pseudo-equivalent widths of absorption lines in stellar spectra. Use was made of a library of stellar spectra made available by Jones (1999). Stars were rejected for which the Hipparcos parallax catalogue gives parallax errors larger than 20% of the parallax itself. Due to this restriction the number of remaining O- and B-type spectra was too small to be included. To close this gap, we decided to launch our own observing program dedicated exclusively to early type stars. A number of recent papers have been dedicated to the subject of quantitative stellar classification. Malyuto & Schmidt-Kaler (1999) classify G-, K-, and M-type stars on the basis of spectral indices derived in the region from 6000 to 10,000 Å. A similar approach is used by Malyuto, Oestreicher, & Schmidt-Kaler (1997) for K- and M-type stars based on spectra in the region from 4800 to 7700 Å.

## 2. OBSERVATIONS

In view of the restriction just mentioned, we started an observing program with the Richardson spectrograph of the CIDA observatory attached to the 1-meter reflector, concentrated on O- and B-type stars which fulfilled the same parallax error restriction used in the previous work. A grating of 600 lines/mm was used in the first order yielding a dispersion of  $1.753 \text{ \AA}/\text{pixel}$ . The detector is a Thompson  $576 \times 384$  CCD with a pixel size of 23 microns.

The spectral range captured by a single exposure is about  $1000 \text{ \AA}$ . Two grating positions were used, one yielding usable spectra from  $3950 \text{ \AA}$  to  $4900 \text{ \AA}$ , the other from  $4800 \text{ \AA}$  to  $5750 \text{ \AA}$ . In the following, we distinguish these as the  $B$  and  $V$  spectra. The grating positioning mechanism was rather crude (it has now been improved) and the different spectra are not always centered at exactly the same wavelength. For this reason some of the lines near the end of the spectra are not always covered.

The observing list was made up with the Bright Star Catalogue in combination with the Hipparcos parallax catalogue, selecting all O and B stars brighter than the 6th apparent  $V$  magnitude. Of these 116 stars were observed, resulting in 72 stars with  $B$  spectra only, 39 stars with  $B$  and  $V$  spectra, and only 5 stars with  $V$  spectra. A typical example of a  $B$  and a  $V$  spectrum is shown in Figure 1. Table 1 contains a list of all observed stars with their HD numbers and additional pertinent information. An intrinsic color-magnitude diagram of the observed stars is shown in Figure 2.

## 3. DATA REDUCTION

As in the previous papers, the data analysis will be based on the equivalent widths of absorption lines. When the true continuum is resolved, as is the case here, the determination of the equivalent width is a standard procedure and need not be described here. The signal/noise ratio naturally depends on the brightness of the star, the exposure time, and a series of other factors. In most cases the value of  $S/N$  is found to be near 50. A total of 11 measurable absorption lines was found in the

TABLE 1  
LIST OF OBSERVED STARS

HD Nr.	$V$	$(B - V)$	Parallax (mas)	Parallax Error (mas)	Spectral Type	$B$ band Spectrum	$V$ band Spectrum
224990	5.04	-0.15	6.40	0.87	B4 V	*	
886	2.83	-0.19	9.79	0.81	B2 IV	*	
3369	4.34	-0.12	4.97	0.82	B5 V	*	
5737	4.30	-0.15	4.85	0.84	B7 IIIp	*	
7374	5.97	-0.08	6.52	0.79	B8 III	*	
14951	5.48	-0.10	5.41	1.04	B7 IV	*	
16582	4.08	-0.21	5.04	0.83	B2 IV	*	
17081	4.24	-0.12	7.40	0.85	B7 IV	*	
18604	4.71	-0.11	7.69	0.76	B6 III	*	*
19356	2.09	0.00	35.14	0.90	B8 V	*	*
22203	4.26	-0.11	11.02	0.75	B9 V	*	
23227	4.99	-0.16	4.45	0.62	B5 III	*	
23302	3.72	-0.10	8.80	0.89	B6 III	*	
23338	4.30	-0.11	8.75	1.08	B6 V	*	
23466	5.34	-0.10	5.54	0.80	B3 V	*	
23630	2.85	-0.09	8.87	0.99	B7 III	*	*
23850	3.62	-0.07	8.57	1.03	B8 III	*	*
23277	5.40	0.10	10.02	0.54	A2 m	*	
24587	4.64	-0.14	8.46	0.75	B5 V	*	
24760	2.90	-0.20	6.06	0.82	B0.5 V	*	
25340	5.28	-0.13	7.20	0.83	B5 V	*	
25330	5.67	0.00	5.77	0.78	B5 V	*	
26326	5.45	-0.15	4.49	0.78	B5 IV	*	*
26912	4.27	-0.05	7.50	1.13	B3 IV	*	*
28375	5.53	-0.10	8.47	1.11	B3 V	*	
29248	3.93	-0.21	5.56	0.88	B2 III sb	*	
29763	4.27	-0.11	8.14	0.78	B3 V	*	*
30211	4.01	-0.15	6.13	1.03	B5 IV	*	*
33802	4.45	-0.10	13.53	0.69	B8 V	*	*
34085	0.18	-0.03	4.22	0.81	B8 Ia	*	*
34503	3.59	-0.12	5.88	0.77	B5 III	*	*
35468	1.64	-0.22	13.42	0.98	B2 III	*	*
35497	1.65	-0.13	24.89	0.88	B7 III	*	*
36267	4.20	-0.14	11.30	1.01	B5 V	*	*
37742	1.74	-0.20	3.99	0.79	O9.5 Ib sb		*
41534	5.65	-0.19	3.01	0.57	B2 V	*	
41753	4.42	-0.16	6.10	0.88	B3 IV	*	*
42560	4.45	-0.18	5.14	0.78	B3 IV	*	*
43157	5.83	-0.16	5.28	0.83	B5 V	*	
43153	5.34	-0.10	6.80	0.93	B7 V	*	*
43955	5.51	-0.16	3.28	0.60	B2/B3 V	*	

