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MHD STABILITY OF A SOLAR PROMINENCE EMBEDDED IN AN EXTERNAL VERTICAL MAGNETIC FIELD

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RESUMEN

Analizamos la estabilidad MHD lineal de una protuberancia solar descrita por el modelo bidimensional de Osherovich (1989). Este modelo de equilibrio considera explícitamente la influencia de un campo magnético vertical externo de tal forma que la protuberancia puede encontrarse en la frontera entre dos regiones de polaridad magnética opuesta. Estudiamos su estabilidad frente a perturbaciones en el marco de la MHD ideal usando un método numérico basado en el Principio de Energía de Bernstein et al. (1958). En nuestro análisis se consideran casos específicos de parámetros físicos observados en protuberancias estacionarias. Por completez presentamos igualmente resultados para otros parámetros que podrían ser aplicables tal vez a estrellas del tipo solar.

ABSTRACT

We examine the linear MHD-stability of a solar quiescent prominence described by the two-dimensional model of Osherovich (1989). This equilibrium model takes into account explicitly the influence of an external vertical magnetic field so that the prominence can be situated on the boundary between two regions of opposite magnetic polarity. We analyze its stability against perturbations within ideal MHD using a numerical method based on the Energy Principle of Bernstein et al. (1958). In our study specific cases of physical parameters observed in quiescent prominence are considered. For completeness we present as well results for other parameters which may perhaps be applicable to solar-type stars.

Key Words: **MHD: WAVES — SUN: OSCILLATIONS — SUN: PROMINENCES**

1. INTRODUCTION

The plasma structure known as quiescent prominence is one of the most stable phenomena on the solar atmosphere. They are usually located above the neutral line which separates polarities of the vertical component of the bipolar magnetic field between two sunspot regions. The prominence plasma is markedly denser and colder by several orders of magnitude than its coronal neighborhood. A magnetic field is thought to be the main cause of the thermal isolation of the prominence plasma from its hostile environment. For recent advances in the diagnostic of solar prominences partially obtained with SOHO, see Vial (2003). MHD- theoretical studies consider quiescent prominences to be in mechanical equilibrium resulting from the balance of Lorenz force, pressure gradients and the external force of gravity. Moreover, the appearance of the prominence resembling a very long vertical sheet allows one to assume a two-dimensional plasma configuration invariant in its longitudinal direction. A theoretical description of a quiescent prominence is strongly constrained, since the required model must necessarily fulfill a stability requirement when the equilibrium configuration is disturbed by small perturbations. Failure to satisfy this condition implies that a configuration is not achievable in nature, i.e., the magnetized plasma of the prominence could not be supported against gravity during the observed lifetime.

Quiescent prominences oscillate in a stable state as a consequence of disturbances which are excited by solar flares (see e.g., Bashkirtsev & Mashnich 1984; Wiehr, Stellmacher, & Balthasar 1984; Molowny-Horas et al. 1998; Oliver 1999; Ballester 2003; Wiehr 2004). The oscillations have a rather horizontal polarization (Kleczek & Kuperus 1969); they also represent an intrinsic characteristic of the prominence which must be explained by any realistic theoretical model of prominences. In general, as reviewed by Vial (1998), the typical periods of such oscillations can be classified into three categories: short periods (less than 5 min), intermediate periods (between 6 and 20 min) and long periods (between 40 and 90 min).

Two-dimensional MHD static equilibrium models of quiescent prominences have been proposed by numerous authors (e.g., Dungey 1953; Kippenhahn & Schlüter 1957; Lerche & Low 1980; Zweibel & Hundhausen 1982; Ballester & Priest 1987; Amari & Aly 1989; Oliver & Ballester 1996; Nagablushana 1998). On the other hand, Osherovich (1985) has constructed an analytical model for an isolated prominence under the influence of an external horizontal magnetic field. He takes into account configurations with continuous magnetic field and finite magnetic energy per unit length of prominence. Previous works have examined the stability of some prominence models (Brown 1958; Anzer 1969; Migliuolo 1982; Zweibel 1982) but they employed quite restricted classes of perturbations, or specific methods which are applicable only to a particular model. Galindo Trejo (1987, 1989, 1990, and 1998) has analyzed linear stability properties of several two-dimensional models. He determined the stability properties in the relevant parameter range of each model; furthermore, he showed that all models are able to describe adequately observed oscillations of the prominence in response to external perturbations originating in a solar flare. In contrast to this fact, excepting the Kippenhahn and Schlüter model, all other models become unstable for values of parameters outside of the observed range. Recently Costa, González, & Sicardi Schifino (2004) have developed a method based on non-equilibrium thermodynamics in order to analyze the stability properties of prominence models considered as dissipative states. In particular, they have found in case of the Kippenhahn and Schlüter model that the short period oscillations can be explained as internal modes of the prominence. Taking into account a particular displacement along the longitudinal axis of a relaxed prominence by heat losses, they showed that this model is not able to describe the intermediate period oscillations.

The purpose of this paper is to report results of our stability study of the prominence model of Osherovich (1989) which assumes that the prominence's body is embedded in an external vertical magnetic field. This model provides simultaneously an adequate description of the internal structure of a quiescent prominence and its magnetic interplay with the surrounding corona. We apply a two-dimensional formalism based on the Energy Principle of ideal MHD (Bernstein et al. 1958; Hain, Lüst, & Schlüter 1957) and we investigate under which circumstances such a magnetic configuration is really capable of stably supporting the mass distribution of a quiescent prominence.

