# A REVISED CALIBRATION OF THE $M_{V^{-}} W$ (O I 7774) RELATIONSHIP USING HIPPARCOS DATA: ITS APPLICATION TO CEPHEIDS AND EVOLVED STARS ${ }^{1}$ 

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Se ha calculado una nueva calibración de la correlación $M_{V}-W$ (O I 7774) usando mejores estimaciones de enrojecimientos y distancias de 27 estrellas calibradoras de tipos A-G, basadas en las paralajes y movimientos propios de los catálogos Hipparcos y Tycho. La nueva calibración predice magnitudes absolutas con precisiones de $\pm 0.38 \mathrm{mag}$ para una muestra que cubre el intervalo de $-9.5 \mathrm{a}+0.35 \mathrm{mag}$. El término de color, usado en calibraciones anteriores, no fue encontrado significativo y fue eliminado. Se estudiaron las variaciones del triplete O I 7774 a lo largo del ciclo de pulsación en la cefeida clásica SS Sct. Calculamos una corrección dependiente de la fase a la intensidad del triplete de OI obtenida en fase al azar en Cefeidas, para determinar su magnitud absoluta media a través de la relación $M_{V^{-}}$ $W$ (O I 7774). Después de aplicar esta corrección a una muestra de 31 Cefeidas, pudimos incrementar la lista de calibradoras a 58. La desviación estandar de la calibración, usando la muestra combinada, es comparable a la obtenida a partir de las 27 calibradoras primarias, lo que indica que el método permite obtener luminosidades medias de Cefeidas a partir de observaciones de OI 7774 obtenidas al azar. Las calibraciones fueron aplicadas a un grupo de estrellas evolucionadas para posicionarlas en el diagrama HR.


#### Abstract

A new calibration of the $M_{V}-W$ (O I 7774) relationship has been calculated using better reddening and distance estimates for a sample of 27 calibrator stars of spectral types A to G, based on accurate parallaxes and proper motions from the Hipparcos and Tycho catalogues. The present calibration predicts absolute magnitude with accuracies of $\pm 0.38 \mathrm{mag}$ for a sample covering a large range of $M_{V}$, from -9.5 to +0.35 mag. The color term included in a previous paper has been dropped since its inclusion does not lead to any significant improvement in the calibration. The variation of the O I 7774 feature in the classical cepheid SS Sct has been studied. We calculated a phase-dependent correction to random phase OI feature strengths in Cepheids, such that it predicts mean absolute magnitudes using the above calibration. After applying such a correction, we could increase the list of calibrators to 58 by adding $M_{V}$ and O I triplet strength data for 31 classical Cepheids. The standard error of the calibration using the composite sample was comparable to that obtained from the primary 27 calibrators, showing that it is possible to calculate mean Cepheid luminosities from random phase observations of the O I 7774 feature. We use our derived calibrations to estimate $M_{V}$ for a set of evolved objects to be able to locate their positions in the HR diagram.


## Key Words: CEPHEIDS - HERTZSPRUNG-RUSSELL DIAGRAM STARS: FUNDAMENTAL PARAMETERS - STARS: AGB AND POST-AGB

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## 1. INTRODUCTION

Positioning a star or a family of stars in the HR diagram is fundamental for understanding the structure and evolution of stars since it enables proper comparison with evolutionary tracks and computed isochrones. When supplemented by chemical composition data, one can obtain a much deeper insight into the evolutionary processes that the star might have undergone before arriving at its present stage. The temperature of the star can be estimated using increasingly accurate methods such as photometry, scanner observations, shapes of hydrogen and helium lines, from excitation equilibrium of species like $\mathrm{Fe}, \mathrm{Cr}, \mathrm{Ti}$, etc. On the other hand, estimating absolute magnitudes is not always so easy. For hot stars, the profiles of helium and hydrogen are employed whereas for cool stars Mg II lines at $2796.3 \AA$, Ca II H, and K lines at 3933 and $3967 \AA$, and the Mg I triplet in the 5167 to 5184 A region are found to be good indicators of luminosity. A summary of spectral features that are good indicators of spectral types and luminosity types can be found in Jaschek \& Jaschek (1987).

For stars of spectral type A to F the O I triplet at $\lambda \lambda 7771.954,7774.177$, and $7775.395 \AA$ is found to be a very good indicator of luminosity.

The sensitivity of the OI 7774 triplet to the stellar luminosity has been well known since Merril $(1925,1934)$ noted striking strength differences of the feature among supergiants and main sequence stars. Keenan \& Hynek (1950) studied the variation of the feature with spectral type and proposed the use of this feature as a luminosity indicator, noticing its large strength in A- and F-type stars. Osmer (1972) performed the first calibration of the OI 7774 triplet in terms of absolute magnitudes using a photolectric approach for 10 F-type supergiants. After his pioneering work, several calibrations of the feature were carried out spectroscopically and photometrically (e.g., Baker 1974; Sorvari 1974; Kameswara Rao \& Mallik 1978; Arellano Ferro et al. 1989; Arellano Ferro, Giridhar, \& Goswami 1991; Arellano Ferro \& Mendoza 1993; Mendoza \& Arellano Ferro 1993; Slowik \& Peterson 1993; 1995). A good compendium of the feature intensity across the HR diagram can be found in the work of Faraggiana et al. (1988). In the paper by Arellano Ferro et al. (1991) (Paper I) a calibration of the feature was made using high-resolution data and a discussion of the role of the resolution was given. It was demonstrated that the equivalent widths at low resolution can be overestimated by as much as $30 \%$ and that a better calibration of the $M_{V}-W$ (O I 7774) relation-
ship can be obtained at resolution $R \sim 18,000$ for the large $M_{V}$ range ( -10 to +2 mag ). We felt that a calibration covering a larger range in spectral type would have wider application. Though we have reduced calibration errors by measuring the $W$ (O I 7774) feature on high resolution data, the $M_{V}$ data on the calibrators (mostly members of clusters and associations) did not meet the required accuracy.

