Revista Mexicana de Astronomía y Astrofísica, **41**, 389–397 (2005)

TWO-BODY RELAXATION AND THE HEATING OF DISK GALAXY MODELS

Héctor Velázquez

Instituto de Astronomía,
Universidad Nacional Autónoma de México, Ensenada, B. C., México

Received 2003 June 26; accepted 2005 April 4

RESUMEN

Se ha realizado una serie de simulaciones de N-cuerpos de galaxias espirales para estudiar los efectos del calentamiento del disco como resultado de los encuentros de dos cuerpos. Se han cuantificado los coeficientes de difusión y los tiempos de relajamiento en las direcciones radial y vertical en un radio de referencia. Las partículas del disco juegan un papel marginal en el calentamiento vertical de disco, mientras que pueden contribuir de manera importante al calentamiento radial siempre que el halo no esté pobremente muestreado; de otra manera, el calentamiento radial y vertical estará dominado por encuentros entre partículas del disco y el halo. Además, se encuentra que el disco es más eficientemente calentado en la dirección radial que en la vertical a un radio dado. Finalmente, se ha derivado una expresión para estimar, *a priori*, el número de partículas del halo (dentro de un radio de referencia) requeridas para mantener el calentamiento del disco a un nivel deseado.

ABSTRACT

A set of N-body simulations of disk galaxies has been carried out to study the heating effects on the disk component, resulting from two-body encounters. We have quantified the diffusion coefficients and relaxation times in the radial and vertical directions at a given reference radius. Disk self-heating plays a marginal role in the vertical heating of the disk while it has an important contribution to the radial heating whenever the halo is not poorly sampled; otherwise, both the radial and vertical heating will be dominated by disk-halo particle encounters. Also, it is found that the disk is more efficiently heated in the radial direction than in the vertical one at a given radius. Finally, an expression has been derived to estimate, *a priori*, the number of halo particles (inside the reference radius) required to maintain the heating of the disc within a desired level.

Key Words: **GALAXIES: KINEMATICS AND DYNAMICS — METHODS: N-BODY SIMULATIONS — METHODS: NUMERICAL**

1. INTRODUCTION

Numerical simulations provide an efficient and robust technique to study and understand, in an approximate way, the dynamics and evolution of gravitating systems, in particular, those involving disk galaxies. Thus, N-body simulations are an invaluable tool needed to study the disruption and spiral-down of satellites onto spiral galaxies and their impact on the disk component (Huang & Carlberg 1997; Font et al. 2001). Although recently some analytical recipes have been proposed for this purpose, some of their parameters need to be adjusted through N-body simulations (Taylor & Babul 2001). Other examples are found in the context of galaxy harassment and tidal heating of disk galaxies in clusters (Moore, Lake, & Katz 1998; Gnedin 2003) bar formation in isolated disk galaxies and their secular evolution (e.g., Athanassoula 2002). To address these issues, it is common to represent disk galaxies by a finite number of softened point particles much smaller than the real number of stars. This discrete representation of the galaxy leads to numerical relaxation, resulting in an artificial heating of the stellar orbits. This relaxation arises from two-body encounters due to Poisson fluctuations of the granular representation of the potential. In the limit of a large number of particles these numerical artifacts would be reduced to a minimum, although this is sometimes computationally expensive. These computational limitations restrict the number of particles used in a numerical simulation.

The aim of the present paper is to asses the numerical relaxation in self-consistent N-body simulations of isolated disk galaxies. We focus our study on how this numerical heating is related to the number of particles. This is important since a galaxy model can be considered to evolve as a collisionless systems whenever the simulation time is shorter than the artificial relaxation timescale. In that case, the results obtained from the simulations should be reliable enough with respect to this numerical relaxation.

The rest of the paper has been organized as follows: in § 2 we describe the numerical models and tools used to carry out our study. A way to quantify the two-body relaxation timescale is given in section § 3. Section 4 contains the results of our numerical simulations and a general summary and conclusions are given in § 5.

2. NUMERICAL EXPERIMENTS

In this section, we briefly describe the galaxy models employed to study the disk heating inherent to their discrete representation. The algorithm used to evolve our galaxy models is also described.

2.1. *Galaxy Models*

Following Hernquist (1993), the disk galaxy models consist essentially of three components: a disk, a bulge and a halo. The disk component is well represented by the following density profile (Freeman 1970; van der Kruit & Searle 1981, 1982):

$$\rho_D(R,z) \;=\; \frac{M_D}{4\pi R_D^2 z_o}\exp(-R/R_D)\,\mathrm{sech}^2(z/z_o), \tag{1}$$

where M_D is the mass of the disk, R_D and z_o are the radial and vertical scale lengths, respectively. The vertical mass distribution of the disk follows an isothermal sheet (Spitzer 1942).