TABLE 1 (CONTINUED)

HD Nr.	$V$	$(B - V)$	Parallax (mas)	Parallax Error (mas)	Spectral Type	$B$ band Spectrum	$V$ band Spectrum
44743	1.98	-0.24	6.53	0.66	B1 II/III	*	*
45813	4.47	-0.17	8.03	0.58	B4 V	*	*
45542	4.13	-0.12	6.49	1.06	B6 III	*	*
46487	5.09	-0.13	6.08	0.79	B5 Vn	*	*
46936	5.62	-0.09	6.70	0.58	B9 V	*	*
47100	5.34	-0.08	4.30	0.76	B8 III	*	
49643	5.75	-0.10	5.84	0.77	B8 III <sub>n</sub>	*	
52089	1.50	-0.21	7.57	0.57	B2 II	*	*
52670	5.64	-0.17	3.17	0.59	B2/B3 III/IV	*	*
53244	4.11	-0.11	8.11	0.63	B8 II	*	*
56342	5.36	-0.16	5.05	0.57	B2 V	*	
57821	4.94	-0.04	6.31	0.69	B5 II/III		*
58715	2.89	-0.10	19.16	0.85	B8 V <sub>var</sub>	*	
59550	5.78	-0.19	2.85	0.56	B2 IV		*
60863	4.65	-0.11	14.50	0.62	B8 V	*	*
61429	4.69	-0.10	5.92	0.72	B8 IV	*	*
61555	3.80	-0.16	7.18	1.06	B5 IV	*	
63975	5.12	-0.12	7.76	1.02	B8 II	*	*
67797	4.40	-0.16	6.90	0.69	B5 V	*	*
78316	5.23	-0.09	6.74	0.91	B8 III <sub>mnp</sub>	*	*
83754	5.07	-0.15	6.33	0.91	B4 IV/V		*
87901	1.36	-0.09	42.09	0.79	B7 V	*	*
90994	5.08	-0.14	9.46	1.16	B6 V	*	*
106625	2.58	-0.11	19.78	0.81	B8 III		*
107348	5.20	-0.09	8.47	0.78	B8 V	*	*
116658	0.98	-0.23	12.44	0.86	B1 V	*	*
120315	1.85	-0.10	32.39	0.74	B3 V sb	*	*
120709	4.32	-0.15	10.96	0.88	B5	*	*
120955	4.75	-0.11	4.87	0.71	B4 IV	*	*
121847	5.20	-0.09	9.61	0.69	B8 V	*	
126769	4.97	-0.07	7.85	0.80	B7/B8 V	*	*
132955	5.45	-0.13	9.32	0.84	B3 V	*	
135742	2.61	-0.07	20.38	0.87	B8 V	*	*
136298	3.22	-0.23	6.39	0.86	B1.5 IV	*	
138485	5.53	-0.15	4.24	0.84	B3 V	*	
138749	4.14	-0.13	10.49	0.66	B6 V <sub>mn</sub>	*	
138764	5.16	-0.09	9.30	0.86	B6 IV	*	
139365	3.66	-0.18	7.33	1.01	B2.5 V	*	
142883	5.84	0.01	7.16	0.87	B3 V	*	
143275	2.29	-0.12	8.12	0.88	B0.2 IV	*	
147394	3.91	-0.15	10.37	0.53	B5 IV	*	
148605	4.79	-0.12	8.30	0.84	B3 V	*	

TABLE 1 (CONTINUED)