Fortunately, great improvement has been made in the last decade in the determination of distances and reddenings to the parent groups that contain the calibrator stars used in Paper I. With accurate parallaxes and proper motions from Hipparcos (ESA 1997), the data on open clusters, such as number of confirmed members, mean proper motions and parallaxes, has vastly improved (e.g., Baumgardt, Dettbarn, \& Wielen (2000); Tadross 2001). We therefore decided to redetermine $M_{V}$ for the calibrators and to calculate a new $M_{V}-W$ (O I 7774) relationship that would help in determining $M_{V}$ for field stars more accuarately. We describe in § 2 our observational material. In $\S 3$ and $\S 4$ the list of calibrators and their new $M_{V}$-values are presented and the new calibration is discussed. In $\S 5$ we discuss the behaviour of the O I 7774 feature in classical Cepheids and explore the possibility of using them as additional calibrators. In $\S 6$, we calculate the luminosities for a group of selected evolved stars, and their position in the HR diagram is given using our estimated luminosities. We summarize our findings in $\S 7$.

## 2. OBSERVATIONS

Most observations were carried out in August 2001 and January 2002, with the 2.1 m telescope of San Pedro Mártir Observatory (SPM), México, equipped with a Cassegran Echelle spectrograph and a CCD Site SI003 of $1024 \times 1024$ pixels. This instrument gives a resolution of $\sim 18,000$ at $7774 \AA$. Along with the stellar observations, bias frames and $\mathrm{He}-\mathrm{Ar}$ lamp spectra were obtainted to carry out background subtraction and wavelength calibration. All reductions were made using standard procedures and tasks contained in the IRAF package.

The observations of the Cepheid $\zeta$ Gem were obtained with the 1.0 m telescope of the Vainu Bappu Observatory at Kavalur, India. This telescope is equipped with a Coudé Echelle spectrograph giving a resolution of 18,000 . The spectra were recorded on a Thompson-CSF7H7882 CCD of $384 \times 576$ pixels.

## 3. THE CALIBRATOR STARS

We have chosen from the calibrator stars employed in Paper I (Table 1) only those objects that are observable from the latitude of SPM. We have retained in our present list of calibrators only those stars for which we have a new determination of $W\left(\right.$ O I 7774), and/or a new $M_{V}$, estimated as discussed below. Several calibrators are members of open clusters or OB associations as listed by Arellano Ferro \& Parrao (1990) (their Table 1). However, their absolute magnitudes have been recalculated as new proper motions studies can be used to confirm their membership (Baumgardt et al. 2000), and new distances and reddenings (Tandross 2001) are available for clusters. Accurate parallaxes and proper motions are now available even for some field stars (Hipparcos, ESA 1997). Therefore, the list of calibrators also contains some field stars for which new values of $M_{V}$ have been estimated.

### 3.1. The Calibrators in Clusters and Associations

The accurate parallaxes and proper motions given in the Hipparcos catalogue, when combined with their ground-based counterparts and with radial velocities of known cluster and associations members, can lead to much improved values of the mean proper motions and parallaxes for a large number of galactic clusters and associations. Baumgardt et al. (2000) have derived these quantities for 205 open clusters. These authors also give a list of confirmed and possible members of these clusters. Their work also indicates a downward revision in the distances by about $12 \%$ compared to the photometric estimates. This paper enabled us to further confirm the membership in clusters of the stars used in the present work and to re-evaluate $M_{V}$.

HD 7927 ( $\phi$ Cas) is a member of open cluster NGC 457. Tadross (2001) estimates $E(B-V)=0.5$ and a distance of 2851 pc for the cluster, leading to $M_{V}=-8.76$.

HD 9973 belongs to the association Cas OB1 with a distance modulus of 12.4 ; with an adopted $E(B-V)=0.54$ Oestreicher \& Schmidt-Kaler (1999) find $M_{V}=-7.36$.

HD 10494 is a member of open cluster NGC 654. Tadross (2001) estimates $E(B-V)=0.90$ and a distance of 2483 pc for NGC 654 , thus $M_{V}=-7.34$.
$H D 14433$ is a member of open cluster NGC 884 ( $\chi$ Persei) for which Tadross (2001) estimates $E(B-$ $V)=0.50$ and a distance of 2483 pc . Therefore we adopted $M_{V}=-7.08$.
$H D 14535$ is also a member of NGC 884 ( $\chi$ Persei). As for HD 14433, we estimated $M_{V}=$ -5.97 .

HD 17971 belongs to IC 1848 with distance modulus of 11.81 ; with an adopted $E(B-V)=0.76$ Oestreicher \& Schmidt-Kaler (1999) find $M_{V}=$ -6.58.

HD 18391 is listed as a possible member of h $\chi$ Per by Schmidt (1984). From the reddenings and $M_{V}$ compilation of Arellano Ferro \& Parrao (1990) we adopt $M_{V}=-6.6$.

HD 20902 ( $\alpha$ Per) is a member of the $\alpha$ Persei cluster. Using the distance and reddening to the cluster, Humpreys (1978) found $M_{V}=-4.7$. Using Hipparcos parallaxes Jaschek \& Gómez 1998 have calculated $M_{V}$ for some MK standards. For HD 20902 they give $M_{V}=-4.9$.

HD 31964 belongs the associations Aur OB1 (Stothers 1972). From the reddenings and $M_{V}$ compilation of Arellano Ferro \& Parrao (1990) we adopt $M_{V}=-8.7$.

HD 54605 belongs to Collinder 121 with distance modulus of 9.4 ; with an adopted $E(B-V)=0.12$ Oestereicher \& Schmidt-Kaler (1999) find $M_{V}=$ -7.97.

