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A 1.4 GHZ AND 14.7 GHZ ANALYSIS OF DENSITY GRADIENTS IN GALACTIC PLANETARY NEBULAE

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RESUMEN

Actualmente hay una considerable incertidumbre acerca de la existencia de gradientes radiales de densidad en Nebulosas Planetarias (NPs). Si bien el trabajo de Taylor et al. (1987) implica la existencia de gradientes de densidad en la mayoría de las planetarias, el de Siódmiak & Tylenda (2001) sugiere precisamente lo contrario. El trabajo de Phillips (2007) sugiere que ambos análisis son dudosos y que al menos en un 10 a 20% de las NPs existen, probablemente, estos gradientes.

En este trabajo extendemos este último análisis para incluir flujos en frecuencias mayores y los utilizamos para llevar a cabo un análisis más sensible de los efectos de los gradientes radiales de densidad. Concluimos que en más del 85% de las NPs parecen existir gradientes de densidad, el 21% de los cuales tienen probablemente un exponente $\beta > 1.5$ y cavidades centrales con radios menores que el 20% del radio externo de la envolvente.

ABSTRACT

There has been a considerable level of uncertainty as to whether planetary nebulae (PNe) contain radial density gradients. Whilst the work of Taylor et al. (1987) implies that gradients exist in most PNe, that of Siódmiak & Tylenda (2001) suggests precisely the reverse. The work of Phillips (2007) suggests that both of these analyses are suspect, however, and that at least $\sim 10 \rightarrow 20\%$ of PNe probably do contain gradients.

We now extend this latter analysis to include significantly higher frequency fluxes, and use these to undertake a more sensitive analysis of the effects of radial density gradients. We conclude than in excess of 85% of PNe appear to contain gradients, of which $\sim 21\%$ are likely to have density exponents $\beta > 1.5$, and central cavity sizes which are small (i.e. have radii < 20% of the outer radius of the shell).

Key Words: **ISM: JETS AND OUTFLOWS — PLANETARY NEBULAE: GENERAL**

1. INTRODUCTION

Considerable uncertainty has attached to the question of whether planetary nebulae do, or do not possess radial density gradients. Taylor, Pottasch, & Zhang (1987), for instance, investigated the radio continuum characteristics of a limited number of sources, and concluded that most of their sample did indeed possess gradients. The authors used observations taken over a broad spread of frequencies, and allied these to limited modeling of the radio continuum fluxes. Such an analysis could only be applied to relatively few well-studied sources, however, and it remained unclear whether the results were applicable to PNe taken in general.

Given this limitation in the observational data base, it was obviously of interest to determine if useful constraints could be imposed using more limited ranges of flux. One such possibility has recently been outlined by Siódmiak & Tylenda (2001), who analysed of the variation of $F(5GHz)/F(1.4GHz)$ flux ratios with 5GHz brightness temperatures $T_B(5GHz)$. They concluded that practically none of their PNe had any gradients at all.

Phillips (2007) has recently shown that neither of these analyses is reliable, however, and that both sets of results should be regarded with caution. A re-analysis of the Siódmiak & Tylenda (2001) data base suggests that $\sim 10 \rightarrow 20\%$ of nebulae possess gradients, although it seems likely that this represents a lower limit estimate, and that the proportion may be much higher.

One of the problems with these analyses was the limited range of frequencies which were considered. Phillips (2007; and Siódmiak & Tylenda 2001, before him) considered the variation of $F(5GHz)/F(1.4GHz)$ with $T_B(5GHz)$. Where shell optical depths are modest, then values of $T_B(5GHz)$ are also expected to be small, and the ratio $F(5GHz)/F(1.4GHz)$ would be sensitive to the assumed gradient in density. Specifically, where electron densities vary as $n_e \propto r^{-\beta}$, and $\beta = 2$, then $F(5GHz)/F(1.4GHz)$ would be as much as ~ 0.4 dex larger than where $\beta = 0$. Where the nebulae have large central cavities however (and this is expected to apply in most PNe), then the level of sensitivity is very much reduced. Where cavity sizes are of order $\sim 40\%$ of the outer shell radius (see e.g. Phillips 1984), then the difference between the $\beta = 0$ and 2 loci becomes no greater than $\log(F(5GHz)/F(1.4GHz)) \simeq 0.1$ dex; a disparity which is not very much greater than the observational scatter in the results. It follows that differences in gradient are sometimes difficult to discern, and estimates of the number of sources having $\beta > 0$ may constitute an extreme lower limit.