The central bulge component can be described by a spherical density profile defined as (Hernquist 1990)

$$\rho_B(r) \;=\; \frac{M_B}{2\pi}\frac{a}{r(a+r)^3}, \tag{2}$$

where M_B is the mass of the bulge and a is its scale length. The half-mass radius for this model is given

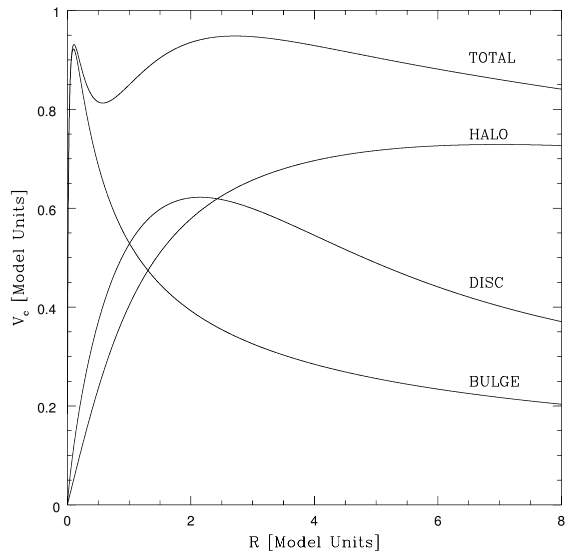

Fig. 1. Rotation curve for our galaxy model.

by $a(1+\sqrt{2})$. The bulge component has been introduced to suppress the bar instability (Sellwood & Moore 1999) so the disk heating is just due to collective phenomena, like spiral arms, and to two-body encounters among the particles.

The dark halo component is modeled by a spherical density profile (Hernquist 1993) which roughly matches a smoothly truncated isothermal halo

$$\rho_H(r) \;=\; \frac{M_H\alpha}{2\pi^{3/2}r_{cut}}\frac{\exp(-r^2/r_{cut}^2)}{r^2+\gamma^2}\,, \tag{3}$$

where M_H is the mass of the halo, γ and r_{cut} are the core and cut-off radii, respectively. Here, α is a normalization constant given by

$$\alpha \;=\; \{1-\sqrt{\pi}q\exp(q^2)[1-\mathrm{erf}(q)]\}^{-1}\,, \tag{4}$$

where $q = \gamma/r_{cut}$. Finally, the velocities for the galactic model are determined through the moments of the collisionless Boltzmann equation (e.g., Binney & Tremaine 1987).

2.2. *Numerical Tools*

A tree algorithm was adopted to evolve the simulations. The force between particles is computed with a tolerance parameter of $\theta_{\rm tol} = 0.8$ and with the quadrupole terms included (Barnes & Hut 1986; Hernquist 1987). These simulations were performed in a Beowulf cluster located at the Instituto de Astronomía, UNAM-Ensenada (Velázquez & Aguilar 2003).

TABLE 1

TWO-BODY RELAXATION EXPERIMENTS

Model	N_D	N_H	ϵ_H
R_1	20480	…	…
R_2	40960	…	…
R_3	81920	…	…
M_1	20480	18432	0.40
M_1a	20480	18432	0.40
M_2	20480	73728	0.40
M_3	20480	147456	0.40
M_4	40960	18432	0.40
M_5	20480	18432	0.05
M_5a	20480	18432	0.05

Notes to Table 1: N_D and N_H refer to the number of particles in the disk and halo components, respectively. ϵ_H is the softening length of halo particles. The lower case letter in first column indicates another random realization. Model simulations R_1 to R_3 use a rigid potential for the halo.

The computations were done assuming a dimensionless system of units such that $G = 1$, $M_D = 1$, and $R_D = 1$. This system can be easily scaled to a Milky Way-like galaxy where time and velocity units are 1.37×10^7 yrs and 262.6 km s^{-1}, respectively; assuming a disk mass of $5.6 \times 10^{10}\, M_\odot$ and a radial scale length of 3.5 kpc (Bahcall, Schmidt, & Soneira 1982). The half-mass radius of the disk is about $1.7\, R_D$ with a period at this radius of about 13 time units. For all our simulations the stability Q parameter was normalized to be 1.5 at $2.43\, R_D$. The value of the vertical scale length is taken to be $z_o = 0.2\, R_D$, and in all cases the halo mass, core and cut-off radii are $7\, M_D$, $\gamma = 1$, and $r_{cut} = 12$, respectively. The bulge is characterized by a mass of $1/3\, M_D$ and a half-mass radius of $0.24\, R_D$. A softening length of 0.05 has been adopted for bulge and disk particles. The simulations were evolved for 360 time units corresponding to about 5 Gyr in the system of physical units indicated above. Table 1 summarizes the rest of the parameters used for all our simulations and the rotation curve is indicated in Figure 1.