HD 62058 is a member of the open cluster NGC 2439. Tadross (2001) estimates $E(B-V)=$ 0.37 and a distance of 3669 pc for the cluster, leading to $M_{V}=-7.32$.
$H D 74180$ is a member of Vel OB1 association (Humpreys 1978). Two recent estimates of the distance are available: 1600 pc (Dambis, Melnik, \& Rostorguev 2001) and 1750 pc (Cameron Reed 2000). The latter author has also provided the value of the total to selective absortion ratio, $R=3.7$, for the association and $E(B-V)=0.478$ for HD 74180. This leads to $M_{V}=-9.1$ and $M_{V}=-8.9$, respectively. We adopted $M_{V}=-9.0$.
$H D{ }^{75276}$ is also a member of VelOB1 (Humpreys 1978). Following the steps taken for HD 74180 and adopting $E(B-V)=0.315$ (Cameron Reed 2000), it is found to give $M_{V}=-6.6$ and $M_{V}=-6.3$, respectively for the two distances. We adopted $M_{V}=-6.45$.
$H D 87283$ is a member of open cluster NGC 3114 (Schmidt 1984) for which Tadross (2001) estimated $E(B-V)=0.50$ and a distance of 2483 pc. Therefore we adopted $M_{V}=-4.01$.

HD 90772 is a member of open cluster IC 2581 (Lloyd Evans 1969). From the reddenings and $M_{V}$ compilation of Arellano Ferro \& Parrao (1990) we adopt $M_{V}=-8.3$.

HD 101947 belongs to Stock 14, (Schmidt 1984).

TABLE 1
CALIBRATOR STARS OF THE $M_{V}-W$ (O I 7774) RELATIONSHIP

| HD | Sp.T. | $W_{71}$ | $W_{74}$ | $M_{V}$ | $(b-y)_{0}$ |
| ---: | :--- | :--- | :--- | :--- | :--- |
| 7927 | F0 Ia | 0.855 | 2.221 | -8.76 | 0.111 |
| 9973 | F5 Iab | 0.541 | 1.399 | -7.36 | 0.225 |
| 10494 | F5 Ia | 0.612 | 1.578 | -7.34 | 0.215 |
| 14433 | A1 Ia | 0.661 | 1.641 | -7.08 | 0.013 |
| 14535 | A2 Ia | 0.496 | 1.198 | -5.97 | 0.124 |
| 17971 | F5 Ia | 0.538 | 1.404 | -6.58 | 0.247 |
| 18391 | G0 Ia | 0.611 | 1.438 | -6.6 | 0.949 |
| 20123 | G6 Ib | 0.124 | 0.279 | -2.0 | 0.645 |
| 20902 | F5 Ib | 0.374 | 1.020 | -4.9 | 0.274 |
| 31964 | F0 Ia | 0.969 | 2.316 | -8.7 | 0.035 |
| 36673 | F0 Ib | 0.449 | 1.249 | -5.1 | 0.116 |
| 48329 | G8 Ib | 0.087 | 0.182 | -1.0 | 0.816 |
| 54605 | F8 Ia | 0.691 | 1.750 | -7.97 | 0.355 |
| 62058 | G0 Ia | 0.579 | 1.485 | -7.32 | 0.981 |
| 62345 | G8 IIIa | 0.042 | 0.084 | +0.35 | 0.541 |
| 65228 | F7 II | 0.216 | 0.547 | -1.9 | 0.46 |
| 74180 | F0 Ia | $\ldots$ | $2.273^{\mathrm{a}}$ | -9.0 | 0.094 |
| 75276 | F2 Iab | $\ldots$ | $1.114^{\mathrm{a}}$ | -6.45 | 0.011 |
| 84441 | G1 II | 0.117 | 0.289 | -1.31 | 0.36 |
| 87283 | F0 II | $\ldots$ | $1.017^{\mathrm{a}}$ | -4.01 | 0.113 |
| 90772 | F0 Ia | $\ldots$ | $2.051^{\mathrm{a}}$ | -8.3 | 0.054 |
| 101947 | G0 Ia | $\ldots$ | $1.757^{\mathrm{a}}$ | -7.9 | 0.439 |
| 102070 | G8 IIIa | 0.040 | 0.1118 | -0.5 | $\ldots$ |
| 164136 | F2 II | 0.286 | 0.753 | -2.73 | 0.253 |
| 194093 | F8 Ib | 0.490 | 1.288 | -6.18 | 0.397 |
| 204867 | G0 Ib | 0.237 | 0.622 | -3.37 | 0.40 |
| 217476 | G0 Ia | 0.910 | 2.174 | -9.2 | 0.674 |

${ }^{\mathrm{a}} W_{74}$ values taken from Paper I.

From the reddenings and $M_{V}$ compilation of Arellano Ferro \& Parrao (1990) we adopt $M_{V}=-7.9$.

### 3.2. The Field Calibrators

We have included in the list of calibrators a few field stars whose $M_{V}$ values can be correctly estimated using Hipparcos data. Hipparcos parallaxes alone can be used to derive distances if the value of the parallax is larger than 5 times the error of the parallax value.

HD 20123. Using its Hipparcos parallax Wallerstein, Machado-Pelaez, \& Gonzalez (1999), have derived $M_{V}=-2.0$.
$H D 36673$ is a field star with $E(B-V)=0.02$. Oestreicher \& Schmidt-Kaler (1999) find $M_{V}=$ -5.1.

HD 48329. As in Paper I, we adopted $M_{V}=$ -1.0 from its parallax value.

HD 62345 . Using its Hipparcos parallax, Allende Prieto \& Lambert (1999) have derived $M_{V}=$ $+0.35 \pm 0.16$.

HD 65228 has a parallax of 6.49 mas, leading to a distance of 154 pc . With $E(B-V)=0.057$ given by Bersier (1996) one gets $M_{V}=-1.9$.
$H D 84441$ has a large parallax of 13.01 mas, indicating a distance of 76 pc . With this distace and $E(B-V)=-0.04$ from Arellano Ferro \& Parrao (1990) one gets $M_{V}=-1.31$.

HD 102070 has parallax of 9.31 mas, leading to a distance of 107 pc . With $E(b-y)=0.02$ or $E(B-$ $V)=0.025$ given in Paper I, we find $M_{V}=-0.5$.

HD 164136 has a parallax of 4.10 mas, leading to a distance of 467 pc . With $E(B-V)=0.07$ (Bersier 1996) then $M_{V}=-2.73$.