2. MHD-EQUILIBRIUM AND STABILITY

The static equilibrium of a magnetized two-dimensional plasma, including uniform external gravity, is described in the frame of the MHD-theory by means of the non-linear elliptic equation (Low 1975; Schindler, Birn, & Janicke, 1983)

$$\begin{aligned} \Delta A &= -4\pi \frac{\partial \Pi(A,\phi)}{\partial A}, \\ \Pi(A,\phi) &= P(A,\phi) + \frac{B_x^2(A)}{8\pi}, \end{aligned} \tag{1}$$

where $\underline{A} = A(y,z)\underline{e}_x$ is the component of the vector potential in x-direction (which is the direction of alignment of the longitudinal axis of the prominence and x is the Cartesian coordinate which is ignorable); ϕ denotes the external potential: $\phi = gz$ and g is the constant gravitational acceleration. The choice for $\Pi(A,\phi)$ is constrained by the condition that the mass density $\rho = -\partial\Pi/\partial\phi = -\partial P/\partial\phi$ must be positive. Moreover, the model functions $P(A,\phi)$ and $B_x(A)$ must be consistent with the required boundary conditions. The magnetic field may be written as

$$\underline{B} = \nabla A(y,z) \times \underline{e}_x + B_x(A)\underline{e}_x \,. \tag{2}$$

A further assumption is that the prominence plasma is composed of ionized hydrogen obeying the equation of state of an ideal gas

$$P = \rho KT/m \,, \tag{3}$$

where K is the Boltzmann constant, T the temperature, and m the proton mass. Any two-dimensional equilibrium model is univocally determined by describing the functions $P(A,\phi)$, $B_x(A)$ and appropriate boundary conditions.

In order to inquire whether an equilibrium model may be applied to reality, we must first analyze its

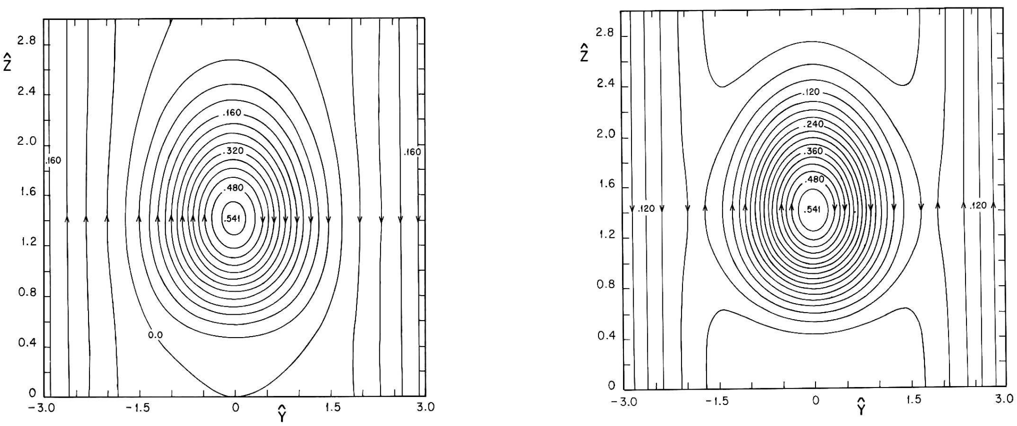

Fig. 1. Magnetic field lines for the ground-state configuration of Osherovich (1989)'s model: a) $\sigma = -5$, $\delta = 0.5$ b) $\sigma = 5$, $\delta = 5$. Numbers on the curves correspond to values of $\widehat{A}$.

linear stability. One of the most convenient procedures for this task is the Energy Principle of ideal MHD (Bernstein et al. 1958; Hain et al. 1957). According with this method, we will have stability if the change in potential energy δW, due to a small perturbation $\underline{\xi}$, is positive for all possible perturbations which satisfy the required boundary conditions. By contrast, if there exists a perturbation, which yields a negative δW, then the equilibrium is unstable. Therefore, the Energy Principle leads to a necessary and sufficient criterion for linear stability. We put into practice the search for stability by determining the sign of the minimum of δW. Due to the conservative character of ideal MHD and the two-dimensionality of the equilibrium, it is allowable to assume the most general three-dimensional complex perturbation displacement of the form

$$\begin{aligned} \underline{\xi}(\underline{r}, t) &= \underline{\xi}(\underline{r}) e^{i\omega t} \\ &= \left[\widehat{\xi}_x(y,z)\underline{e}_x + \widehat{\underline{\xi}}_\perp(y,z)\right] e^{i(kx+\omega t)} \\ &= \left[\widehat{\xi}_x(y,z)\underline{e}_x + \widehat{\xi}_y(y,z)\underline{e}_y + \right. \\ &+ \left. \widehat{\xi}_z(y,z)\underline{e}_z\right] e^{i(kx+\omega t)} . \end{aligned} \tag{4}$$

Here we have assumed periodic boundary conditions for $\underline{\xi}$ along the longitudinal axis of the prominence. Depending on the sign of the imaginary part of the frequency ω, the amplitude of $\widehat{\xi}$ can oscillate, decrease, or grow with time. The energy functional δW is given by

$$\delta W(\underline{\xi}, \underline{\xi}^*) = -\frac{1}{2}\int \underline{\xi}^* \cdot \underline{F}(\underline{\xi}) d^3r , \tag{5}$$

where $\underline{F}$ is the (Hermitian) force density operator defined by

$$\begin{aligned} \underline{F}(\underline{\xi}) &= \frac{1}{4\pi}(\nabla \times \underline{B}) \times \underline{Q} - \frac{1}{4}\underline{B} \times (\nabla \times \underline{Q}) + \\ &+ \nabla\left[\Gamma P \nabla \cdot \underline{\xi} + (\underline{\xi} \cdot \nabla) P\right] + \\ &+ \nabla \cdot (\rho\underline{\xi}) \nabla\phi . \end{aligned} \tag{6}$$

The integral is taken over the volume of the system; $Q = \nabla \times (\underline{\xi} \times \underline{B})$ and we have assumed an adiabatic energy law denoting by Γ the ratio of specific heats. An alternative form of the energy functional δW reads (Bernstein et al. 1958)

$$\begin{aligned} \delta W(\underline{\xi}, \underline{\xi}^*) &= \frac{1}{2}\int \left\{ \frac{1}{4\pi}|\underline{Q}|^2 - \frac{1}{4\pi}(\nabla \times \underline{B}) \cdot \right. \\ &\cdot (\underline{Q} \times \underline{\xi}^*) + \Gamma P |\nabla \cdot \underline{\xi}|^2 + (\underline{\xi} \cdot \nabla P)\nabla \cdot \\ &\cdot \left. \underline{\xi}^* - (\underline{\xi}^* \cdot \nabla\phi)\nabla \cdot (\rho\underline{\xi}) \right\} d^3r . \end{aligned} \tag{7}$$

In this equation we have considered rigid boundary conditions, i.e., $\underline{\xi}\ |_{\text{plasma boundary}} = 0$. Only in such a case, there is no constraint on the orientation of the equilibrium magnetic field. The minimization of δW is carried out in practice by additionally using a normalization constraint: $\frac{1}{2}\int \rho|\underline{\xi}|^2 d^3r = 1$.