3. TWO-BODY RELAXATION TIMESCALE

The grainy nature of our N-body simulations has profound consequences to their dynamics and long-term evolution; the cumulative effects of two-body encounters induce orbital deflections and changes on energy of the particles (Chandrasekhar 1942). For spherical systems composed of equal-mass particles the relaxation time estimate is given by (Binney & Tremaine 1987),

$$\tau_{\rm rel} \;=\; \frac{N}{8 \ln N}\, \tau_{\rm cross}\,, \qquad (5)$$

where $\tau_{\rm cross} = R/V$ is the crossing time, R and V are the characteristic radius and velocity of the system (e.g., the half-mass radius of the system and the mean square velocity within such a radius). This analysis has been extended for a system of particles represented by the softened pair-wise potential $\phi_{ij} = -G m_i m_j / (r_{ij}^2 + \epsilon^2)^{1/2}$ (ϵ is the softening length) more commonly used in numerical simulations (Theis 1998). In this case, the relaxation time is found to be (White 1982; Farouki & Salpeter 1982, 1994; Huang, Dubinski, & Carlberg 1993)

$$\tau_{\rm rel} \;=\; \frac{N}{16 \ln (R/\epsilon)}\, \tau_{\rm cross}\,. \qquad (6)$$

However, given the axisymmetric nature of galaxy disks, they experience an anisotropic heating, in isolation, as a result of their spiral activity and two-body encounters.

The view has been adopted that the disk heating is a diffusion process in velocity space and the relaxation timescale can be estimated by the time evolution of the velocity dispersions of disk particles. In general, this process can be roughly modeled by the following equation (Lacey 1991)

$$\frac{d\sigma_i^2}{dt} \;=\; \frac{D_i(t)}{\sigma_i^n}\,, \qquad (7)$$

whose solution is reduced to the form

$$\sigma_i \;=\; (\sigma_{0,i}^{1/p} + D_i t)^p\,, \qquad (8)$$

in the case where the diffusion coefficient D_i and $\sigma_{0,i}$ are time-independent. Here, $i = R, z$ and $p = 1/(2+n)$. Observations suggest that $0.2 \lesssim p \lesssim 0.5$ where the upper value corresponds to the well-known velocity-age relation obtained by Wielen (1974) while the lower one fits the sample by Gómez et al. (1990). In particular, the value $p = 0.25$ corresponds to the heating of the disk component by giant molecular clouds (e.g., Lacey 1984; Villumsen 1985; Jenkins & Binney 1990), while $p = 0.5$ is more related to the heating of the disk by supermassive black holes (Lacey & Ostriker 1985, hereafter Lacey-Ostriker model; however, see Carlberg & Sellwood 1985). Figure 2 shows the radial and vertical disk heating measured at $2.43\, R_D$ for model M_1 in Table 1. Equation (8) was used to make the fits in this figure after the model has been relaxed

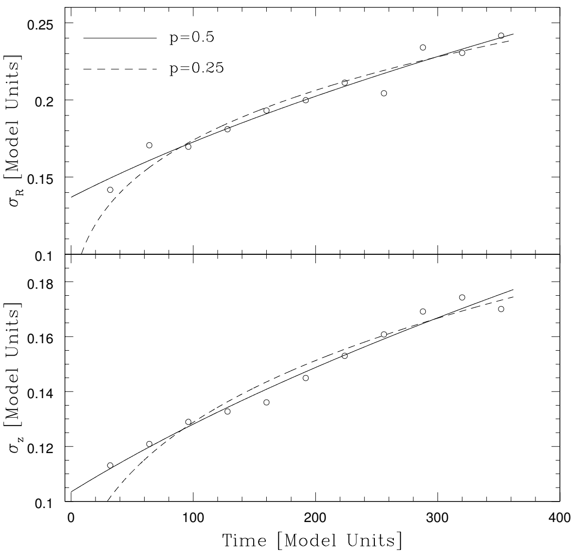

Fig. 2. Disk heating in the radial and vertical directions for model M_1 produced by numerical relaxation at a reference radius of $2.43\,R_D$. The solid line refers to a fitting as given by Eq. (8) with $p = 0.5$ and the dashed one corresponds to $p = 0.25$. Initially, models are allowed to relax so as to suppress any transients for about two and a half rotation periods at the disk half-mass radius.

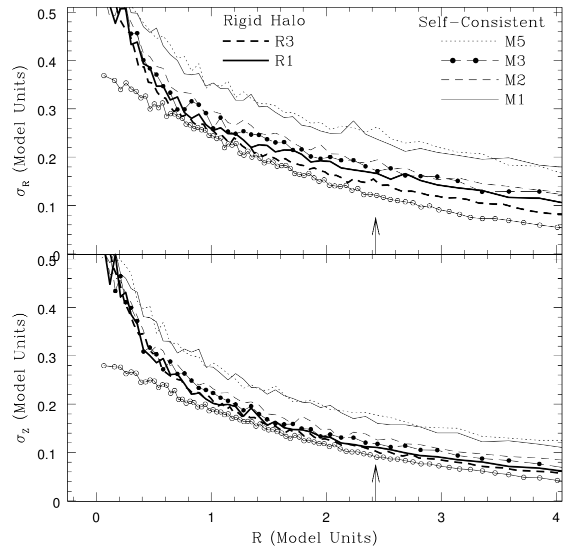

Fig. 3. Final velocity dispersion in the radial (upper panel) and vertical (lower panel) directions, respectively, at the end of each simulation. Lower solid lines with open circles refer to the initial state of the disk. Thick lines correspond to a galaxy model with a rigid halo and thin lines to self-consistent numerical galaxy models. The arrow indicates a reference radius $R = 2.43\,R_D$.