HD 194093 has a paralax of 2.14 mas, leading to a distance of 244 pc . With $E(B-V)=0.026$ (Bersier 1996) then $M_{V}=-6.18$.
$H D 204867$ has a parallax of 5.33 leading to a distance of 187.6 pc. With $E(B-V)=0.026$ (Bersier 1996) one finds $M_{V}=-3.37$.

HD 217476 is considered a hypergiant. Its $\log \left(L_{*} / L_{\odot}\right)$ is estimated to be 5.6 by de Jager (1998), which leads to $M_{V}=-9.2$.

In Table 1 we sumarize the newly calculated values of $M_{V}$ for each calibrator. We also tabulate in Table 1 the individual strength of OI at $7771.95 \AA$, along with the strength of 7774 blend comprising the three components. It should be noted that between the two O I lines at $7771.95 \AA$ and $7774.17 \AA$ features of $\mathrm{Fe} \mathrm{I}(7772.59 \AA)$ and $\mathrm{CN}(7772.9 \AA)$ are present. These lines are weak or non-existent in Atype stars but become prominent in stars of spectral type G or later. The contributions from these lines could cause overestimation of triplet strengths in relatively cooler stars. However, for O I $7771.95 \AA$, the blue side of the profile remains largely unaffected, hence a more accurate estimate of its strength can be made. At our resolution, O I $7771.954 \AA$ was distinctly separated from the rest of the blend. In an attempt to get a calibration with smaller dispersion and a possible extension of the calibration to the cooler stars we calculated two separate calibrations, one using only the first component at O I $7771.957 \AA$ (hereinafter called $W_{71}$ ), and the other, using the combined strength of three components (hereinafter called $W_{74}$ ). The $W_{74}$ value for those stars with a new estimate of $M_{V}$ but not observed in SPM has been adopted from Paper I-these cases are marked with an asterisk in Table 1 and plotted as circles in Figs. $1 b$ and $6 b$. It should be noted that the spectra used in Paper I were also of resolution very similar to that of SPM spectra. The $(b-y)_{0}$ values were calculated from Strömgren data and color excesses given in the literature (e.g., Arellano Ferro \& Parrao 1990 and Paper I).

## 4. THE $M_{V^{-}} W$ (O I 7774) CALIBRATION

Figure 1 shows the distribution of the calibrators in an equivalent width versus absolute magnitude diagram for $W_{71}$ and $W_{74}$. The solid lines are the least


Fig. 1. Calibration of the equivalent widths of O I 7771 ( $W_{71}$ ) and O I 7774 ( $W_{74}$ ) (blend of three components) in terms of the absolute magnitude $M_{V}$. The solid lines are represented by eqs. (1) and (2), which hold for the absolute magnitude range -9.5 to +0.35 mag. Open circles are the stars with adopted $W_{74}$ from Paper I.
square fits to the points and can be represented by the equations

$$
\begin{align*}
M_{V}= & (0.604 \pm 0.282)-(17.079 \pm 1.346) W_{71}  \tag{1}\\
& +(7.227 \pm 1.383) W_{71}^{2} \\
M_{V}= & (0.427 \pm 0.292)-(6.234 \pm 0.572) W_{74}  \tag{2}\\
& +(0.907 \pm 0.243) W_{74}^{2} .
\end{align*}
$$

The standard errors of the fits are 0.38 mag for both calibrations. Figure $1 b$ and eq. (2) can be compared with Fig. 3 and eq. (1) of Paper I,

$$
\begin{align*}
M_{V}= & (0.49 \pm 0.75)-(6.33 \pm 1.57) W_{74} \\
& +(0.85 \pm 0.68) W_{74}^{2}, \tag{3}
\end{align*}
$$

where the standard error of the fit was 1.5 mag.
It is evident from the standard deviations of the fit and the reduced standard errors of the coefficients that the new calibrations of eqs. (1) and (2) are better, by more than a factor of three, than the old calibration, which is undoubtedly due to the improved values of $M_{V}$ calculated from the Hipparcos data.


Fig. 2. Residuals in Fig. $1 a$ and Fig. $1 b$ in terms of the color $(b-y)_{0}$. As no trend is seen, it is evident that there is no significance of the color in the $M_{V}-W$ (O I 7774) relationship.

As in Paper I, we have investigated the effect of the temperature variation over the spectral types, represented by the color $(b-y)_{0}$, in the relationship. The equations including the color term are of the form

$$
\begin{equation*}
M_{V}=A-B W+C W^{2}-D(b-y)_{0} \tag{4}
\end{equation*}
$$

with $A=1.015 \pm 0.403, B=-17.961 \pm 1.489$, $C=7.924 \pm 1.468, D=-0.424 \pm 0.407$ for $W_{71}$, and $A=0.947 \pm 0.402, B=-6.544 \pm 0.603$, $C=0.986 \pm 0.246, D=-0.734 \pm 0.414$ for $W_{74}$. The standard deviations of the fit are 0.35 and 0.33 mag respectively, and, although a bit smaller than for eqs. (1) and (2), the color term does not appear to be significant. Further, the plots of the residuals versus $(b-y)_{0}$ show no trend with color, as demonstrated in Figure 2, thus the color term was dropped from the calibration. Eqs. (1) and (2) hold good for absolute magnitudes in the large range -9.5 to +0.35 mag and spectral types between A1 and G8.

In Fig. $7 a$ examples of the O I 7774 triplet are given for three calibrator stars. They illustrate the considerable range of variation of the O I 7774 strength between low- and high-luminosity stars.


Fig. 3. $W_{71}$ and $W_{74}$ curves of the Cepheid SS Sct. The phases were calculated using a period of $P=3.671280 \mathrm{~d}$ and epoch of maximum light HJD 2444398.419. The amplitudes of $W_{71}$ and M74 variations are 0.098 and $0.220 \AA$, respectively.


Fig. 4. $W_{71}$ curve of the Cepheid $\zeta$ Gem. The phases were calculated using a period of $P=10.15078 \mathrm{~d}$ and epoch of maximum light HJD 2444232.443. The amplitude of the $W_{74}$ variation is $0.078 \AA$.