One way out of this dilemma might be to employ measurements taken at higher radio frequencies. Where one considers fluxes at 14.7 GHz, as in the analysis below, then the ratio $F(14.7GHz)/F(1.4GHz)$ would be more sensitive to variations in β. This permits the effects of source density gradients to be more readily determined, and distinguished from the scatter associated with observational error. We shall show, as a result, that most PNe shells have $\beta > 0$.

2. OBSERVATIONAL AND THEORETICAL DATA BASE

We have employed fluxes at 5 GHz and 14.7 GHz deriving from the extensive compilation of Acker et al. (1992). Although these are based upon 21 or so differing sets of measurements, they derive principally from Milne & Aller (1982), Purton et al. (1982), and Zijlstra, Pottasch, & Bignell (1989) (in the case of the 14.7 GHz fluxes), and Zijlstra et al. (1989), Aaquist & Kwok (1990), Calabretta (1982), Milne (1979) and Milne & Aller (1975) (for the 5 GHz fluxes). The 14.7 GHz brightness temperatures are taken to be given by

$$T_B(14.7GHz) = 8.159 \, \frac{F(14.7GHz)}{\theta^2} \quad , \qquad (1)$$

where F(14.7GHz) is the flux density in mJy, and θ is the source radius in arcseconds. We have, for this latter parameter, taken results from a variety of publications. First and foremost are those deriving from the 5 GHz observations compiled by Acker et al. (1992) (the number of 14.7 GHz measurements is relatively small). Where these are not available, then we have used harmonic mean dimensions $\theta_{HAR} = (\theta_{MAX}\theta_{MIN})^{0.5}$ based upon the optical results of Tylenda et al. (2003), and the visual estimates summarized by Cahn, Kaler, & Stanghellini (1992) and Acker et al. (1992).

The use of such dimensions may cause errors in the estimate of $T_B(14.7GHz)$, although these are not expected to radically affect our overall conclusions. In particular, where the sources are optically thick at 5 GHz, but optically thin (or partially optically thick) at 14.7 GHz, then the values of $T_B(14.7GHz))$ may turn out to be somewhat understated.

These various results are plotted in Figure 1, where we represent the variation of $\log(F(14.7GHz)/F(1.4GHz))$ with respect to $\log(T_B(14.7GHz))$. We have eliminated sources having $F(14.7GHz) < 20$mJy, many of which have less reliable flux measurements. We also show a variety of model trends based upon a radiative transfer analysis similar to that of Phillips (2007). The specific intensity of radiation along a line of sight through the source is determined through numerical solution of the relation

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \frac{j_\nu}{\kappa_\nu} \quad , \qquad (2)$$

where τ_ν is the optical depth, and j_ν and κ_ν are the emission and absorption coefficients for the radio regime. It is adequate, under such circumstances, to assume that $j_\nu/\kappa_\nu = 2\nu 2kTc^{-2}$, whilst

$$d\tau_\nu = \kappa_\nu ds \simeq 8.24 \; 10^{-2} T_e^{-1.35} \nu^{-2.1} n(H^+) n_e ds \quad , \qquad (3)$$

where emission is primarily due to H^+ ions (see e.g. Pottasch 1984). Overall flux levels are then determined by integrating over the projected surface of the source.