for about two rotation periods so as to suppress any transients arising from the way the initial conditions were set up. Solid lines correspond to a value of $p = 0.5$, the dashed one to $p = 0.25$. Notice that the fits are almost indistinguishable in the range of time showed for the radial heating; however, the differences between fits are more noticeable in the vertical direction. A simple Kolmogorov-Smirnov test (Press et al. 1992) indicates that a fitting with $p = 0.5$ provides a better representation of the data, resembling Lacey-Ostriker's model. Heating by other physical processes is nicely reviewed in Binney (2000).

4. RESULTS

4.1. *Heating Due to Disk Particles*

To ensure that the measured heating of the disk comes solely from disk particles, a rigid halo has been implemented for some of our numerical simulations (models R_1 to R_3 in Table 1). Figure 3 shows the numerical relaxation effect on both the radial, σ_R, and vertical, σ_z, velocity dispersions. Notice that in both cases a rigid halo (thick lines) significantly reduces the heating of the disk. As the number of disk particles is increased, it can be appreciated that the numerical heating is clearly lowered in the radial

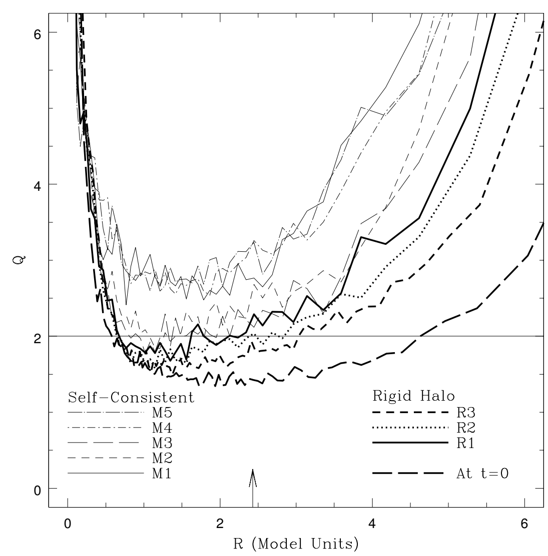

Fig. 4. Toomre's Q parameter for the disks at the end of each simulation. The lower thick line corresponds to the initial state of the disk. As in Fig. 3, thick solid lines indicate that a rigid potential has been employed and thin lines refer to self-consistent numerical simulations.

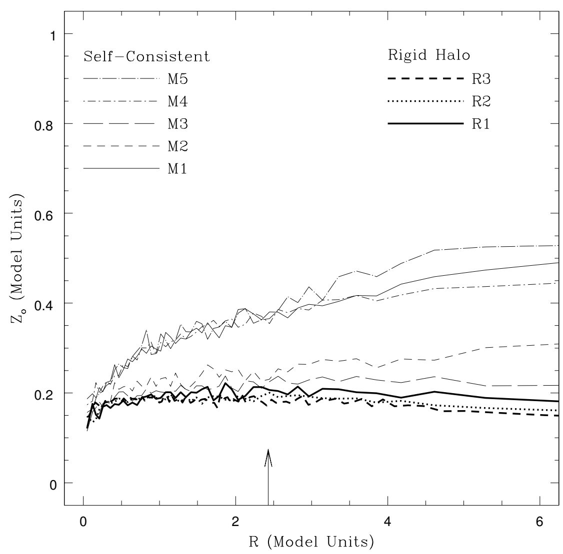

Fig. 5. The final vertical scale length of the disks, z_o, is plotted in this figure. Its initial value is $0.2\,R_D$ and is independent of R. As before, thick lines refer to a rigid potential for the halo. Clearly, disk-disk interactions do not play an important role in the vertical heating of the disk.

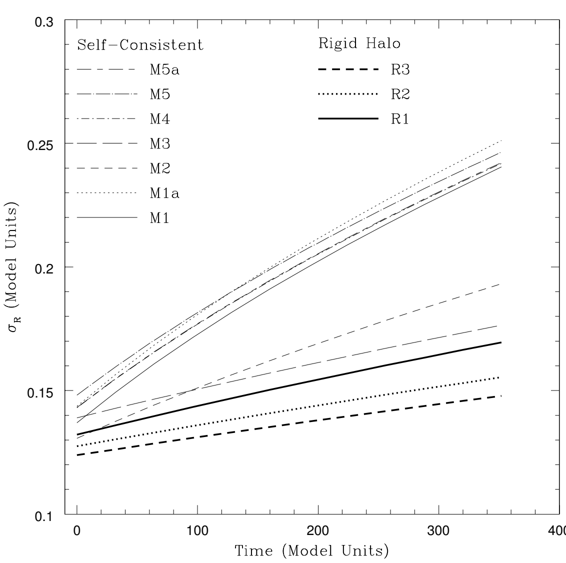

Fig. 6. Radial heating of the disk component for all the simulations built up. A value of $p = 1/2$ has been adopted which is consistent with Lacey-Ostriker's model. The radial velocity dispersions were measured at the reference radius $2.43\,R_D$. Thick lines correspond to the numerical simulations with a rigid potential and the thin ones to fully self-consistent galaxy models.

direction but not so much in the vertical one. This suggests that disk-disk particle encounters and collective phenomena associated to them, such as spiral arms, are responsible for some of the radial heating observed in self-consistent simulations (see below) but it seems that disk-disk interactions only play a marginal role in the vertical heating of the disk component.