Therefore, one has the following variational principle

$$\delta\left[\delta W(\underline{\xi}, \underline{\xi}^*) + \lambda\frac{1}{2}\int \rho|\underline{\xi}|^2 d^3r\right] = 0 , \tag{8}$$

where $\lambda = -\omega^2$ represents the associated Lagrange multiplier. On the other hand, the Euler-Lagrange equation for this variational principle is given by the eigenvalue equation

$$-\rho\omega^2 \underline{\xi}(\underline{r}) = \underline{F}(\underline{\xi}(\underline{r})) , \tag{9}$$

where $\underline{\xi}(\underline{r}) = \widehat{\underline{\xi}}(y,z)e^{ikx}$. It is straightforward to show that the minimum eigenvalue ω_1^2 equals the minimum of δW, i.e., $\omega_1^2 = \mathrm{Min}\ \delta W$. In the case of two-dimensional equilibria, the specialized energy functional δW becomes (Schindler et al. 1983)

$$
\begin{aligned}
\delta W \ =\ & \frac{1}{2}\int\left\{\frac{1}{4\pi}\left[|\nabla_\perp a|^2 - 4\pi\frac{\partial^2\Pi}{\partial A^2}|a|^2\right]+\right.\\
+\ & \frac{1}{4\pi}|\underline{B}_\perp\cdot\nabla_\perp\widehat{\xi}_x - B_x\nabla_\perp\cdot\widehat{\underline{\xi}}_\perp|^2 + \\
+\ & \frac{1}{4\pi}k^2|B_x\widehat{\underline{\xi}}_\perp - \widehat{\xi}_x\underline{B}_\perp|^2 + \\
+\ & \gamma P\left[1+\frac{\rho^2}{\gamma P(\partial\rho/\partial\phi)}\right]|\nabla\cdot\underline{\xi}|^2 - \\
-\ & \frac{\partial\rho}{\partial\phi}\left|\widehat{\underline{\xi}}_\perp\cdot\nabla_\perp\phi + \frac{\rho}{(\partial\rho/\partial\phi)}\nabla\cdot\underline{\xi}\right|^2 + \\
+\ & 2k\mathrm{Im}\left[\frac{1}{4\pi}(\nabla_\perp a\times\underline{e}_x)\cdot(B_x\widehat{\underline{\xi}}^*_\perp - \widehat{\xi}^*_x\underline{B}_\perp) - \right.\\
-\ & \frac{1}{4\pi}\Delta A\widehat{\xi}_x a^* - \frac{1}{8\pi}B_x\Delta A(\widehat{\underline{\xi}}_\perp\times\widehat{\underline{\xi}}^*_\perp)\cdot\\
\cdot\ & \left.\left.\underline{e}_x\right]\right\}d^3r\,,
\end{aligned}
\tag{10}
$$

where the current density is given by $J_x = -(e/4\pi)\Delta A$, $\nabla_\perp = \underline{e}_y\frac{\partial}{\partial y} + \underline{e}_z\frac{\partial}{\partial z}$, $\widehat{\underline{\xi}}_\perp = \widehat{\underline{\xi}}_y + \widehat{\underline{\xi}}_z$, $a = -\widehat{\underline{\xi}}_\perp\cdot\nabla A$, $\underline{B}_\perp = \nabla_\perp A\times\underline{e}_x$. The x-integration is to be carried out over one period. In order to determine the stability properties of a specific plasma equilibrium we need to introduce further in δW the model depending functions P and B_x and the corresponding solution A. Then we only need to carry out the minimization process to arrive at a stability statement.

3. STABILITY ANALYSIS OF THE OSHEROVICH (1989) MODEL

We now perform the stability analysis of the model of Osherovich (1989). Before presenting it, it is convenient to define a normalization for all variables

$$
\begin{aligned}
\widehat{\underline{r}} &= \underline{r}\kappa^{1/2}, \quad \widehat{A} = A/A_0,\\
\widehat{\underline{B}} &= \underline{B}/(A_0\kappa^{1/2}), \quad \widehat{P} = P/(A_0^2\kappa/8\pi)\,,
\end{aligned}
$$

$$
\begin{aligned}
\widehat{J}_x &= J_x/(cA_0\kappa/4\pi), \quad \widehat{T} = T/T_0,\\
\widehat{\phi} &= \phi/(g\kappa^{-1/2}), \quad \widehat{\rho} = \rho/(A_0^2\kappa^{3/2}/8\pi g)\,,
\end{aligned}
$$

$$
\begin{aligned}
\widehat{\underline{\xi}} &= \underline{\xi}\kappa^{1/2}, \quad \widehat{k} = k\kappa^{-1/2}, \widehat{\omega}^2 = \omega^2/(g\kappa^{1/2}),\\
\delta\widehat{W} &= \delta W/(A_0^2\kappa^{-1/2}/8\pi)\,,
\end{aligned}
$$

where $\underline{r} = x\underline{e}_x + y\underline{e}_y + z\underline{e}_z$ and A_0, T_0, $\kappa^{-1/2}$ are characteristic values of the vector potential, temperature and a characteristic length related to the width of the prominence.