HD Nr.	$V$	$(B - V)$	Parallax (mas)	Parallax Error (mas)	Spectral Type	$B$ band Spectrum	$V$ band Spectrum
148703	4.24	-0.17	4.37	0.80	B2 III-IV	*	
149438	2.82	-0.21	7.59	0.78	B0 V	*	
149757	2.54	0.04	7.12	0.71	O9.5 V	*	
154204	6.29	-0.04	8.21	0.76	B7 IV/V	*	
158408	2.70	-0.18	6.29	0.81	B2 IV	*	
158926	1.62	-0.23	4.64	0.90	B1.5 IV+...	*	
160762	3.82	-0.18	6.58	0.56	B3 V sb	*	
160578	2.39	-0.17	7.03	0.73	B1.5 III	*	
172910	4.86	-0.17	7.23	1.09	B2 V	*	
173300	3.17	-0.11	14.14	0.88	B8.5 III	*	
175191	2.05	-0.13	14.54	0.88	B2.5 V	*	
176162	5.51	-0.04	6.34	0.80	B4 V	*	
180163	4.43	-0.15	3.13	0.51	B2.5 IV	*	
180554	4.76	-0.06	3.58	0.60	B4 IV	*	
182255	5.22	-0.12	8.10	0.68	B6 III	*	
182568	4.99	-0.12	4.21	0.58	B3 IV	*	
184171	4.74	-0.15	5.20	0.55	B3 IV	*	
184930	4.36	-0.08	10.61	0.94	B5 III	*	
186500	5.51	0.02	5.98	1.00	B8 III	*	
189103	4.37	-0.15	5.28	0.90	B2.5 IV	*	
189944	5.88	-0.13	4.85	0.66	B4 V	*	
190993	5.08	-0.16	6.68	0.71	B3 V	*	
196740	5.06	-0.13	6.61	0.66	B5 IV	*	
202671	5.40	-0.12	5.63	0.95	B5 II/III	*	
207330	4.23	-0.12	2.82	0.52	B3 III	*	
207971	3.00	-0.08	16.07	0.77	B8 III	*	
209409	4.74	-0.10	8.56	0.81	B7 IVe	*	
210424	5.43	-0.12	5.81	0.75	B5 III	*	
210934	5.45	-0.12	6.42	0.85	B7 V	*	
214748	4.18	-0.10	4.38	0.87	B8 V	*	
214923	3.41	-0.09	15.64	0.75	B8.5 V	*	
216831	5.73	-0.05	3.90	0.70	B7 III	*	
219688	4.41	-0.14	10.13	1.04	B5 Vn	*	

$B$  spectra and only 4 in the  $V$  spectra. Three of these are common to both the  $B$  and the  $V$  spectra. Thus, only one additional line was added by including the  $V$  band in the observing program. Even so, the inclusion of the  $V$  band turned out to be quite fortunate as will be seen in the analysis. In the first place, it gave us a convenient handle to determine the accuracy with which the equivalent widths are

determined. Also, it turned out that the line added by the  $V$  band provides an important classification criterion.

For the lines in common to both bands we can carry out an analysis of the accuracy with which the equivalent widths are determined. Only three lines are available for the test. There is a pronounced dependence of the accuracy on the equivalent width

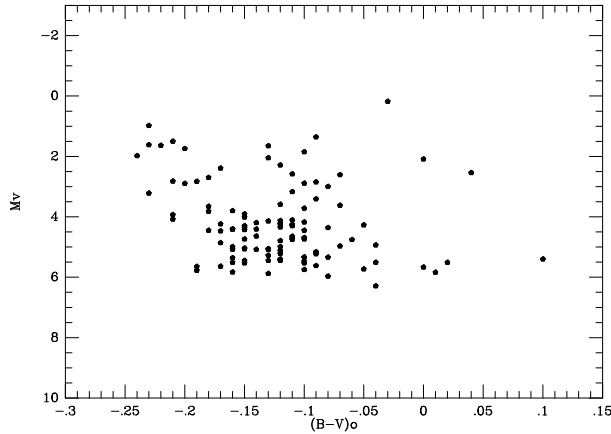


Fig. 2. H-R diagram of observed stars.

itself. The relation found is best described by

$$e = 0.035 + 0.088w, \quad (1)$$

where  $e$  is the average accuracy with which a line of equivalent width  $w$  is determined. Units are Angstroms. A complete list of the lines is given in Table 2, with their wavelength taken from the Multiplet Table by Moore (1972), as well as their identification. Furthermore, for each line, an inner region and two outer regions were selected. The two outer regions, one on each side of the line—each at least several Angstroms wide—were used to determine a “continuum” or “pseudo-continuum”, while the integration of the inner region yielded the equivalent width. Also it is indicated whether they were measured in the  $B$  or in the  $V$  spectra or in both. All this information is shown in Table 2. For two lines the relation between the equivalent widths measured in the  $B$  and the  $V$  spectra is shown in Figure 3. Likewise the relation between the intrinsic colors  $(B-V)_0$  and the equivalent widths for  $H\delta$  and  $He\ 4143\ \text{\AA}$  is shown in Figure 4. In these plots, the size of the symbol represents the luminosity of the star in the sense that the larger the symbol the more luminous the star. The plots demonstrate the known fact that the He lines reach their maximum equivalent width around the spectral type B2, while the equivalent width of the hydrogen lines increases all the way through the B class.

#### 4. ANALYSIS

The principal purpose of this work is to find means by which physical parameters, namely the absolute magnitude  $M_V$  and the intrinsic color  $(B-V)_0$ , can be predicted from the equivalent widths of absorption lines. The external data used are the spectral types in the MK system, the apparent  $V$

magnitudes, and the observed  $(B-V)$  colors given in the Bright Star Catalog, and the parallaxes and parallax errors in the Hipparcos Catalog. Intrinsic colors  $(B-V)_0$  were deduced from the MK-types. The respective relations may be found for instance in tables given by Allen (1973). These colors can be compared with the observed colors and a reddening effect can be found. If interpreted as due to absorption by interstellar or circumstellar material the effect on the apparent  $V$  magnitude can be estimated. For this purpose, we use the commonly adopted relation