4.2. *Heating Due to Disk-Halo Particle Encounters*

In Figure 4 the change induced in Toomre's Q parameter by the discrete representation of the halo can be appreciated. In this case, the thick long-dashed line corresponds to the initial state of the disk and the thin lines clearly show how sensitive this stability parameter is to disk-halo particle encounters. Also, it can be seen that an increase in the number of halo particles reduces the change of Q. Notice, however, that the stability Q parameter still increases its value even if the self-consistent halo is replaced by a rigid one (thick lines). This behavior is expected since the radial heating is related to the change of Toomre's parameter, $\Delta Q^2 \propto \sigma_R^2$. Here, it has been assumed that the radial surface density of the disk and the epicyclic frequency do not change, which it is roughly true since the radial surface density, at the end of the simulations, remains exponential with a radial scale length that changes at most by about 5%. Once the galaxy models reach a limit value of $Q = 2$, spiral activity ends, also, it can be observed from Fig. 4 that the change in Q is weakly dependent in the number of disk particles (Sellwood & Carberg 1984; Carlberg & Sellwood 1985). As of now, there is no complete satisfactory theory to explain the existence of spiral arms in disk galaxies (Sellwood 2000).

Figure 5 shows how the vertical structure of the disk, represented by its vertical scale length z_0, is affected as a result of numerical relaxation. Clearly, the vertical heating is quite sensitive to the number of particles in the halo component. An increase in the number of halo particles reduces significantly the vertical heating. Increasing the number of disk particles seems not to have a major effect on the vertical structure of the disk, as can be seen in the galaxy models with rigid halos (thick lines). This suggests that disk-disk encounters and any collective phenomenon involving disk particles, like spiral arms, do not play an important role in the vertical heating of the disk component, in agreement with the change shown by the vertical velocity dispersion, σ_z.

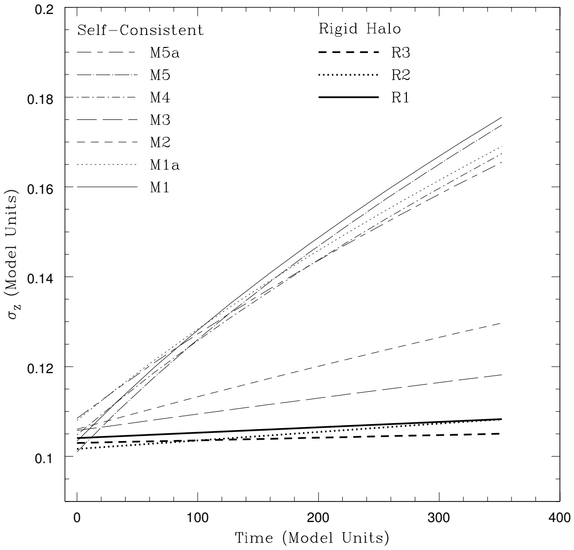

Fig. 7. Same as in Fig. 6 but for the vertical heating of the disk.

4.3. *Relaxation Timescale and Diffusion Coefficients*

In Figures 6 and 7 the relations $t-\sigma_R$ and $t-\sigma_z$, at the reference radius $2.43\,R_D$, have been plotted for all the simulations listed in Table 1. Again, the relation (8) has been used to fit data with $p = 1/2$.

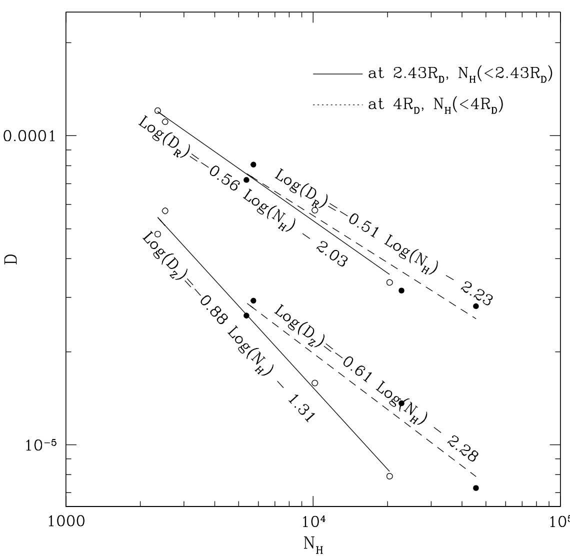

Fig. 8. Correlation between the diffusion coefficients with the halo number of particles at the radius indicated for models M_1, M_1a, M_2, and M_3.