## 5. THE BEHAVIOR OF THE O I 7774 FEATURE IN CLASSICAL CEPHEIDS

The $M_{V}-W$ (O I 7774) calibrations discussed so far had been obtained using A to G giant and supergiant calibrators with a large $M_{V}$ range of nearly 10 magnitudes. Therefore, they are expected to be also valid for classical Cepheids that are generally F-G supergaints. But it should be noted that calibrating $M_{V}-W$ (O I 7774) independently from Cepheids alone may not be possible due to their small range in $M_{V}$ ( -1 to -5 mag ) and variations in $W_{74}$ as a function of phase. On the other hand, Cepheid luminosities are believed to be well determined from the Period-Luminosity (P-L) relation, hence they can be used to enlarge the list of calibrators.

Since the luminosity of a Cepheid changes as the star pulsates, the values of $W_{71}$ and $W_{74}$ also change along the cycle, thus $W_{71}(\phi)$ and $W_{74}(\phi)$ at a given phase $\phi$ must be corrected to be brought to the mean

TABLE 2
O I 7774 DATA FOR CLASSICAL CEPHEIDS

| Name | $W_{71}(\phi)$ <br> (A) | $W_{74}(\phi)$ <br> (A) | $\begin{gathered} P \\ \text { (days) } \end{gathered}$ | $\begin{gathered} M_{V} \\ (\mathrm{mag}) \end{gathered}$ | $\phi$ | $W_{71,0}$ <br> (A) | $W_{74,0}$ <br> (A) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DT Cyg | 0.268 | 0.679 | 2.499035 | -2.548 | 0.45 | 0.278 | 0.702 |
| V532 Cyg | 0.320 | 0.826 | 3.283612 | -2.881 | 0.92 | 0.289 | 0.755 |
| SS Sct | 0.262 | 0.665 | 3.671280 | -3.017 | 0.44 | 0.280 | 0.718 |
| RT Aur | 0.187 | 0.447 | 3.728220 | -3.036 | 0.61 | 0.238 | 0.563 |
| SU Cyg | 0.205 | 0.485 | 3.845733 | -3.074 | 0.40 | 0.240 | 0.564 |
| CM Sct | 0.218 | 0.566 | 3.916977 | -3.096 | 0.60 | 0.257 | 0.653 |
| BQ Ser | 0.251 | 0.632 | 4.316700 | -3.215 | 0.74 | 0.247 | 0.621 |
| T Vul | 0.165 | 0.477 | 4.435532 | -3.248 | 0.65 | 0.247 | 0.722 |
| VZ Cyg | 0.259 | 0.687 | 4.864504 | -3.361 | 0.19 | 0.226 | 0.611 |
| V Lac | 0.289 | 0.716 | 4.983149 | -3.390 | 0.17 | 0.232 | 0.587 |
| AP Sgr | 0.198 | 0.542 | 5.057936 | -3.408 | 0.75 | 0.295 | 0.760 |
| V350 Sgr | 0.193 | 0.583 | 5.154557 | -3.431 | 0.51 | 0.234 | 0.741 |
| V386 Cyg | 0.338 | 0.946 | 5.257655 | -3.455 | 0.90 | 0.314 | 0.894 |
| $\delta$ Cep | 0.242 | 0.577 | 5.366316 | -3.480 | 0.32 | 0.252 | 0.633 |
| X Lac | 0.209 | 0.561 | 5.444990 | -3.498 | 0.26 | 0.208 | 0.557 |
| Y Sgr | 0.239 | 0.589 | 5.773400 | -3.570 | 0.34 | 0.238 | 0.588 |
| FM Aql | 0.314 | 0.838 | 6.114240 | -3.640 | 0.08 | 0.232 | 0.656 |
| X Vul | 0.219 | 0.655 | 6.319562 | -3.680 | 0.70 | 0.344 | 0.932 |
| RR Lac | 0.219 | 0.599 | 6.416190 | -3.698 | 0.75 | 0.295 | 0.771 |
| AW Per | 0.190 | 0.470 | 6.463589 | -3.707 | 0.70 | 0.267 | 0.645 |
| U Aql | 0.362 | 0.921 | 7.024100 | -3.809 | 0.91 | 0.322 | 0.830 |
| $\eta$ Aql | 0.228 | 0.519 | 7.176779 | -3.835 | 0.67 | 0.303 | 0.687 |
| V600 Aql | 0.298 | 0.471 | 7.238748 | -3.846 | 0.71 | 0.254 | 0.652 |
| V459 Cyg | 0.160 | 0.444 | 7.251250 | -3.848 | 0.71 | 0.231 | 0.602 |
| W Sgr | 0.210 | 0.532 | 7.595080 | -3.904 | 0.55 | 0.294 | 0.760 |
| U Vul | 0.343 | 0.930 | 7.990736 | -3.966 | 0.92 | 0.296 | 0.829 |
| S Sge | 0.270 | 0.680 | 8.382044 | -4.025 | 0.31 | 0.254 | 0.644 |
| YZ Sgr | 0.291 | 0.738 | 9.553606 | -4.184 | 0.63 | 0.305 | 0.769 |
| Y Sct | 0.306 | 0.822 | 10.341650 | -4.281 | 0.05 | 0.224 | 0.639 |
| TT Aql | 0.316 | 0.842 | 13.755290 | -4.629 | 0.92 | 0.287 | 0.777 |
| CD Cyg | 0.344 | 0.764 | 17.073967 | -4.893 | 0.85 | 0.420 | 0.935 |

values of $W_{71}$ and $W_{74}$. Knowing that the lines contributing to the O I triplet are luminositiy sensitive, it is reasonable to assume that the amplitudes of the $W_{71}$ and $W_{74}$ curves are proportional to the light curve amplitude in the visual $(V)$ band. Thus, the size of the equivalent width correction to be applied to bring it to the mean value depends on the phase of the observation and the $V$ amplitude. To properly estimate the scale between $V$ and $W_{74}$ and $W_{71}$ variation amplitudes, one should measure $W_{71}$ and $W_{74}$ at several phases for a group of Cepheids. However,
due to observing time limitations we were able to measure $W_{71}$ and $W_{74}$ only at a few phases in SS Sct. The variations are shown in Figure 3, where we have used the ephemeris $\phi_{i}=\left(t_{i}-2444398.419\right) / 3.671280$.