$$A_V = 3.0 E(B-V), \quad (2)$$

where  $A_V$  is the absorption in the  $V$  band, and  $E(B-V)$  the reddening of the  $(B-V)$  color. This correction is applied when the color excess is greater than 0.03 magnitudes. Using the parallax given in the Hipparcos parallax catalogue the corrected magnitudes were converted into absolute magnitudes  $M_V$ . For the relation between the physical parameters  $M_V$  or  $(B-V)_0$  and the equivalent widths we adopt second order polynomials with two or three independent variables, the latter being the equivalent widths of two or three absorption lines. Thus the equation for the absolute magnitude with three lines has the form

$$\begin{aligned} M_V = & a_{000} + a_{100}w_1 + a_{010}w_2 + a_{001}w_3 \\ & + a_{200}w_1^2 + a_{110}w_1w_2 \\ & + a_{101}w_1w_3 + a_{020}w_2^2 \\ & + a_{011}w_2w_3 + a_{002}w_3^2, \end{aligned} \quad (3)$$

where  $w_1$ ,  $w_2$ , and  $w_3$  are the equivalent widths of the three respective absorption lines. The coefficients  $a_{ijk}$  have to be determined by least squares, forming equation (3) for all spectra in which values for the three different equivalent widths were obtained. With 3 lines out of 12 a total of 220 combinations can be made up. With two lines a total of 66 combinations can be made up. Equation (3) can also be used for the intrinsic colors, introducing these instead of the absolute magnitudes. For the three-line combinations, 10 coefficients have to be determined by least squares. If only two lines are used (skipping in equation (3) all terms containing  $w_3$ ), the number of unknowns is 6. Likewise, omitting the square and mixed terms, the expression for the linear dependence of  $M_V$  on two or three lines is obtained. Replacing in all equations  $M_V$  by  $(B-V)_0$ , expressions are found which relate the intrinsic colors to the equivalent widths. We have also tested solutions based on four absorption lines without obtaining a significant improvement with respect to the previous solutions.

TABLE 2  
LIST OF SELECTED LINES

Line	Cont1(Å)		Line(Å)		Cont2(Å)		$\lambda$ (Å)	Id.
1 <i>B</i>	4006.2	4020.3	4022.1	4032.7	4036.2	4050.3	4026.4	He I
2 <i>B</i>	4067.9	4082.1	4085.6	4119.1	4124.4	4142.0	4101.7	H $\delta$
3 <i>B</i>	4122.6	4140.2	4142.0	4156.1	4157.9	4168.4	4143.8	He I
4 <i>B</i>	4298.9	4313.0	4316.5	4357.1	4362.4	4374.7	4340.5	H $\gamma$
5 <i>B</i>	4364.1	4378.2	4381.8	4392.3	4397.6	4410.0	4387.9	He I
6 <i>B</i>	4439.9	4462.9	4454.0	4477.0	4478.7	4487.5	4471.7	He I
7 <i>B</i>	4475.2	4477.0	4478.7	4482.3	4485.8	4494.6	4481.0	Mg II
8 <i>B</i>	4686.8	4702.6	4709.7	4716.7	4720.3	4729.1	4713.4	He I
9 <i>B - V</i>	4820.8	4833.1	4836.6	4868.0	4889.5	4905.4	4861.3	H $\beta$
10 <i>B - V</i>	4900.1	4912.4	4916.0	4926.5	4931.8	4945.9	4921.3	He I
11 <i>B - V</i>	5005.9	5011.2	5014.7	5020.0	5023.5	5032.3	5015.7	He I
12 <i>V</i>	5101.1	5112.4	5116.9	5129.3	5136.3	5148.7	5047.7	He I