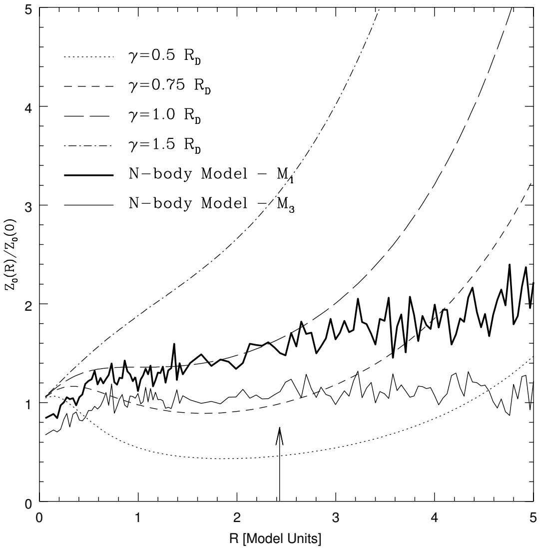

Fig. 9. Radial dependence of the vertical scale length of the disk, normalized to its value at the center, as predicted by Lacey & Ostriker's model for a pseudo-isothermal halo. γ represents the core of the halo and the reference position is indicated by an arrow. For comparison, the final vertical scale lengths of models M_1 (thick solid line) and M_3 (thin solid line) have been included.

It can be seen that for a halo poorly sampled the heating in both cases is mainly dominated by disk-halo encounters and in second place by disk-disk interactions. However, in the limit of a completely smooth potential, disk heating is diminished substantially and the heating in the radial direction is determined by the number of disk particles. Since disk heating is anisotropic and depends on radius, it has been characterized at two radii: at the reference radius, $2.43\,R_D$, and at $4\,R_D$. The measured diffusion coefficients and relaxation timescales for these regions have been summarized in Table 2. The relaxation time T_{D_i} has been defined as the time at which $\Delta\sigma_i^2 \simeq \sigma_{0,i}^2$, i.e.,

$$T_{D_i} = \frac{\sigma_{0,i}^2}{D_i}\,, \tag{9}$$

where $i = R, z$. As would be expected, two-body encounters heat the outer regions of the disk more easily because self-gravity is less important there and, hence, the relaxation time is shorter than near the center.

In Figure 8 a correlation between the diffusion coefficients, D_i, and the number of halo particles enclosed within a sphere of a given radius is observed.

Solid lines refer to the diffusion measured at the reference radius, $2.43\,R_D$, and dotted lines at a radius of $4\,R_D$. This relation can be used to estimate the number of halo particles required to reduce the heating of the disk to a given value as follows; the diffusion coefficients scale as

$$D_i(R) \,=\, \eta_i N_H^{-\alpha_i}(<R)\,, \qquad (10)$$

where $N_H(<R)$ is the number of halo particles inside a sphere of radius R and $D_i(R)$ is the diffusion coefficient at this radius. η_i and α_i are constants given by a specific fit. By substituting this expression in Eq. (8) with $p = 1/2$ and designating by δ_i the heating ratio $\Delta\sigma_i^2/\sigma_{0,i}^2$ we obtain the number of halo particles inside a radius R required to reduce the heating of the disk by an amount $\Delta\sigma_i^2$ during a time interval $t = T$,

$$N_H(<R) \,\simeq\, \left[\frac{\eta_i T}{\delta_i \sigma_{0,i}^2}\right]^{1/\alpha_i}. \qquad (11)$$

For instance, for a change of $\delta_R = 0.5$ at the reference radius during an interval of time of $T = 352$ (about 27 rotation periods at the half-mass radius of the disk), it is found from Fig. 8 that $\eta_R \sim 9.3 \times 10^{-3}$ and $\alpha_R \sim 0.56$ and hence, about $28/\sigma_{0,R}^{(2/\alpha_R)}$ halo particles will be needed inside the reference radius. The same amount of heating in the vertical direction will require about $57/\sigma_{0,z}^{(2/\alpha_z)}$ halo particles. Assuming reasonable values for $\sigma_{0,R}$ and $\sigma_{0,z}$ (see Fig. 3), it is found that to keep the vertical direction at the same amount of radial heating will require fewer halo particles by almost an order of magnitude.

Lacey-Ostriker model predicts a vertical scale length dependence on R. For a pseudo-isothermal halo (IS) this dependence translates into $z_o \propto \rho_H/\Sigma_D \propto \exp(R/R_D)/(R^2+\gamma^2)$ where γ is the core radius of the halo. Figure 9 shows this dependency of z_o, normalized to its value at the galaxy center for different core radii of the halo model. It is evident that a pseudo-isothermal halo needs to be concentrated ($\gamma \sim 0.75 - 1\,R_D$) in order to match a constant vertical scale length inside about $3.5\,R_D$; larger core radii exhibit a strong flaring of the disk which is in complete disagreement with observational data (van der Kruit & Searle 1982, 1983). Notice that the vertical scale length in model M_3 (thin solid line) remains almost constant by the end of the simulation, while model M_1 (thick solid line) presents a systematic increase with respect to its initial value and a modest flaring in the outer skirts of the disk.