For a sample of Cepheids we can represent the light curve using the Fourier coefficients calculated by Arellano Ferro et al. (1998). The expression used is of the form,

$$
\begin{equation*}
V(\phi)=V_{0}+\sum_{k=1}^{n} A_{k} \cos \left(2 \pi k \phi+\Phi_{k}\right) \tag{5}
\end{equation*}
$$



Fig. 5. Distribution of Cepheids in the $W_{71}-M_{V}$ plane. The solid line is the calibration in Fig. $1 b$ obtained from the calibrators in Table 1. (a) Distribution of Cepheids before $W_{71}$ correction. (b) The small crosses are the Cepheid positions after their $W_{71}$ values were corrected according to pulsational phase; see text for details.


Fig. 6. Same as Fig. 4. for $W_{74}$. Open circles as in Fig. 1.
where $A_{k}$ and $\Phi_{k}$ are the amplitude and the displacement of each harmonic $k$. Thus we can calculate $\Delta V(\phi)=V(\phi)-V_{0}$. We now assume that $F_{74}=A_{V} / A_{W_{74}}$, i.e., the ratio of the $V$ amplitude to the $W_{74}$ variation amplitude $A_{W_{74}}$, is constant over the complete cycle, or

$$
\begin{equation*}
F_{74}=A_{V} / A_{W_{74}}=\Delta V(\phi) / \Delta W_{74}(\phi) \tag{6}
\end{equation*}
$$

where $\Delta \mathrm{W}_{74}(\phi)=\mathrm{W}_{74}(\phi)-\mathrm{W}_{74,0}$. Then the estimated mean value of $W_{74}$ would be given by

$$
\begin{equation*}
W_{74,0}=W_{74}(\phi)-\Delta V(\phi) / F_{74} \tag{7}
\end{equation*}
$$

which can be used to estimate the mean absolute magnitude for a Cepheid from eq. (2). Similar arguments hold for $W_{71}$ and eq. (1).

The estimated $F$-values for the reference star SS Sct are $F_{71}=4.388$ and $F_{74}=1.955$. The uncertainties of these values are proportional to the scatter of both $V$ and $\mathrm{W}(\mathrm{O}$ I) curves. In Figure 4 we have presented the $W_{71}$ curve for the Cepheid $\zeta$ Gem for which $W_{74}$ curve is very noisy. This star shows a large variation in temperature and during the cooler phase the contributions from the Fe I and CN features mentioned in $\S 3$ are large, hence the errors in $W_{74}$ also become large. For this star we calculate $F_{71}=6.01$ and it would therefore, produce similar corrections to those using SS Sct. For the corrections to the other Cepheids we have used SS Sct as a reference for both $W_{71}(\phi)$ and $W_{74}(\phi)$.

It should be noted that the above approach may have limitations since the amplitude scale factor calculated in this manner may not be applicable to Cepheids with highly asymmetrical light curves. However, this should be considered as a first effort with considerable room for improvement. We intend to carry out a similar calculation for a large sample of Cepheids.

This approach to random-phase correction in Cepheids is, nevertheless, presented here as a preliminary result and as a promising method to estimate the $M_{V}-W$ (O I 7774) from random-phase observations of Cepheids.

The above method has been applied to both $W_{71}$ and $W_{74}$ data of 31 Cepheids, listed in Table 2 along with their random phase $W_{71}(\phi), W_{74}(\phi), M_{V}$, and the mean values $W_{71,0}$ and $W_{74,0}$. The $M_{V}$ values were obtained from the Feast \& Catchpole (1997) $P-L$ relationship $M_{V}=-2.81 \log P-1.43$. This calibration is brighter than other solid calibrations (e.g., Sandage \& Tammann 1968; Feast \& Walker 1987; Madore \& Freedman 1991), but only at the level of $\sim 0.1 \mathrm{mag}$ (Sandage \& Tammann 1998; Madore \&


Fig. 7. (a) Three examples of calibrator stars from very low to very large luminosity. (b) Three examples of evolved stars. These stars known for having $\mathrm{H} \alpha$ in emission do not show emission in the OI 7774 feature. Other stars in Table 3 were inspected and no emission was found, thus their values $W_{71}$ and $W_{74}$ are reliable for the estimation of $M_{V}$. (c) The three upper spectra are from single Cepheids while the three examples in the bottom are of Cepheids known for having hot companions (see Table 2 of Evans (1995) for details). Numbers between parentheses are their periods in days. The presence of the companion does not seem to affect the nature of the OI 7774 in Cepheids.

Freedman 1998); thus, adopting a calibration of the $P$ - $L$ relationship with a slightly smaller zero point would have a very minor effect on our calculations and conclusions, especially considering that the uncertainty of our O I $7774-M_{V}$ calibration is of the order of 0.4 mag. The periods and epochs are usually known with large precision, the values in Table 2 were adopted from the sources listed in the paper by Arellano Ferro et al. (1998) in their Table 2. In the
present Table 2, we include the phase at which the OI observation was obtained, and on the basis of which the absolute magnitude is corrected.

Figure $5 a$ shows the distribution of uncorrected $W_{71}$ measurements in the $W_{71}-M_{V}$ plane. The solid curve is the calibration in eq. (1). While the Cepheids fall along the path defined by the nonCepheid calibrators, their dispersion is, as expected, unacceptably large as the phase effect is yet to be
corrected. In Fig. $5 b$ the corrected $W_{71}$ values for the sample of Cepheids are plotted along with the nonCepheid calibrators. The process has been repeated for $W_{74}$ and is shown in Figure 6. The present corrections of $W_{71}$ and $W_{74}$ in fact brought the Cepheids closer to the mean trend, and their dispersion is now comparable to that of cluster, associations and field calibrators. Once the Cepheids are included the calibrations take the forms

$$
\begin{align*}
M_{V}= & (0.260 \pm 0.281)-(15.889 \pm 1.387) W_{71} \\
& +(6.393 \pm 1.390) W_{71}^{2}  \tag{8}\\
M_{V}= & (0.131 \pm 0.287)-(5.831 \pm 0.563) W_{74} \\
& +(0.789 \pm 0.231) W_{74}^{2} \tag{9}
\end{align*}
$$

The standard deviations in $M_{V}$ are 0.42 and 0.43 mag, which are comparable to the calibrations in eqs. (1) and (2).