This model considers a solar filament in a stratified atmosphere with a vertical magnetic field. Such an external magnetic field can redistribute the plasma density above the prominence, so that a magnetohydrostatic equilibrium is achieved. The described filament is on the boundary between two regions of opposite magnetic polarity. Using the chosen normalization, we may write the gas pressure and the longitudinal component of the magnetic field

$$
\begin{aligned}
\widehat{P} \ =\ & \widehat{P}_\infty - 2\widehat{z}^6\left[6+\widehat{z}^2(2\widehat{z}^2+2\widehat{y}^2-9)\right]\cdot\\
\cdot\ & e^{-2(\widehat{y}^2+\widehat{z}^2)} + \frac{1}{\sigma^2}\left[1-\mathrm{erf}^2(\delta\widehat{y})\right] + \\
+\ & \frac{4\widehat{z}^2}{\sigma}\left[(6-9\widehat{z}^2+2\widehat{z}^4)\widehat{I}(\widehat{y},\delta)e^{-\widehat{z}^2} - \right.\\
-\ & \left.\widehat{y}\widehat{z}^2\mathrm{erf}(\delta\widehat{y})e^{-(\widehat{y}^2+\widehat{z}^2)}\right]\,.
\end{aligned}
\tag{11}
$$

$$\widehat{B}_x = \mathrm{const}\,, \tag{12}$$

where $\delta = a^{1/2}/\kappa^{1/2}$ defines the relative width of the filament and the constant quantity $a^{1/2}$ characterizes the width of the transition region between the domains of opposite magnetic polarity; $\sigma = A_0\kappa^{1/2}/H$ is the relative strength of the filament's internal magnetic field and H is a constant denoting a typical value of the external vertical magnetic field. The function $\widehat{I}(\widehat{y},\delta)$ is given by the integral expression $\int_{\widehat{y}}^{\infty} e^{-t^2}\ \mathrm{erf}(\delta t)dt$ and the free function $\widehat{P}_\infty(\widehat{z})$ represents the gas pressure of the surrounding atmosphere. This equilibrium is the ground state solution in the eigenvalue approach of Osherovich (1989). The associated magnetic flux function is given by

$$
\begin{aligned}
\widehat{A}(\widehat{y},\widehat{z}) \ =\ & \widehat{z}^4e^{-(\widehat{y}^2+\widehat{z}^2)} + \frac{1}{\sigma}\widehat{y}\ \mathrm{erf}(\delta\widehat{y}) + \\
+\ & \frac{1}{\sqrt{\pi}\delta\sigma}e^{-\delta^2\widehat{y}^2}\,.
\end{aligned}
\tag{13}
$$

Furthermore, the components of the magnetic field in the $\widehat{y},\widehat{z}$–plane are

$$\widehat{B}_y = 2\widehat{z}^3(2-\widehat{z}^2)e^{-(\widehat{y}^2+\widehat{z}^2)}\,, \tag{14}$$

$$\widehat{B}_y = 2\widehat{y}\widehat{z}^4e^{-(\widehat{y}^2+\widehat{z}^2)} - \frac{1}{\sigma}\mathrm{erf}(\delta\widehat{y})\,. \tag{15}$$

This solution represents a closed configuration with one magnetic axis along the x–direction. In

TABLE 1

STABILITY PROPERTIES OF A QUIESCENT PROMINENCE FOR OSHEROVICH (1989) MODEL[a]

$\widehat{k}$	$\widehat{Y}$	$\widehat{Z}$	δ	σ	$\widehat{B}_x$	$\widehat{\omega}_1^2$	ν_1 ($10^{-3}\mathrm{s}^{-1}$)	$P(s)$	P (s)	P (min)
0.314	5.	5.	0.01	−1.	5.	0.41879	1.70487	586.55	9.77	
0.314	5.	5.	0.001	0.01	5.	0.46084	1.78842	559.15	9.32	
0.314	5.	5.	0.001	0.01	1.	0.03337	0.48126	2077.87	34.63	
0.078	5.	5.	0.001	0.01	1.5	0.00754	0.22888	369.06	72.81	
0.314	5.	5.	0.001	−0.001	1.	0.47327	1.81238	548.30	9.13	
0.078	5.	5.	0.001	−0.001	1.	0.48249	1.82995	546.46	9.10	
0.314	5.	5.	0.01	−1.	2.	0.07638	0.72809	1373.45	22.89	
0.314	5.	5.	0.01	−1.	1.	0.01957	0.36854	2713.37	45.22	
0.314	5.	5.	0.01	−1.	3.	0.16759	1.07849	927.21	15.45	
0.314	5.	5.	0.01	−1.	4.	0.28254	1.40034	714.10	11.90	
0.078	5.	5.	0.001	0.01	5.	0.03513	0.49383	2024.98	33.75	
0.1	5.	5.	0.001	0.01	5.	0.05382	0.60728	1646.66	27.44	
0.15	5.	5.	0.001	0.01	5.	0.11210	0.88206	1133.70	18.89	
0.175	5.	5.	0.001	0.01	5.	0.14932	1.01801	982.30	16.37	
0.2	5.	5.	0.001	0.01	5.	0.19328	1.15821	863.39	14.39	
0.25	5.	5.	0.001	0.01	5.	0.29530	1.43161	698.51	11.64	
0.3	5.	5.	0.001	0.01	5.	0.42339	1.71421	583.35	9.72	

[a]Considering observed parameters. Frequency $\nu_1 = (5g/Y)^{1/2}\widehat{\omega}_1/2\pi$ and period P of stable oscillations.

case of a negative σ the polarities of both (internal and external) magnetic fields are the same. However, for positive σ two x–type neutral points are created.