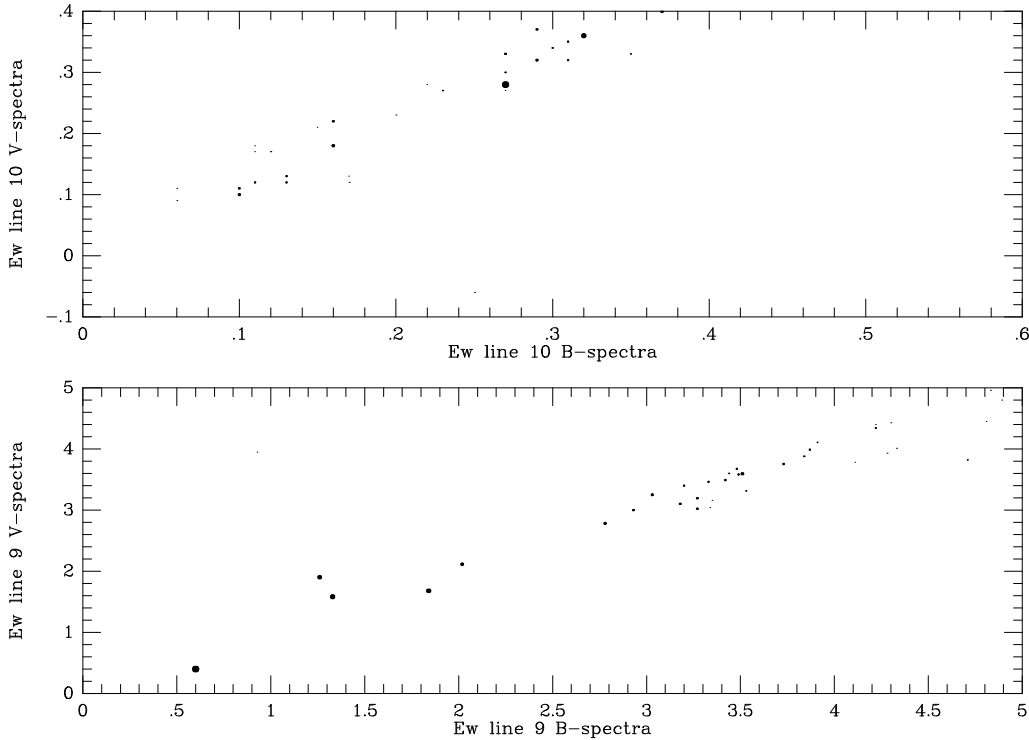


Fig. 3. Relation between the equivalent widths Ew measured in the *B* and *V* spectra.

We should point out here that all spectra entered the solution with the same weight. It would be possible to assign weight according to the respective relative parallax errors, i.e., the parallax error divided by the parallax itself, which in our selection is limited to a maximum of 20%. This idea was discarded in view of the biased selection of stars in the Hipparcos catalogue, which may favor stars of certain absolute magnitudes, and of the fact that, in general, low-

luminosity stars have more accurate parallaxes due to their proximity.

### 5. RESULTS FOR THE ABSOLUTE MAGNITUDES

Two- and three-line solutions were calculated for all possible line combinations. For each combination the average residual  $r_{av}$  (Hipparcos absolute magnitude minus polynomial absolute magnitude) was cal-

TABLE 3  
BEST COMBINATIONS FOR THE DETERMINATION OF  
ABSOLUTE MAGNITUDES BASED ON TWO LINES

Linear Solutions									
<i>B</i> Spectra					<i>B</i> and <i>V</i> Spectra				
L1	L2	$r_{av}$	$N_u^a$	$N_e^b$	L1	L2	$r_{av}$	$N_u^a$	$N_e^b$
2	4	0.517	54	5	2	4	0.517	54	5
2	9	0.504	54	5	2	9	0.504	54	5
3	9	0.529	91	9	3	9	0.529	91	9
4	7	0.553	102	9	4	7	0.553	102	9
4	11	0.496	74	7	4	11	0.496	74	7
5	9	0.588	103	8	4	12	0.381	34	5
6	9	0.581	102	9	7	9	0.556	102	9
7	9	0.556	102	9	9	10	0.568	106	10
9	10	0.568	106	10	9	11	0.563	82	4
9	11	0.563	82	4	9	12	0.482	41	3
Quadratic Solutions									
<i>B</i> Spectra					<i>B</i> and <i>V</i> Spectra				
L1	L2	$r_{av}$	$N_u^a$	$N_e^b$	L1	L2	$r_{av}$	$N_u^a$	$N_e^b$
2	9	0.460	54	5	2	9	0.460	54	5
3	9	0.550	93	7	3	9	0.550	93	7
4	7	0.604	104	7	4	8	0.578	102	9
4	8	0.578	102	9	4	9	0.561	103	8
4	9	0.561	103	8	4	11	0.513	74	7
4	11	0.513	74	7	7	9	0.572	104	7
7	9	0.572	104	7	8	9	0.536	102	9
8	9	0.536	102	9	9	10	0.567	108	8
9	10	0.567	108	8	9	11	0.586	84	2
9	11	0.586	84	2	9	12	0.412	41	3

<sup>a</sup> $N_u$ : used stars; <sup>b</sup> $N_e$ : eliminated stars.



TABLE 4  
BEST COMBINATIONS FOR THE DETERMINATION OF  
ABSOLUTE MAGNITUDES BASED ON THREE LINES

Linear Solutions											
<i>B</i> Spectra						<i>B</i> and <i>V</i> Spectra					
L1	L2	L3	$r_{av}$	$N_u^a$	$N_e^b$	L1	L2	L3	$r_{av}$	$N_u^a$	$N_e^b$
1	2	9	0.482	53	5	1	4	11	0.421	26	2
1	4	11	0.421	26	2	1	4	12	0.387	26	2
1	9	11	0.333	26	2	1	9	11	0.333	26	2
2	4	11	0.451	28	1	1	9	12	0.326	26	2
2	9	10	0.498	54	5	2	4	11	0.451	28	1
3	4	11	0.464	64	6	2	4	12	0.440	27	1
3	9	11	0.475	65	5	2	9	12	0.384	27	1
4	5	11	0.500	74	7	3	4	11	0.464	64	6
4	8	11	0.499	74	7	4	8	12	0.424	35	4
4	10	11	0.486	74	7	4	10	12	0.387	34	5
Quadratic Solutions											
<i>B</i> Spectra						<i>B</i> and <i>V</i> Spectra					
L1	L2	L3	$r_{av}$	$N_u^a$	$N_e^b$	L1	L2	L3	$r_{av}$	$N_u^a$	$N_e^b$
2	4	9	0.465	56	3	1	9	12	0.395	27	1
2	5	9	0.489	55	4	2	6	9	0.443	54	5
2	6	9	0.443	54	5	3	9	11	0.447	66	4
2	7	9	0.484	55	4	3	9	12	0.334	35	4
2	9	10	0.483	55	4	4	6	11	0.456	75	6
3	9	11	0.447	66	4	4	8	12	0.457	38	1
4	6	11	0.456	75	6	4	9	12	0.417	37	2
4	8	11	0.473	75	6	7	9	12	0.454	38	1
4	9	11	0.471	76	5	8	9	12	0.440	38	1
9	10	11	0.480	81	5	9	10	12	0.342	40	4

<sup>a</sup> $N_u$ : used stars; <sup>b</sup> $N_e$ : eliminated stars.