Many Cepheids have hot companions, generally late B-type main sequence stars, although contamination of the Cepheid O I 7774 feature is unlikely, since hot main sequence stars have very weak O I 7774 relative to supergiant stars (Faraggiana et al. 1988). We have compared in Figure $7 c$ the O I 7774 profiles of three Cepheids with B- or Atype main sequence companions (SU Cyg, AW Per, and S Sge) with those of presumably single Cepheids of similar periods (RT Aur, $\delta$ Cep, and U Vul). We do not see any peculiarity introduced by the companions, hence it is concluded that the O I 7774 feature in Cepheids is not affected by their companions.

## 6. ESTIMATED LUMINOSITIES FOR AGB CANDIDATE STARS

We felt it would be important to estimate the $M_{V}$ of possibly evolved objects from their $W_{71}$ and $W_{74}$ using the calibration derived in the present work. In a different program, in search of post-AGB stars, we had chosen a sample of A to G stars with high Galactic latitude and detected IR flux. Not all of them turned out to be objects showing very significant chemical peculiarities caused by evolutionary processes, but we were interested in determining their locations in the HR diagram and hence have used their $W_{71}, W_{74}$ to estimate $M_{V}$ for each. Their temperatures are those estimated using fine spectral analysis as described in Arellano Ferro et al. 2001. In the absence of such data we relied upon $u v b y \beta$ or 13 -color photometry calibrations, or on their spectral types. The stars under consideration are listed in Table 3 along with their $W_{71}$ and $W_{74}$ values. Also


Fig. 8. HR diagram with the positions of selected evolved stars (open circles) and five established post-AGB stars (dots). The evolutionary tracks are from Schaller et al. (1992) for $Z=0.2$ and $Y=0.30$.
reported in Table 3 is $\left\langle M_{V}\right\rangle$, the mean of the absolute magnitudes obtained from eqs. (1) and (2), which agree within 0.2 mag. The corresponding $\log L / L_{\odot}$ is also given in Table 3. At the bottom of the table, we have presented the data for five well-established post-AGB stars and their O I derived luminosities. Previous $M_{V}$ values are given between parentheses, and their sources are listed in Table 3.

The O I 7774 profiles of evolved stars in Table 3 were inspected to make sure that equivalent widths were not affected by emission often present in evolved or unusual stars. To demonstrate that this is not the case, in Figure $7 b$ three spectra, of stars HD 224014 ( $\rho$ Cas, G2 Ia0e), HD 163506 ( 89 Her, F2 Ibe) and HD 161796 (F3 Ib) are shown. These stars were selected for illustration since they are well known to show $\mathrm{H} \alpha$ in emission. Nevertheless, the OI 7774 feature appears to be in absorption. There could be weak underlying emission, but its effect, if any, would be negligible.

The positions of the stars on the HR diagram are shown in Figure 8. The evolutionary tracks for several masses of Schaller et al. (1992) for $Z=0.02$ and $Y=0.30$ are presented for reference. These models do not show blue loops for stars with $4 M_{\odot}$ and below, hence do not cross the instability strip. On the other hand, lower metallicity models $(Z=0.001)$ do

TABLE 3
O i 7774 DATA AND LUMINOSITIES OF SELECTED EVOLVED STARS AND ESTABLISHED POST-AGB STARS.

| HD | Sp.T. | $W_{71}$ | $W_{74}$ | $\left\langle M_{V}\right\rangle$ | $\log L / L_{\odot}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 725 | F5 Ib-II | 0.450 | 1.220 | -5.723 | 3.905 |
| 1457 | F0 Iab | 0.461 | 1.198 | -5.737 | 3.927 |
| 4266 | F2 Iab |  | 1.028 | -5.023 | 3.633 |
| 9167 | F1 II | 0.544 | 1.397 | -6.530 | 4.240 |
| 9233 | A4 Iab | 0.489 | 1.328 | -6.136 | 4.142 |
| 12533 | K3 IIb | 0.070 | 0.146 | -0.510 | 2.008 |
| 12545 | G5 | 0.148 | 0.345 | -1.691 | 2.316 |
| 15257 | F0 III |  | 0.655 | -3.267 | 2.939 |
| 15788 | G8 III | 0.048 | 0.129 | -0.281 | 1.772 |
| 27381 | F2 | 0.505 | 1.343 | -6.244 | 4.121 |
| 54605 | F8 Iab: | 0.691 | 1.750 | -7.726 | 4.710 |
| 55612 | F0 III/IV | 0.068 | 0.211 | -0.686 | 1.906 |
| 55661 | A7:V | 0.198 | 0.558 | -2.632 | 2.705 |
| 57321 | F2 II | 0.661 | 1.027 | -6.273 | 4.133 |
| 61227 | F0 Ib | 0.148 | 0.383 | -1.796 | 2.343 |
| 62058 | F8/G0Ia | 0.456 | 1.144 | -5.599 | 3.864 |
| 137569 | B5 III | 0.290 | 0.804 | -3.870 | 3.704 |
| 191635 | F0 | 0.234 | 0.610 | -3.018 | 2.839 |
| 194093 | F8 Iab: | 0.436 | 1.149 | -5.504 | 3.821 |
| 202240 | F0 III | 0.322 | 0.987 | -4.494 | 3.430 |
| 209747 | K4 III | 0.046 | 0.087 | -0.137 | 1.899 |
| 216756 | F5 II | 0.172 | 0.477 | -2.230 | 2.508 |
| 224014 | G2 Ia0e |  | 1.468 | -6.777 | 4.340 |
| 112374 | F3 Ia |  | 1.026 | $\begin{aligned} & -5.014 \\ & (-4.44) \end{aligned}$ | $3.630^{\text {a }}$ |
| 161796 | F3 Ib | 0.747 | 2.008 | $\begin{gathered} -8.278 \\ (-8.5)^{\mathrm{b}} \end{gathered}$ | 4.935 |
| 163506 | F2 Ibe | 0.634 | 1.663 | $\begin{gathered} -7.375 \\ (-8.1)^{\mathrm{b}} \end{gathered}$ | 4.574 |
| 172324 | B9 Ib | 0.440 | 1.153 | $\begin{aligned} & -5.533 \\ & (-5.98) \end{aligned}$ | $4.021^{\text {c }}$ |
| 172481 | F2 Ia0 | 0.458 | 1.121 | $\begin{aligned} & -5.562 \\ & (-6.44) \end{aligned}$ | $3.849^{\text {c }}$ |