In order to attain a better understanding of the stability states of Osherovich's model we will undertake a numerical evaluation of the Energy Principle. For this purpose we examine the minimum eigenvalue $\widehat{\omega}_1^2$ and its associated minimizing displacement $\underline{\widehat{\xi}}_{\rm min}$. In addition, we will separate single energy contributions to the functional $\delta\widehat{W}_{\rm min}$, namely

$$\delta\widehat{W}_{\rm min} = \delta M + \delta K + \delta G = \widehat{\omega}_1^2\,, \qquad (16)$$

where δM, δK, and δG denote the contributions of the Lorenz force, pressure force and gravitational force to the total potential energy resulting from the minimizing displacement $\underline{\widehat{\xi}}_{\rm min}$, respectively (Galindo Trejo 1987 gives the explicit expressions for these contributions). Numerical minimization of $\delta\widehat{W}$ with respect to general displacements was carried out by the variational method of finite elements (see for example, Zienkiewicz 1977). With the assumed normalization constraint the minimization of the result-

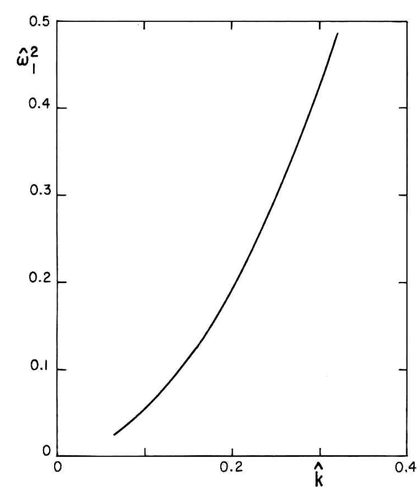

Fig. 2. Smallest eigenvalue $\widehat{\omega}_1^2$ as a function of the wave number $\widehat{k}$ along the prominence's axis. Observed parameters: $\widehat{Y} = \widehat{Z} = 5$, $\delta = 0.001$, $\widehat{B}_x = 5$ and $\sigma = 0.01$.

ing quadratic form in the discretized partial displacements, one obtains a generalized matrix eigenvalue

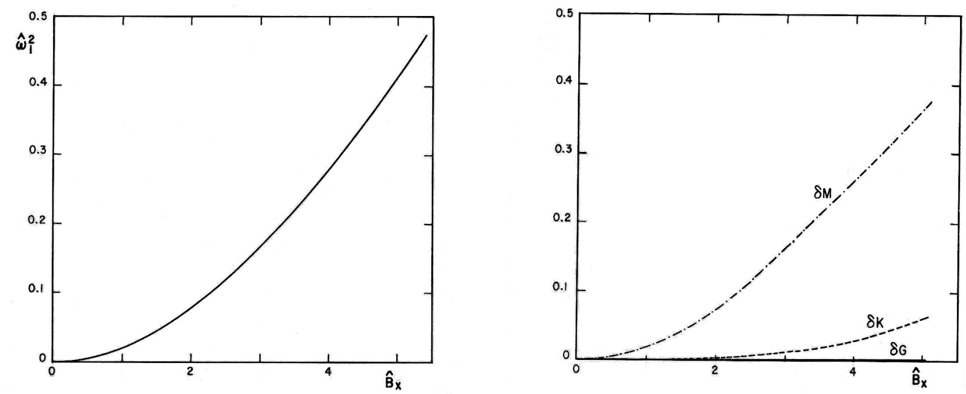

Fig. 3. a) Smallest eigenvalue $\widehat{\omega}_1^2$, b) The (compressional) δK, the (electromagnetic) δM and the (gravitational) δG contributions to the perturbed potential energy as a function of the axial magnetic field $\widehat{B}_x$ for observed parameters, $\widehat{k} = 0.314$, $\widehat{Y} = \widehat{Z} = 5$, $\delta = 0.01$ and $\sigma = -1$.

problem

$$\tilde{A}\chi = \lambda \tilde{B}\chi \,, \tag{17}$$

where $\tilde{A}$ and $\tilde{B}$ are symmetric matrices. Moreover $\tilde{B}$ is a positive definite matrix and the vector χ consists of the values of the displacements for a set of predetermined points on the $\widehat{y}, \widehat{z}$–plane. The resulting computer code was extensively tested by applying it to simple two-dimensional equilibria (e.g., Alfvén waves in a homogeneous cold plasma, plane current sheets, and acoustic gravity waves) whose dynamic behavior can be determined by analytic methods (see Galindo Trejo 1987).

We first present our results concerning the parameter range, observed in solar quiescent prominences. We then give stability results for other parameter ranges, eventually valid for other stellar atmospheres. We take $\Gamma = 5/3$ which describes an ideal monoatomic gas. We assume typical dimensions of a quiescent prominence, i.e., width $Y = 5 \times 10^3$ km, height $Z = 1.5 - 5 \times 10^4$ km and length $X = 10^5$ km. Observations of periodic structures in quiescent prominences suggest that the wave number k along the longitudinal axis falls within the range 0.78×10^{-4} km^{-1} $\leq k \leq 3.1 \times 10^{-4}$ km^{-1} (Nakagawa & McKim Malville 1969). The average magnetic field obtained by the Hanle depolarization is $B = 5G$ (Leroy 1978; Leroy, Bommier, & Sahal-Bréchot 1984; Wiehr & Bianda 2003). The acceleration in the corona due to gravity is $g = 2.74 \times 10^4$ cm s^{-2}.

Recent measurements of magnetic fields in quiescent prominences (Leroy, Bommier, & Sahal-Bréchet 1983; Leroy et al. 1984; Nikolskii et al. 1985; Ballester 2003) suggest that the horizontal magnetic field intersects the longitudinal axis of a prominence at small angles α_p. According to the prominence

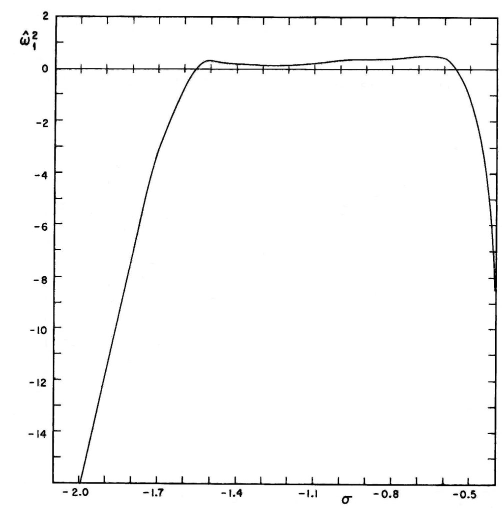

Fig. 4. Smallest eigenvalue $\widehat{\omega}_1^2$ as a function of the relative strength of the filament's internal magnetic field σ. Observed parameters: $\widehat{k} = 0.314$, $\widehat{Y} = \widehat{Z} = 5$, $\delta = 0.01$ and $\widehat{B}_x = 5$.