TABLE 5  
 COEFFICIENTS FOR THE BEST LINEAR COMBINATIONS OF  
 ABSOLUTE MAGNITUDES BASED ON THREE LINES

Blue Spectra						
L1	L2	L3	$a_{000}$	$a_{100}$	$a_{010}$	$a_{001}$
1	2	9	-5.1881	0.1720	-0.3517	0.9100
1	4	11	-6.1041	-1.1267	0.8186	4.2092
1	9	11	-5.3635	-0.2788	0.6886	0.0740
2	4	11	-6.2617	-0.5080	1.2510	1.1302
2	9	10	-4.5249	-0.3179	0.8207	-0.5408
3	4	11	-5.4876	0.5196	0.7536	-1.4535
3	9	11	-5.0903	0.5743	0.6669	-1.4267
4	5	11	-5.0534	0.6887	0.0973	-1.8178
4	8	11	-4.9882	0.6807	-0.3788	-1.4161
4	10	11	-5.4992	0.7414	0.8885	-2.7578
Blue and Visual Spectra						
L1	L2	L3	$a_{000}$	$a_{100}$	$a_{010}$	$a_{001}$
1	4	11	-6.1041	-1.1267	0.8186	4.2092
1	4	12	-5.6824	-0.1997	0.7742	5.1345
1	9	11	-5.3635	-0.2788	0.6886	0.0740
1	9	12	-5.3137	-0.2258	0.6787	1.6170
2	4	11	-6.2617	-0.5080	1.2510	1.1302
2	4	12	-5.8773	-0.2922	1.0317	4.0268
2	9	12	-5.4125	-0.1425	0.7869	0.9973
3	4	11	-5.4876	0.5196	0.7536	-1.4535
4	8	12	-5.4024	0.7490	-1.1068	5.4555
4	10	12	-5.9997	0.8121	0.1954	5.1382

TABLE 6  
 BEST COMBINATIONS FOR THE DETERMINATION OF  
 $(B - V)_0$  COLORS BASED ON TWO LINES

Linear Solutions									
<i>B</i> Spectra					<i>B</i> and <i>V</i> Spectra				
L1	L2	$r_{av}$	$N_u^a$	$N_e^b$	L1	L2	$r_{av}$	$N_u^a$	$N_e^b$
1	10	0.020	55	3	1	10	0.020	55	3
3	5	0.018	87	13	3	5	0.018	87	13
3	6	0.019	91	9	3	6	0.019	91	9
3	10	0.017	88	12	3	10	0.017	88	12
4	10	0.020	102	9	5	10	0.020	99	12
5	10	0.020	99	12	5	12	0.018	36	3
6	10	0.021	101	10	7	10	0.020	98	13
7	10	0.020	98	13	8	12	0.019	37	2
9	10	0.020	108	8	10	11	0.019	74	12
10	11	0.019	74	12	10	12	0.017	41	3
Quadratic Solutions									
<i>B</i> Spectra					<i>B</i> and <i>V</i> Spectra				
L1	L2	$r_{av}$	$N_u^a$	$N_e^b$	L1	L2	$r_{av}$	$N_u^a$	$N_e^b$
1	10	0.021	55	3	1	10	0.021	55	3
2	6	0.022	58	1	2	6	0.022	58	1
2	10	0.022	58	1	3	10	0.021	92	8
3	10	0.021	92	8	4	6	0.021	103	8
4	5	0.023	103	8	4	10	0.021	102	9
4	6	0.021	103	8	6	9	0.022	105	6
4	10	0.021	102	9	8	10	0.021	99	12
6	9	0.022	105	6	8	12	0.020	38	1
8	10	0.021	99	12	9	10	0.020	109	7
9	10	0.020	109	7	10	12	0.019	41	3

<sup>a</sup> $N_u$ : used stars; <sup>b</sup>  $N_e$ : eliminated stars.