Finally, Figure 10 shows the evolution of the velocity ellipsoid $\sigma_R : \sigma_\phi : \sigma_z$ at the reference radius.

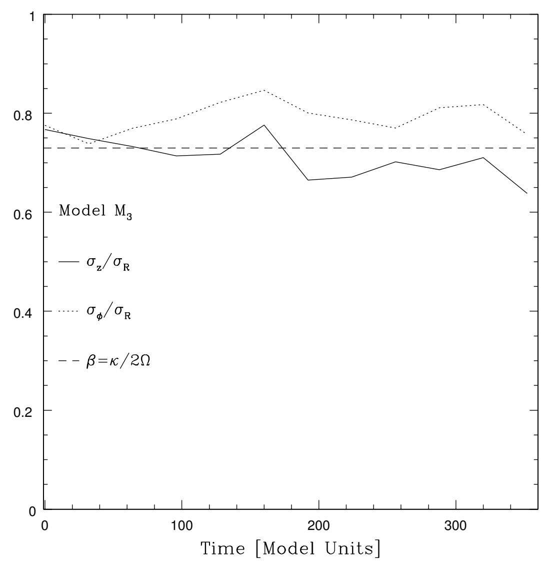

Fig. 10. Evolution of the velocity ellipsoid at the reference radius for model M_3. The value of β^{-1} indicated has been measured at the end of the simulation.

Notice that the ratio (σ_ϕ/σ_R) (dotted line) remains almost constant with a value of ~ 0.8 while the ratio (σ_z/σ_R) (solid line) shows a decreasing tendency resulting from the fact that the disk is more efficiently heated in the radial direction than in the vertical one. Also, in the epicyclic approximation and for a stellar population which is phase-mixed, it is predicted that $\sigma_\phi/\sigma_R = 1/\beta$ where $\beta = 2\Omega/\kappa$ (Lacey 1991). To check this, β^{-1} has been calculated at the end of the simulation and has been indicated by a dashed line. The agreement is fairly good.

5. SUMMARY AND DISCUSSION

Heating of the disk by two-body encounters poses a serious limitation to simulations involving realistic models of spiral galaxies. Despite of the complex results of the diffusion of stars in velocity space our simulations allow us to point to some general trends:

Disk particles play only a marginal role in the vertical heating of the disk, as is clearly demonstrated by our numerical simulations with a rigid halo. Thus, the time evolution of the vertical scale length of the disk is, mainly, dominated by disk-halo particle encounters.

Disk self-interaction has an important contribution to the radial heating of the disk component as is clearly seen in the change suffered by Toomre's Q parameter. However, if the halo is poorly sampled the radial heating will also be dominated by disk-halo particle encounters.

TABLE 2

DIFFUSION COEFFICIENTS AND RELAXATION TIMES ($P = 0.5$)[a]

	2.43 R				$4R_D$			
Model	T_{D_R} ($\times 10^3$)	D_R ($\times 10^{-5}$)	T_{D_Z} ($\times 10^3$)	D_Z ($\times 10^{-5}$)	T_{D_R} ($\times 10^3$)	D_R ($\times 10^{-5}$)	T_{D_Z} ($\times 10^3$)	D_Z ($\times 10^{-5}$)
R_1	0.55	3.20	4.20	0.26	0.25	2.06	1.08	0.28
R_2	0.73	2.24	2.62	0.39	0.23	1.60	0.99	0.27
R_3	0.83	1.85	8.70	0.12	0.39	0.95	1.67	0.16
M_1	0.17	11.10	0.19	5.71	0.05	8.06	0.09	2.93
M_1a	0.17	12.05	0.24	4.80	0.07	7.19	0.12	2.62
M_2	0.30	5.74	0.71	1.58	0.16	3.15	0.19	1.36
M_3	0.58	3.35	1.41	0.79	0.17	2.81	0.38	0.72
M_4	0.19	10.81	0.23	4.84	0.05	7.61	0.14	2.47
M_5	0.20	11.02	0.18	5.68	0.07	7.15	0.09	3.64
M_5a	0.19	10.85	0.26	4.44	0.05	7.65	0.10	3.09

[a] The diffusion coefficients and relaxation times, in the radial and vertical direction, computed at two radii: at 2.43 R_D and at $4R_D$. All quantities are in model units.