${ }^{\mathrm{a}}$ Luck et al. (1983); $\quad{ }^{\mathrm{b}}$ Arellano Ferro \& Parrao (1990); ${ }^{\mathrm{c}}$ Arellano Ferro et al. (2001).
show blue loops for masses down to about $2-3 M_{\odot}$ (Schaller et al. 1992) and cross the lower part of the Cepheid instability strip. While such selection of metallicity would be adequate for old low mass stars, it would be inappropriate for Population I Cepheids. To produce longer blue loops at this low mass range, Alongi et al. (1991) have introduced an extra over-
shoot parameter that extends the outer convective envelope towards the interior. Post-RGB stars located on the HR diagram might serve as land-marks for theoretical work.

The stars from the Table 3 with $\log L / L_{\odot}$ in the range of 3.5 to 5.0 have certainly passed the red giant phase and populate the blue loop for masses between

7 and $10 M_{\odot}$. Blue loops for higher masses are not populated in part due to the specific sample considered, but also due to the fact that evolution in this region of the diagram is fast (Blöcker 1995). However, it is not possible to say if a given star would evolve to the left or to the right. Stars like HD 137569 and HD 172324 might be useful observational input that can be used to examine the extent of blue loops in this mass range.

The five established post-AGB stars, plotted as dots in Fig. 8, have been plotted according to their OI luminosities and their spectroscopically determined temperatures. Although the O I 7774 feature is sensitive to the luminosity, it is also partially sensitive to the oxygen abundance. The calibrations have been established using nearly solar abundance calibrators. Therefore, its application to highly evolved stars with peculiar oxygen abundances is a little uncertain, and hence might give luminosities with large error bars. The oxygen $[\mathrm{O} / \mathrm{H}]$ abundances for these five post-AGB are: -0.33 (HD 112374; Luck et al. 1983) +0.08 and -0.27 (HD 161796 and HD 163506; Luck, Lambert, \& Bond 1990), +0.41 and -0.58 (HD 172324 and 172481; Arellano Ferro et al. 2001); therefore, their positions on the HR diagram may have larger uncertainties. Mildly evolved stars like those given in the upper part of Table 3 have essentially solar $[\mathrm{O} / \mathrm{H}]$, hence the calibration certainly gives good $M_{V}$ estimates for them. For C-rich strongly evolved objects the relation might give low values of the luminosity.

Also, as a reference, the position of the instability strip has been indicated in Fig. 8. The upper strip is the classical Cepheid strip from Sandage \& Tammann (1969) and the lower strip comes from Marconi \& Palla (1998). Given the 0.4 mag uncertainty in $M_{V}$ produced by eqs. (1) and (2), we cannot be sure whether borderline cases are inside or outside the instability strip. It is worth, however, pointing at the variables sitting well inside the strip, HD 112374 whose variability was discovered by Arellano Ferro (1981), HD 62058 (R Pup) and HD 194093 (37 Cyg). Variable stars in the upper part of the diagram, like HD 54605 ( 25 CMa ), HD 163506 ( 89 Her ), HD 161796 (V814 Her), and HD 224014 ( $\rho \mathrm{Cas}$ ) lie in a region where the instability strip is ill-defined.

## 7. CONCLUSIONS

The newly estimated values of the absolute magnitude $M_{V}$ for a group of selected A to G supergiant calibrators enabled us to revise the $M_{V}-W$ (O I 7774)
relationship. The results show an improvement in $M_{V}$ predictions accuracies of at least a factor of three relative to the previous calibration from high resolution data in Paper I, over a large absolute magnitude range -9.5 to +0.35 mag . The calibrations presented in eqs. (1) and (2) are our final calibrations and they predict $M_{V}$ values with an accuracy of $\pm 0.38$ mag.

It is shown that the O I 7774 feature in Cepheids follows the basic calibration. A method to correct $W$ (O I 7774) obtained at random phases is described and succesfully applied to a sample of 31 Cepheids. At our resolution, the bluemost component of the triplet at $7771.954 \AA$ is resolved from the two redder components and, being unaffected by FeI and CN lines, may be more useful for G-type stars and later. We have calculated the calibrations for both the blue component $\left(W_{71}\right)$ and for the blend of the triplet ( $W_{74}$ ) including the Cepheids along with the primary calibrators. The calibrations for composite data, given by eqs. 8 and 9 , predict $M_{V}$ values within $\pm 0.42$ and $\pm 0.43 \mathrm{mag}$ and therefore, are comparable to the calibrations based solely on A to G non-variable supergiants. Hence, the present calibration with phase-corrected $W_{71}$ or $W_{74}$ shows that the OI 7774 feature in Cepheids is as sensitive to luminosity as in non-variable supergiants.

The new calibrations have been applied to a group of intermediate temperature, high Galactic latitude stars with detected IR fluxes that are considered good candidates to post-AGB stars. The luminosities determined by the present work not only help in ascertaining the evolutionary status of the sample of stars, but can also be used by theorists doing evolutionary calculations of the post-red-giant evolution, in order to establish the loci of the blue loops for stars of five solar masses and below.

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