We have assumed, for simplicity, that the nebular shells can be approximated by spherical shells, that they have radial density gradients $n_e \propto r^{-\beta}$,

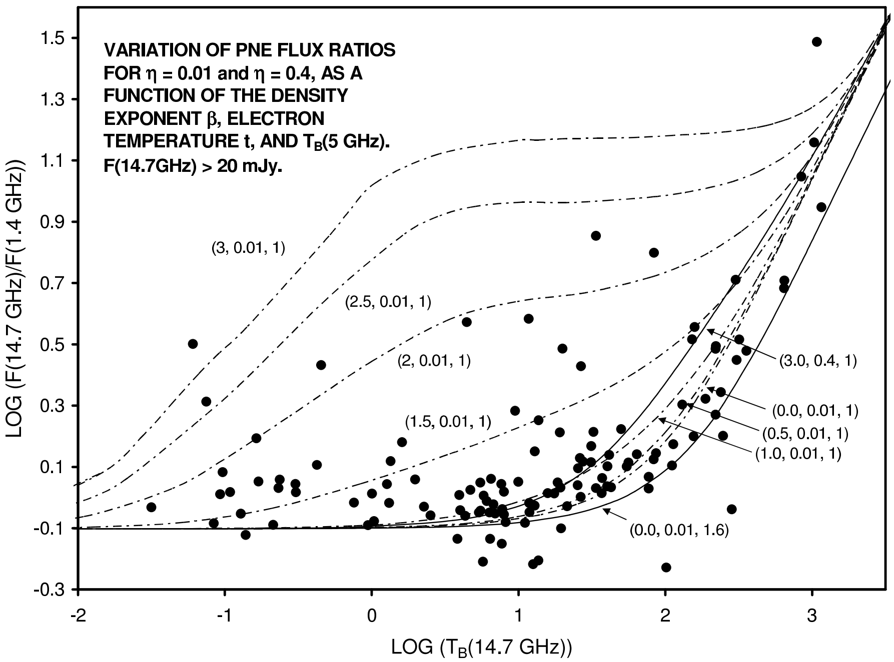

Fig. 1. The distribution of PNe within the $\log(F(14.7GHz)/F(1.4GHz))$-$\log(T_B(14.7GHz))$ plane, where we have limited the sample of sources to those having $F(14.7GHz) > 20$ mJy. The dot-dashed curves correspond to the trends predicted where cavity sizes are small ($\eta = 0.01$), normalized temperatures $t = 1$, and density exponents β vary between 0 and 3. They are labeled with the identifiers (β, η, t). The right-hand solid curve corresponds to $t = 1.6$, $\eta = 0.01$, and $\beta = 0$, whilst the left-hand solid curve indicates the trend where $\eta = 0.4$, $t = 1$, and $\beta = 0$.

and that they possess central cavities with fractional radii η. This model is clearly somewhat too simple for many PNe (e.g. those having bipolar and irregular morphologies), but represents a tolerable, statistical mean approximation for the sources taken as a whole. The curves correspond to differing combinations of the parameters (β, η, t), where $t = T_e/10^4$ K is the normalized electron temperature.

3. THE FRACTION OF SOURCES HAVING DENSITY GRADIENTS

It is apparent from Figure 1 that where values of η are small, and density exponents β are significant, then ratios $F(14.7GHz)/F(1.4GHz)$ may be appreciable at intermediate values of $T_B(14.7GHz)$ (say $-1 < \log(T_B(14.7GHz)) < 2$). In the extreme case where $\beta = 3$ and $t = 1$, then the flux ratio is ~ 1.2 dex greater than where $\beta = 0$ (and ~ 0.8 dex greater than one would expect for the ratio $F(5GHz)/F(1.4GHz)$). This permits a more sensitive analysis of trends in density gradient, as noted in our discussion in Section 1.

The solid curve at higher values of $T_B(14.7GHz)$ corresponds to $t = 1.6$ and $\eta = 0.01$, and represents the upper limit temperature range for most observed PNe (see e.g. Phillips 2007). The solid curve towards the left represents the trend expected where $t = 1$, but $\eta = 0.4$; that is, for PNe having significantly larger central cavities.