class, the mean value of $\alpha_p = \arctan(B_y/B_x)$ ranges from 15° to 25°. For the Osherovich's model, we study the interval $10° \leq \alpha_p \leq 25°$. If we consider the maximum values of B_y as given by (14), the longitudinal component of the magnetic field will vary in the range: $1 \leq \widehat{B}_x \leq 5$. To take into account in our analysis transition regions where internal and external magnetic fields interact, we choose as normalization width $\kappa^{-1/2} = Y/5$. Thus, the $\widehat{k}$–interval becomes: $0.078 \leq \widehat{k} \leq 0.314$ and the considered prominence has $\widehat{Y} = 5$. As shown in Figure 1, basic characteristics of the magnetic configuration are contained in a prominence region with $\widehat{Z} = 5$. We have as-

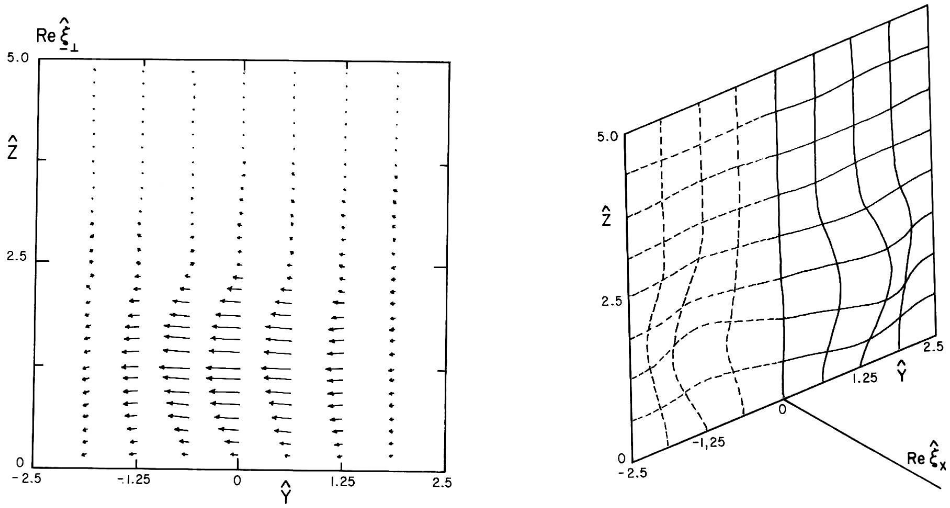

Fig. 5. Minimizing mode for observed parameters: $\widehat{k} = 0.314$, $\widehat{Y} = \widehat{Z} = 5$, $\delta = 0.001$, $\sigma = -0.001$, $\widehat{B}_x = 1$; $\widehat{\omega}_1^2 = 0.47327$. a) Re$\widehat{\underline{\xi}}_{\perp}$; b) Re$\widehat{\xi}_x$; $|\max \mathrm{Re}\widehat{\xi}_x|/|\max \mathrm{Re}\widehat{\xi}_y| \approx 12$; $|\max \mathrm{Re}\widehat{\xi}_x|/|\max \mathrm{Re}\widehat{\xi}_z| \approx 851$; ____ Re$\widehat{\xi}_x > 0$, – – – – Re$\widehat{\xi}_x < 0$ (this convention to visualize the longitudinal component of the mode will be used in the figures that follow).

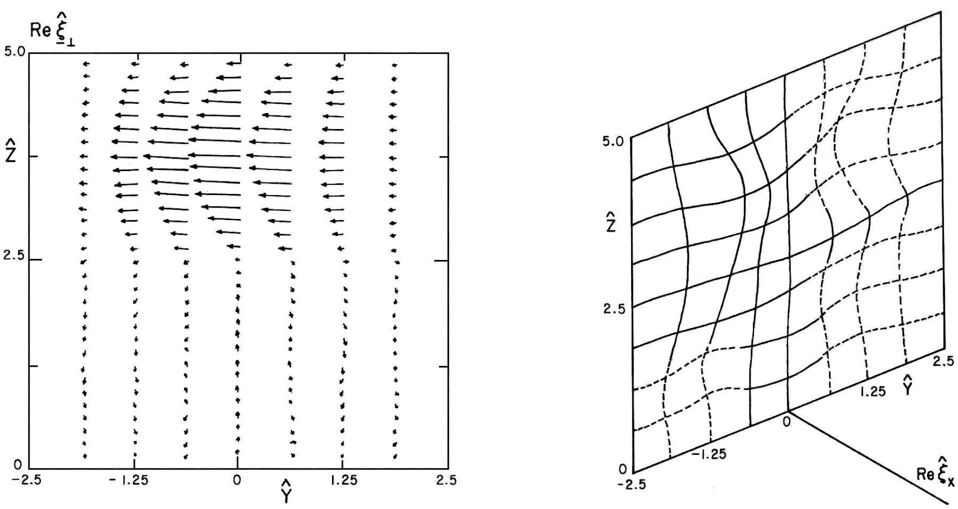

Fig. 6. Minimizing mode for observed parameters: $\widehat{k} = 0.078$, $\widehat{Y} = \widehat{Z} = 5$, $\delta = 0.001$, $\sigma = 0.01$, $\widehat{B}_x = 5$; $\widehat{\omega}_1^2 = 0.03513$. a) Re$\widehat{\underline{\xi}}_{\perp}$; b) Re$\widehat{\xi}_x$ $|\max \mathrm{Re}\widehat{\xi}_x|/|\max \mathrm{Re}\widehat{\xi}_y| \approx 21$; $|\max \mathrm{Re}\widehat{\xi}_x|/|\max \mathrm{Re}\widehat{\xi}_z| \approx 147$.

sumed this value for the prominence height. On the other hand, we will consider prominences which are far from an active region. In this case the interaction between internal and external fields is weak so that $\delta \ll 1$. Due to the difficulty to measure parameter σ we will assume it as a free parameter.