TABLE 7  
 BEST COMBINATIONS FOR THE DETERMINATION OF  
 $(B - V)_0$  COLORS BASED ON THREE LINES

Linear Solutions											
<i>B</i> Spectra						<i>B</i> and <i>V</i> Spectra					
L1	L2	L3	$r_{av}$	$N_u^a$	$N_e^b$	L1	L2	L3	$r_{av}$	$N_u^a$	$N_e^b$
1	3	6	0.018	53	5	1	8	12	0.017	27	1
1	3	10	0.018	54	4	2	8	11	0.014	27	2
1	8	11	0.017	27	1	2	8	12	0.016	27	1
2	8	11	0.014	27	2	3	5	12	0.016	36	3
3	5	7	0.018	87	13	3	6	12	0.016	36	3
3	5	10	0.019	91	9	3	10	12	0.016	37	2
3	6	10	0.019	92	8	4	8	12	0.013	35	4
3	6	11	0.019	63	7	6	10	12	0.016	36	3
3	7	10	0.018	89	11	8	9	12	0.015	36	3
7	10	11	0.018	69	12	9	10	12	0.017	42	2
Quadratic Solutions											
<i>B</i> Spectra						<i>B</i> and <i>V</i> Spectra					
L1	L2	L3	$r_{av}$	$N_u^a$	$N_e^b$	L1	L2	L3	$r_{av}$	$N_u^a$	$N_e^b$
1	4	6	0.018	56	2	1	8	11	0.012	27	1
1	8	11	0.012	27	1	1	8	12	0.015	27	1
3	4	10	0.017	91	9	3	7	10	0.017	89	11
3	5	11	0.018	63	7	3	10	12	0.016	37	2
3	7	10	0.017	89	11	4	10	12	0.014	38	1
3	8	10	0.018	92	8	5	8	12	0.016	38	1
3	9	10	0.017	92	8	6	8	12	0.015	38	1
5	8	11	0.018	71	10	6	10	12	0.017	38	1
6	9	10	0.019	105	6	7	8	12	0.014	38	1
9	10	11	0.017	82	4	8	10	12	0.013	38	1

<sup>a</sup> $N_u$ : used stars; <sup>b</sup>  $N_e$ : eliminated stars.

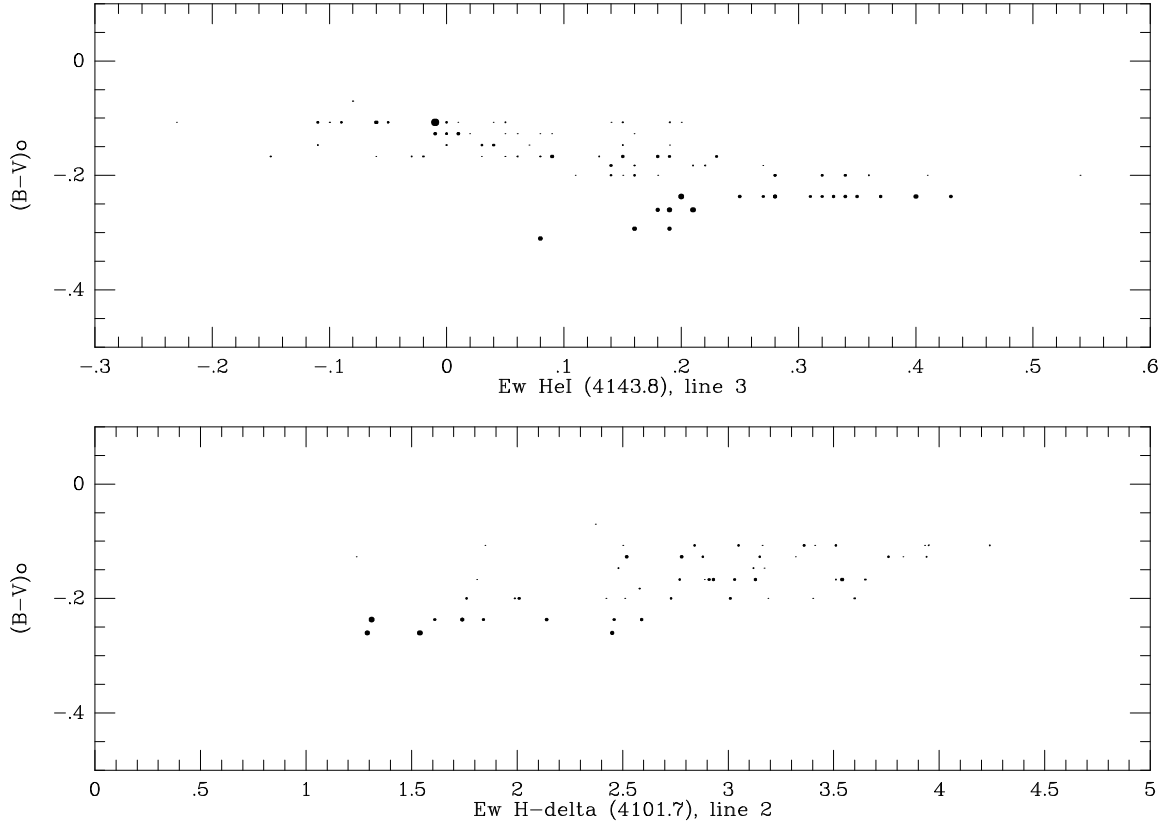


Fig. 4. Relation between the intrinsic colors  $(B - V)_0$  and the equivalent widths Ew for  $H\delta$  and HeI 4143Å.