Two things are apparent from these latter curves. The first is that most sources lie within the range expected for observed PNe temperatures. The second is that ratios $\log(F(14.7GHz)/F(1.4GHz))$ depend sensitively upon both β and η. If η is large, then the flux ratio may be very much reduced. One therefore only expects to see large ratios $\log(F(14.7GHz)/F(1.4GHz))$ where η is small, β is appreciable, and/or source optical depths (and brightness temperatures) are reasonably large (see Phillips 2007 for a more detailed analysis of these trends).

Two things are immediately apparent from the distribution of sources in Figure 1. The first is that there is a scattering of 25 or so nebulae in the low η/high β regime; $\sim 20\%$ of the total sample. It seems likely that most of these sources possess appreciable density gradients, even exceeding (in a very few cases) $\beta \sim 2$.

A list of sources having $\beta > 1.5$ is provided in Table 1, where we have assumed that cavity sizes η are

TABLE 1

CANDIDATE SOURCES HAVING LARGE DENSITY EXPONENTS β

Source	PNG	Log (T_B)	β	Source	PNG	Log (T_B)	β
IC 4634	000.3+12.2	1.528	2.3	NGC 6879	057.2−08.9	1.137	1.5
M 1-38	002.4−03.7	1.426	1.7	NGC 6905	061.4−09.5	−0.516	1.6
M 1-28	006.0+03.1	−0.786	2	CRL 618	166.4−06.5	3.032	>3
M 1-31	006.4+02.0	0.977	1.6	NGC 2610	239.6+13.9	−0.964	1.7
Th 4-7	006.8+02.3	0.648	2	NGC 2818	261.9+08.5	−1.032	1.7
M 2-9	010.8+18.0	−0.77	1.7	NGC 4361	294.1+43.6	−0.373	1.7
A 51	017.6−10.2	−1.218	>3	NGC 6072	342.1+10.8	−0.626	1.7
K 3-4	032.7+05.6	0.13	1.6	NGC 6337	349.3−01.1	−0.343	2.2
NGC 6772	033.1−06.3	−1.015	1.9	M 2-22	357.4−04.6	1.07	1.9
Sh 2-71	035.9−01.1	−1.127	2.6	M 1-26	358.9−00.7	1.923	2.1
NGC 6804	045.7−04.5	−0.519	1.6	Hb 5	359.3−00.9	1.301	1.8
M 1-67	050.1+03.3	−0.635	1.6	M 3-9	359.9+05.1	0.207	1.6

≤ 0.01. Where values of η are larger, then the estimates of β would be required to be increased. More accurate values of β might be determined where nebular structures (and cavity sizes) are known. Similarly, errors in the flux ratios and/or values of $T_B(14.7GHz)$ may cause errors in β of ~ 0.2 or so.

A surprisingly large fraction of these sources (30 % or so) appear to have bipolar morphologies (viz. M 1-28, M 2-9, K 3-4, CRL 618, NGC 2818, NGC 6072, and Hb 5). This may imply that these sources are particularly prone to large density gradients, or that the analysis is inappropriate for such outflows — that their structures are too greatly at variance with the assumptions behind our modeling. If this is so, then these values of β should be discarded. Such outflows are well known for containing at least two primary structures, however, the very large bilobal wings, and compact interior spheroidal shells. These latter formations are normally responsible for the primary radio emission (the lobes are quite weak), and it is with these that this analysis is primarily concerned. The large values in these outflows are therefore likely to be valid, and apply to those sectors of the structures which are more in conformity with our modeling.

Even more interesting than these results, however, is the tendency for most sources having $\log(T_B(14.7GHz)) < 1.25$ to be located above the $\beta = 0$ locus (designated as (0.0, 0.01, 1) in Figure 1). This tendency is further illustrated in Figure 2, where we show the (normalized) distribution of sources as a function of $\log(F(14.7GHz)/F(1.4GHz))$. The dashed lines indicate the range of ratios corresponding to the $\beta = 0$ trajectory. It can be seen that the trend is strongly skewed with respect to $\beta = 0$, with the large majority of PNe having $\log(F(14.7GHz)/F(1.4GHz)) > -0.1$.