In order to make sure of a non-negative pressure everywhere we have chosen $\widehat{P}_\infty = \frac{2.10^2}{\sigma}\left(1 + \frac{\delta}{\sigma}\right) =$ constant. This value guarantees a positive $\widehat{P}$ for wide ranges of parameters δ and σ.

We have performed the numerical minimization of $\delta\widehat{W}$ for the above parameters and have obtained mainly stability, with the exception of some combinations of δ and σ. In the case of stability, Table 1 exemplifies some typical values of the minimum eigenvalue $\widehat{\omega}_1^2$, the ground frequency $\nu_1 = (5g/Y)^{1/2}\widehat{\omega}_1/2\pi$ and the associated oscillation period P. Dependence of $\widehat{\omega}_1^2$ on $\widehat{k}$ is clearly monotonous. Figure 2 shows the typical behavior of $\widehat{\omega}_1^2$, for a changing $\widehat{k}$. Although we obtain a wide

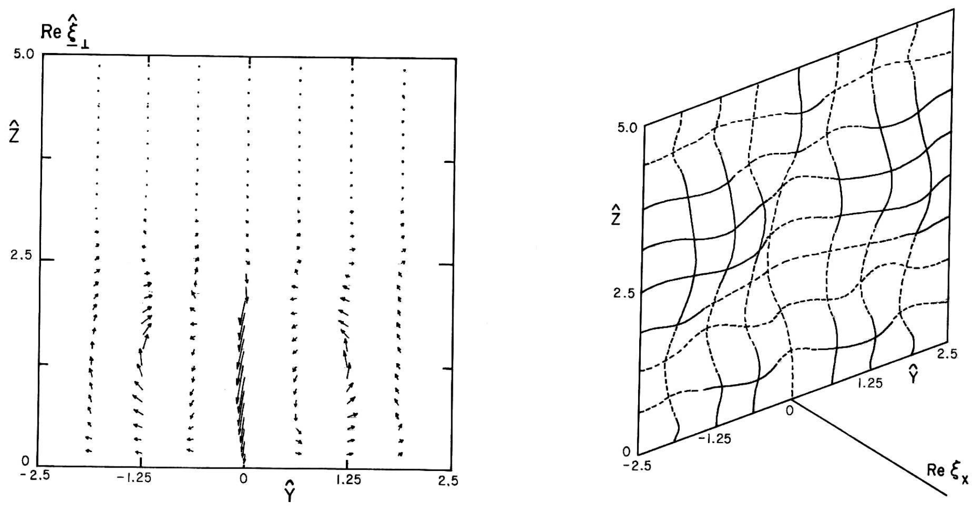

Fig. 7. Minimizing mode in the case of instability for: $\widehat{k} = 1$, $\widehat{Y} = \widehat{Z} = 5$, $\delta = 10$, $\sigma = 1$, $\widehat{B}_x = 1.$; $\widehat{\omega}_1^2 = -5.9852$; $|\max \mathrm{Re}\widehat{\xi}_x|/|\max \mathrm{Re}\widehat{\xi}_y| \approx 7$, $|\max \mathrm{Re}\widehat{\xi}_x|/|\max \mathrm{Re}\widehat{\xi}_z| \approx 1.7$.

range of periods, i.e., usually $P = 9 - 73$ min, it is necessary to analyze the polarity of each oscillation. In order to inquire into the essential characteristics of such oscillations, we will analyze in detail some situations, which display features belonging equally to all cases considered.

Changing the height of the integration region, we get the typical wall-effect on the eigenvalue $\widehat{\omega}_1^2$. However, instability can arise as soon as the prominence's dimensions become large enough.

Figure 3a shows the eigenvalue $\widehat{\omega}_1^2$ as a function of the axial field $\widehat{B}_x$ in case of observed parameters. As expected, an increasing axial field has a stabilizing influence on the equilibrium. This result is consistent with that for pinch discharges in laboratory (see, e.g., Shafranov 1957). As shown by Figure 3b stable oscillations are mainly driven by electromagnetic forces. The compressional contribution becomes substantial only for larger magnetic strengths. Gravitational effects are in this case energetically unimportant.

Figure 4 shows an example of a stability/instability transition region. The eigenvalue $\widehat{\omega}_1^2$ is positive only for a relatively narrow σ–interval. Outside of this interval, the equilibrium becomes severely unstable.

In order to inquire into further characteristics of these oscillations we will analyze some eigenmodes. The more general eigenmode has the form $\underline{\xi} = \left[(\mathrm{Re}\widehat{\xi}_x + i\mathrm{Im}\widehat{\xi}_x) + (\mathrm{Re}\widehat{\underline{\xi}}_\perp + i\mathrm{Im}\widehat{\underline{\xi}}_\perp)\right] e^{ikx}$. On the other hand, one can always find a phase kx so that one of both statements is valid: $\mathrm{Re}\underline{\xi} = \mathrm{Re}\widehat{\underline{\xi}}$ or $\mathrm{Im}\underline{\xi} = \mathrm{Im}\widehat{\underline{\xi}}$. Moreover, both possibilities are physically acceptable in describing the minimizing perturbation. Due to the internal structure of $\widehat{\delta W}$ in case of a non-zero $\widehat{B}_x$ the symmetry of $\widehat{\underline{\xi}}$ is slightly disturbed.

Figure 5a exhibits the component $\mathrm{Re}\widehat{\underline{\xi}}_\perp$ as a function of the position on the y, z–plane for observed parameters of quiescent prominences; arrows indicate the direction and intensity of motion of single elements of fluid. Note that the upper region of the prominence remains practically quiet during the oscillations. The stable oscillations on the y, z–plane have almost entirely horizontal polarization. The component $\mathrm{Re}\widehat{\xi}_x$ shown in Figure 5b attains its maximum amplitude approximately at the same position like $\mathrm{Re}\widehat{\underline{\xi}}_\perp$. Thus, the prominence oscillates almost horizontally but not entirely perpendicular to its longitudinal plane. The horizontal polarization described above remains if one takes into account other observed parameters. However, it is possible that the lower region of the prominence remains nearly quiet. Figures 6a and 6b show an example of such a situation. A common property in both cases is that their component $\mathrm{Re}\widehat{\xi}_x$ has a much larger amplitude compared with that of $\mathrm{Re}\widehat{\xi}_z$ and $\mathrm{Re}\widehat{\xi}_y$. Therefore, for both possible polarizations of the external vertical field the prominence can oscillate horizontally almost along its longitudinal axis.