TABLE 8

FREQUENCY  $N$  WITH WHICH LINES  $L$  WERE USED FOR THE BEST  $M_V$  SOLUTIONS

$M_V$ Solutions		$(B - V)_0$ Solutions	
L	$N$	L	$N$
1	8	1	8
2	13	2	3
3	7	3	18
4	23	4	5
5	2	5	6
6	4	6	9
7	4	7	6
8	6	8	16
9	26	9	5
10	6	10	20
11	22	11	10
12	14	12	17

culated. Outliers with residuals larger than  $2.5 r_{av}$  were eliminated and the solution was repeated. For both the two-line and the three-line solution those with the smallest  $r_{av}$  were selected. The respective combinations for two and three lines and their  $r_{av}$  value are given in Table 3 and Table 4. The full information with the corresponding coefficients is found in Tables 3a-3d and Tables 4a-4d, available at the CDS<sup>4</sup>. We also tried the linear dependence of the absolute magnitude on the equivalent widths of two or three lines. The respective data are also given in Tables 3, 3a-3d, 4, and 4a-4d. As may be seen in these tables, average residuals of 0.40 magnitudes can be obtained. For the best combination of the linear solutions with three lines, we also give the respective coefficients in Table 5.

The importance of the line 12 (HeI 5047.736) is evident. Since this line is present only in the  $V$  spectra, which are considerably less numerous than the  $B$  spectra, we have also calculated solutions based entirely on the 11 lines in the  $B$  spectra. The re-

<sup>4</sup><http://vizier.u-strasbg.fr/cgi-bin/VizieR?-source=J/other/RMxAA/38.127> or via ftp on <ftp://cdsarc.u-strasbg.fr/pub/cats/J/other/RMxAA/38.127>.

spective results are also given in Tables 3, 3a, 4, and 4a. Comparing the data for the solutions with 11 and with 12 lines, the importance of the inclusion of the  $V$  spectra is clearly demonstrated. For most of the stars on our list, comments are given in the Bright Star Catalogue, some actually rather extensive. We have looked at the comments for the outliers eliminated in the calculation of the coefficients. They do have comments, but these are shared with other stars that were not found to be outliers. It appears that we would need more stars if we were to determine whether the “outlier” condition can be predicted on the basis of measurements of the equivalent widths of absorption lines only. We should point out here that only two supergiants are included in our sample of stars, one a B8.Ia, the other a O9.5Ib. Both turned out to be outliers. Solutions based on three lines consistently gave better results than those based on two lines. The differences between the linear and the second order solutions, however, indicate no clear advantage of one or the other.

## 6. RESULTS FOR THE INTRINSIC COLORS

We have already indicated that equation (3) can readily be modified to be applied to the intrinsic colors as function of the equivalent widths of two or three lines in a linear or second-order polynomial. The data for the combinations which gave the smallest average residual are given in Tables 6, 7, and Tables 6a, 7a (the last two are available also at the CDS).

As may be seen in these tables, average residuals of 0.016 magnitudes can be obtained. Just as in the case of the absolute magnitudes, a significant improvement is obtained with the three-line dependence as compared to the two-line dependence. On the other hand, linear or second order solutions do not consistently favor one or the other. The lines 10 (He I 4921.929 Å) and 12 (He I 5047.736 Å) occur most frequently among the ten best solutions.

## 7. CONCLUSIONS

For the recovery of the physical stellar parameters  $M_V$  and  $(B - V)_0$  from the equivalent widths

of absorption lines we have tested linear and second order polynomials with two and with three lines as independent variables, making use of all possible line combinations. Significant improvement was obtained by switching from a two-line dependence to a three-line dependence, but no consistent improvement was found by switching from linear to second order polynomials for both the absolute magnitudes and the intrinsic colors. Table 8 shows how often the different lines were used in the best solutions for  $M_V$  and for  $(B - V)_0$  colors. Taking into account that line 12 only occurs in the  $V$  spectra, which are far less numerous than the  $B$  spectra, the lines 4, 9, 11, and 12 are the most important lines for the recovery of the absolute magnitudes. For intrinsic colors, again one has to allow for the fact that line 12 is far less observed than all the other lines. The lines 3, 8, 10, and 12 are the most important ones for the recovery of the intrinsic colors. The usefulness of the hydrogen lines  $H\beta$  (9) and  $H\gamma$  (4) has long been known. The sensitivity of the green helium lines 5015.7 (11) and 5047.7 (12) for classification purposes was not widely known because they are not available in the spectra used for MK classification.

This research has been partially supported by CONDES of the Universidad del Zulia and by CONICIT from Venezuela.

## REFERENCES

- Allen, C. W. 1973, *Astrophysical Quantities* (London: The Athlone Press, University of London)
- Jones, L. A. 1999, Ph.D. thesis, University of North Carolina
- Malyuto, V., Oestreicher, M., & Schmidt-Kaler, Th. 1997, *MNRAS*, 286, 500
- Malyuto, V., & Schmidt-Kaler, Th. 1999, in *ASP Conf. Ser. Vol. 167, Harmonizing Cosmic Distance Scales in a Post-Hipparcos Era*, eds. D. Egret & A. Heck (San Francisco: ASP), 271
- Moore, Ch. 1972, *Table of Multiplets* (Washington: Nat. Stand. Ref. Data Series., National Bureau of Standards)
- Stock, J., & Stock, M. J. 1999, *RevMexAA*, 35, 143

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