We interpret this trend in the following terms. Sources to the left of the $\beta = 0$ locus are likely scattered there through observational error. There is also the possibility that some of these fluxes are affected by source confusion, and that a proportion of the continuum may arise from non-thermal components of emission. Those to the right of this locus are affected by errors in the fluxes, and/or have exponents $\beta > 0$. The observed asymmetry in the distribution therefore implies that a very large fraction of PNe ($\geq 85\%$) are likely to have appreciable values of β. This proportion is much larger than was determined by Phillips (2007), and reflects the higher sensitivity of the procedure employed here

We therefore conclude that most PNe are likely to possess density exponents $\beta > 0$, but that a more precise analysis requires knowledge of the structures of the nebulae. In particular, it would be interesting to undertake a similar in-depth analysis for nebulae in which the cavity dimensions η have been determined.

Finally, we note that such results are consistent with a broad variety of interacting wind models. These have improved considerably over the years, from the earlier modeling of Okorokov et al. (1985), Ferch & Salpeter (1975), and Schmidt-Voigt & Koppen (1987), through to the more sophisticated analyses by Mellema (1994), Marigo et al. (2001),

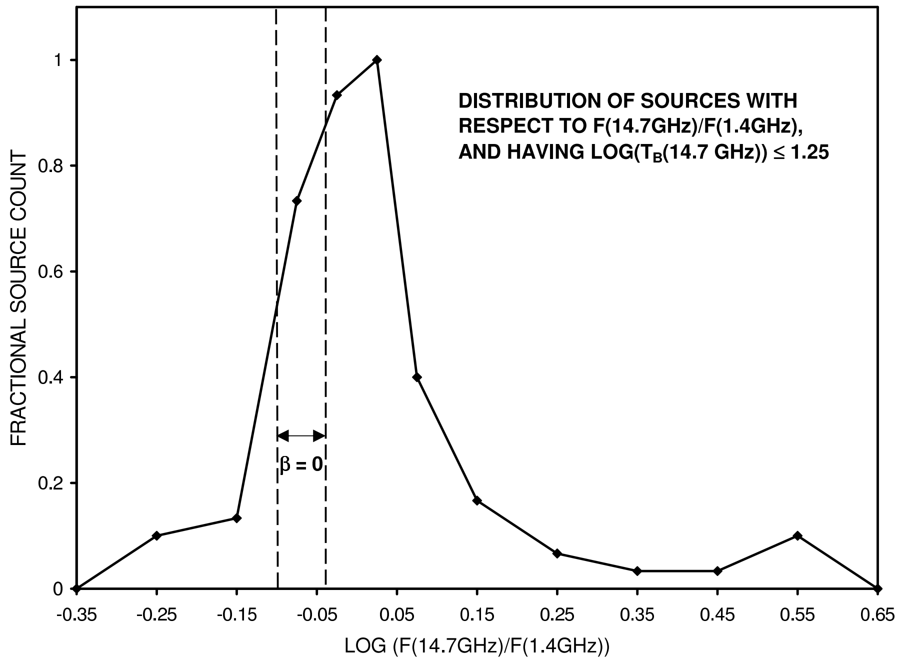

Fig. 2. The normalized distribution of sources as a function of $\log(F(14.7GHz)/F(1.4GHz))$, and where $\log(T_B(14.7GHz)) < 1.25$. It will be noted that the trend is strongly skewed with respect to the $\beta = 0$ locus (vertical dashed lines). This asymmetry is interpreted as implying that most ($\sim$ 85%) nebulae have $\beta > 0$.