Oscillatory phenomena in prominences have been observed by many authors (see, e.g., Balthasar et al. 1986; Tsubaki et al. 1987; Wiehr 2003). Large-scale oscillations are often detected following a major flare and they are mainly horizontal. Wiehr et al. (1984) reported long-period oscillations in the range from

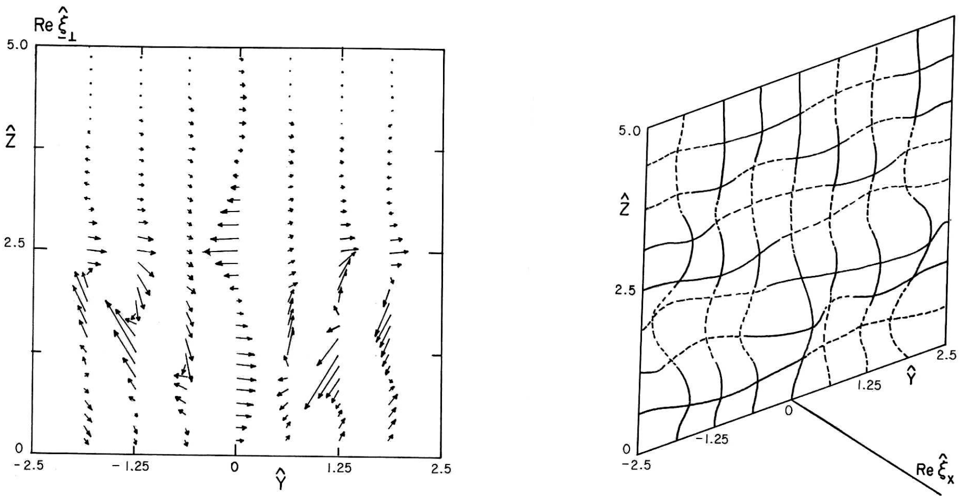

Fig. 8. Minimizing mode for observed parameters: $\widehat{k} = 1$, $\widehat{Y} = \widehat{Z} = 5$, $\delta = 1$, $\sigma = 10$, $\widehat{B}_x = 1$; $\widehat{\omega}_1^2 = -11.3013$; $|\max \mathrm{Re}\widehat{\xi}_x|/|\max \mathrm{Re}\widehat{\xi}_y| \approx 0.01$, $|\max \mathrm{Re}\widehat{\xi}_x|/|\max \mathrm{Re}\widehat{\xi}_z| \approx 0.006$.

40 to 80 min. In addition, they found indications of oscillations with short periods around 3 and 5 min. Balthasar et al. (1986) confirmed such findings obtaining the period range 3.5 − 6.5 min. Solovjev (1985) has interpreted such oscillations as the propagation of Alfvén waves along the prominence axis. For adequate combinations of $\delta \ll 1$ and σ we obtain rather horizontal oscillations whose periods are marginally consistent (see Table 1) with those of the short period oscillations reported by Balthasar et al. (1986). On the other hand, for other combinations of $\delta \ll 1$ and σ it is possible to have also horizontal oscillations with periods (see Table 1) which are located in the range of intermediate and even long period oscillations as established by Wiehr et al. (1984).

Let us examine briefly the stability characteristics of Osherovich's model in other parameter ranges which could be relevant for plasma structures in other stellar atmospheres. A striking feature is the generalized appearance of instability. It is convenient to notice that instability arises in parameter ranges, which do not correspond to the observed ones in solar quiescent prominences.

Figure 7 and Figure 8 illustrate the minimizing eigenmode $\mathrm{Re}\widehat{\underline{\xi}}$ for two unstable situations; one may recognize the expected tendency in the motion of the elements of fluid. It is interesting to note that for the case depicted in Figs. 7a and 7b the amplitudes of $\mathrm{Re}\widehat{\underline{\xi}}_\perp$ and $\mathrm{Re}\widehat{\xi}_x$ are of the same order of magnitude, so that matter in motion generates two strong eddies.

On the other hand, in the case shown in Figs. 8a and 8b the amplitude of $\mathrm{Re}\widehat{\underline{\xi}}_\perp$ is much larger than that of $\mathrm{Re}\widehat{\xi}_x$. The motion pattern now becomes very complicated.

Indirect observational methods have made possible to identify plasma structures in other stellar atmospheres (Cameron et al. 2003; Petit et al. 2005); however, a more detailed knowledge of the physical parameters of such prominence-like structures is necessary in order to confidently apply the solar results to another, similar, star.

4. CONCLUSIONS

In this paper we have presented a stability analysis of the solar prominence model of Osherovich (1989). This theoretical model reproduces suitably the basic oscillatory features of quiescent prominences. We have considered parameter ranges observed in these plasma structures. We found stability. Both the polarization of the stable oscillations and their periods are consistent with reported short, intermediate and long period oscillations in prominences (Balthasar et al. 1986; Wiehr et al. 1984; Oliver 1999; Ballester 2003). As a second stage in our search for general stability properties of Osherovich's model, we have also taken into account other parameter ranges, which not appear in solar plasma structures. In such cases, instability appears. The plasma displacements in the onset of the instability show a clearly intricate pattern which cannot be subsequently described by a linear theory.

However, it is obvious that the stellar prominence's spectacular end will occur shortly. Further investigation on the stability of more realistic models will yield important insight into the fundamental nature of solar prominences.

The alternative of studying the stability of multiple magnetic tubes in other astrophysical contexts, like in interplanetary magnetic clouds, within the framework of Osherovich's model is a relatively easy matter. MHD excited states coming from Osherovich's formalism lead to multiple plasma configurations in a natural way.

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