In general, the disk is more efficiently heated in the radial direction than in the vertical one (i.e., $D_z < D_R$ or $T_{D_z} > T_{D_R}$ at a given radius). This indicates that the survival of spiral arms in N-body simulations, in absence of any gas cooling contribution or any external agent as a satellite being accreted, requires a large number of particles to keep $Q < 2$. For instance, assuming that only σ_R is affected by these encounters (which is not completely true), then the change in Toomre's Q parameter is given by $\Delta Q^2/Q_0^2 \approx \Delta\sigma_R^2/\sigma_{0,R}^2$. If spiral arms should survive for around 30 rotation periods at the half-mass radius of the disk, then $\Delta\sigma_R^2 \approx 0.15\sigma_{0,R}^2$ at the reference radius. This change in σ_R^2 translates into a number of particles for the halo inside this radius of about 6×10^5, for $Q_0 = 1.5$ and $\sigma_{0,R} = 0.12$.

A useful expression has been derived to estimate the number of particles required for a simulation of a disk galaxy to keep the disk heating, $\Delta\sigma_{R,z}^2$, to a certain level at a reference radius of 2.43 R_D.

Lacey & Ostriker's model for a population of massive black holes for the halo is in fair agreement with the evolution of the dispersion velocities ($\sigma \propto t^{1/2}$). However, this model requires concentrated halos, so they should dominate the inner regions of the galaxy in order to produce a roughly constant vertical scale length within $\sim 3.5R_D$ (see Wielen et al. 1992). This last result is in conflict with current observed rotation curves of spiral galaxies (e.g., Persic, Salucci, & Stel (1996).

I thank Luis Aguilar and an anonymous referee for some helpful suggestions to improve the present work. This work was supported by CONACyT under grant 27678-E.

REFERENCES

Athanassoula, E. 2002, ApJ, 569, L83
Bahcall, J. N., Schmidt, M., & Soneira, R. M. 1982, ApJ, 258, L23
Barnes, J. E., & Hut, P. 1986, Nat., 324, 446
Binney, J. 2000, in ASP Conf. Ser., Vol. 230, Galaxy Disks and Disk Galaxies, eds. J. G. Funes & E. M. Corsini (San Francisco: ASP), 63
Binney, J., & Tremaine, S. 1987, in Galactic Dynamics (Princeton, NJ: Princeton University Press)
Carlberg, R. G., & Sellwood J. A. 1985, ApJ, 292, 79
Chandrasekhar, S. 1942, Principles of Stellar Dynamics (Chicago: Univ. Chicago Press)
Farouki, R. T., & Salpeter, E. E. 1982, ApJ, 253, 512
_________. 1994, ApJ, 427, 676
Font, A. S., Navarro, J. F., Stadel, J., & Quinn, T. 2001, ApJ, 563, L1
Freeman, K. C. 1970, ApJ, 160, 811
Gnedin, O. Y. 2003, ApJ, 582, 141
Gómez, A. E., Delhaye, J., Grenier, S., Jaschek, C., Arenou, F., & Jaschek, M. 1990, A&A, 236, 95
Hernquist, L. 1987, ApJS, 64, 715
_________. 1990, ApJ, 356, 359
_________. 1993, ApJS, 86, 389
Huang, S., & Carlberg, R. G. 1997, ApJ, 480, 503
Huang, S., Dubinski, J., & Carlberg, R. G. 1993, ApJ, 404, 73

Jenkins, A., & Binney, J. 1990, MNRAS, 245, 305
Lacey, C. G., 1984, MNRAS, 208, 687
__________. 1991, in Dynamics of Disc Galaxies, ed. B. Sundelius (Göteborg, Sweden), p. 257
Lacey, C. G., & Ostriker, J. P. 1985, ApJ, 299, 633
Moore, B., Lake, G., & Katz, N. 1998, ApJ, 495, 139
Persic, M., Salucci, P., & Stel, F. 1996, MNRAS, 281, 27
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, Numerical Recipes in Fortran (Cambridge: CUP)
Sellwood, J. A. 2000, Ap&SS, 272, 31
Sellwood, J. A., & Carlberg, R. G. 1984, ApJ, 282, 61
Sellwood, J. A., & Moore, E. M. 1999, ApJ, 510, 125
Spitzer, L. 1942, ApJ, 95, 329
Taylor, J. E., & Babul, A. 2001, ApJ, 559, 716
Theis, C. 1998, A&A, 330, 1180
van der Kruit, P. C., & Searle, L. 1981, A&A, 95, 105
__________. 1982, A&A, 110, 61
Velázquez, H., & Aguilar, L. 2003, RevMexAA, 39, 197
Villumsen, J. V. 1985, ApJ, 290, 75
White, S. D. M. 1982, in Dynamical Structure and Evolution of Stellar Systems, eds. L. Martinet & M. Mayor (SAAS-FEE, Geneva Observatory), p. 291
Wielen, R. 1974, Highlights of Astronomy, 3, 395
Wielen, R., Dettbarn, C., Fuchs, B., Jahreiβ, H., & Radons, G. 1992, in IAU Symp. 149, The Stellar Population of Galaxies, eds. B. Barbuy & A. Renzini (Dordrecht: Kluwer), p. 81

Héctor Velázquez: Instituto de Astronomía, UNAM, Apdo. Postal 877, 22860 Ensenada, B. C., México (hmv@astrosen.unam.mx).