Perinotto et al. (1998) and Schoenberner, Jacob, & Steffen (2005). Most of these show the presence of steep increases in density associated with interior and exterior shocks, together with more gradual variations in n_e throughout the primary shell mass. Such results also depend upon specific details of the modeling, however, including the assumed ages of the shells, the history of pre- and post-AGB central star mass-loss, the mass (and evolution) of the central stars, and so forth. These differences in input parameters lead to variations in the deduced radial fall-offs in density.

Taken as a whole, however, it is clear that gradual radial variations in n_e are expected to be common, and characterised by exponents β which are in many cases appreciable. In the case of the models investigated by Schoeberner et al. (2005), for instance, it is found that where central star masses $M_{CS} = 0.595 M_\odot$, and nebular ages are of order 7 10^3 yrs $< t <$ 9 10^3 yrs, then mean density exponents would be of order $\beta \sim 1.1 \rightarrow 1.4$.

Finally, one further point is worth noting about the comparisons described above. None of the models predicts variations $n_e(r)$ which are very closely comparable, although all of them appear similar at the qualitative level. Whilst a power-law fall off $n_e \propto r^{-\beta}$ is therefore simpler than implied by the models cited above, it represents what is probably the least contentious approximation given the uncertainties in such analyses.

4. CONCLUSIONS

An analysis of the distribution of sources within the $\log(F(14.7GHz)/F(1.4GHz))$-$\log(T_B(14.7GHz))$ plane, together with a radiative transfer analysis similar to that of Phillips (2007), shows that in excess of 85 % PNe are likely to have density exponents $\beta > 0$. A few sources ($\sim$ 20% or so) also appear to have small central cavities, and values $\beta > 1.5$. This work represents an advance upon the lower frequency analysis of Phillips (2007), and permits the role of gradients to be more clearly discerned. A combination of the present analysis, and an investigation of cavity sizes should permit values of β to be more clearly defined.

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REFERENCES

Aaquist, O. B., & Kwok, S. 1990, A&AS, 84, 229
Acker, A., Ochsenbein, F., Stenholm, B., Tylenda, R., Marcout, J., & Schohn, C. 1992, Strasbourg-ESO Catalogue of Planetary Nebulae (Garching: ESO)

Cahn, J. H., Kaler, J. B., & Stanghellini, L. 1992, A&AS, 94, 399
Calabretta, M. R. 1982, MNRAS, 199, 141
Ferch, R. L., & Salpeter, E. E. 1975, ApJ, 202, 195
Marigo, P., Girardi, L., Groenewegen, M. A. T., & Weiss, A. 2001, A&A, 378, 958
Mellema, G. 1994, A&A, 290, 915
Milne, D. K. 1979, A&AS, 36, 227
Milne, D. K., & Aller, L. H. 1975, A&A, 38, 183
_________. 1982, A&AS, 50, 209
Okorokov, V. A., Shustov, B. M., Tutukov, A. V., & Yorke, H. W. 1985, A&A, 142, 441
Perinotto, M., Kifonidis, K., Schonberner, D., & Marten, H. 1998, A&A, 332, 1044
Phillips, J. P. 1984, A&A 137, 92
_________. 2007, MNRAS, 378, 231
Pottasch, S. R. 1984, Planetary Nebulae (Dordrecht: Reidel)
Purton, C. R., Feldman, P. A., Marsh, K. A., Allen, D. A., & Wright, A. E. 1982, MNRAS, 198, 321
Schmidt-Voigt, M., & Koppen, J. 1987, A&A, 174, 211
Schoenberner, D., Jacob, R., & Steffen, M. 2005, A&A, 441, 573
Siódmiak, N., & Tylenda, R. 2001, A&A, 373, 1032
Taylor, A. R., Pottasch, S. R., & Zhang, C. Y. 1987, A&A, 171
Tylenda, R., Siódmiak, N., Gorny, S. K., Corradi, R. L. M., & Schwarz, H. E. 2003, A&A, 405, 627
Zijlstra, A. A., Pottasch, S. R., & Bignell, C. 1989, A&AS, 79, 329

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