METALLICITY EFFECTS ON THE MODIFIED WIND MOMENTUM OF CSPN

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ABSTRACT

Recent investigations on the central stars of planetary nebulae (CSPN) indicate that the masses based on model atmospheres can be much larger than the masses derived from theoretical mass-luminosity relations. Also, the dispersion in the relation between the modified wind momentum and the luminosity depends on the mass spread of the CSPN, and is larger than observed in massive hot stars. Since the wind characteristics probably depend on the metallicity, we analyze the effects on the modified wind momentum by considering the dispersion in this quantity caused by the stellar metallicity. Our CSPN masses are based on a relation between the core mass and the nebular abundances. We conclude that these masses agree with the known mass distribution both for CSPN and white dwarfs, and that the spread in the modified wind momentum can be explained by the observed metallicity variations.

Key Words: planetary nebulae: general — stars: mass loss — stars: winds, outflows

1. INTRODUCTION

Stellar winds are observed in practically all the HR diagram, from dwarf, solar-like stars to the hot supergiants and massive stars on the main sequence. In particular, AGB stars have slow, massive winds leading to the ejection of the stellar outer layers as a planetary nebula (PN). At this stage, the stellar remnant is extremely hot, originating a fast, massive wind which is similar to the radiative winds observed in hot, massive young stars. These winds are observed in central stars of planetary nebulae (CSPN), both for H-rich stars and for stars with Wolf-Rayet characteristics. The origin of the winds of these stars is attributed to the stellar radiation pressure on metallic lines, a similar mechanism known to be active in hot, massive galactic stars.

In the last few years, some work has been done in order to use the available information on the stellar winds of CSPN to determine physical properties of the stars, such as their radii, luminosities and masses (see for example Kudritzki & Puls 2000; Pauldrach, Hoffmann, & Méndez 2003). In particular, observed
or inferred properties such as the mass loss rate and the wind terminal velocity can be used in order to constrain some physical properties of the stars, such as their luminosities and masses. As a result, there seems to exist a discrepancy between recent mass determinations for CSPN based on model atmospheres affected by winds and the corresponding determinations from mass-luminosity relations derived from evolutionary tracks of post-AGB stars. There is a much larger mass spread obtained from model atmospheres as compared with the predictions of standard mass-luminosity relations and with the known mass distribution of CSPN and white dwarfs.

For a stellar wind with mass loss rate $\dot{M} = dM/dt$ and terminal velocity $v_{\infty}$ in a star with radius $R_*$, a correlation can be obtained between the modified wind momentum, defined as $p_w = \dot{M} v_{\infty} \sqrt{R_*}$ and the stellar luminosity $L/L_\odot$. This correlation is well defined for massive stars, and is also approximately valid for CSPN. However, the use of a standard mass-luminosity relation implies a relatively large spread in the modified wind momentum at a given luminosity for CSPN, in contrast with the results from model atmospheres, which show a better agreement with the relation derived from massive stars (Kudritzki et al. 1997; Pauldrach, Hoffmann, & Méndez 2004).

In the present work, we analyze the available information on the wind momentum in a sample of well-studied CSPN in the Galaxy, and use this information in order to constrain basic stellar properties. In particular, we look for correlations involving the observed wind and nebular properties, especially the chemical abundances. In fact, the stellar metallicity (and as a consequence the nebular chemical composition) has some effect on the wind properties, which can be seen for example by comparing galactic stellar winds with the corresponding quantities in stars of the Magellanic Clouds, which are more metal-poor than the Milky Way. This can be explained by the fact that the radiative mechanism operates essentially on the metal absorption lines, so that the stellar metallicity plays a role in the mass loss process. On the other hand, there is a clear correlation between the nebular chemical composition and the stellar mass, especially regarding those element ratios that are affected by the evolution of the PN progenitor stars, such as the N/O and He/H ratios (cf. Maciel 2000). As a consequence, the observed wind properties, such as the wind momentum or some related quantity, should in principle be related to stellar mass and metallicity. The IAG/USP group has a considerable experience in the determination and analysis of the chemical composition of PN in the Galaxy and the Magellanic Clouds (see for example Costa, Uchida, & Maciel 2004, and references therein). In this work we use this database and examine the effects of the nebular metallicity in the observed properties of the corresponding stellar winds in a sample of well studied galactic CSPN.

2. Masses of CSPN

2.1. Previous mass determinations

As recently discussed by Pauldrach et al. (2003, 2004), there is a clear discrepancy between the mass-luminosity relation for CSPN as derived from their improved model atmospheres and the relation previously obtained by Kudritzki et al. (1997), which results from the application of post-AGB evolutionary tracks. The new results produce a larger spread in the stellar masses, $0.4 < M/M_\odot < 1.4$, as compared with the range $0.6 < M/M_\odot < 1$ from Kudritzki et al. 1997. Moreover, the predicted luminosities are lower for a given mass in the new models, and the overall correlation is less well defined, as can be seen from Figure 1. In this figure, empty circles are the results by Kudritzki et al. (1997) and the filled circles refer to the data by Pauldrach et al. (2004). The objects included in the figure are NGC 2392, NGC 3242, IC 4637, IC 4593, He2-108, IC 418, Tc 1, He2-131, and NGC 6826. The values of the corresponding quantities (stellar radius, mass, luminosity, terminal velocity, mass loss rate, etc.) are given in the original papers by Kudritzki et al. (1997) and Pauldrach et al. (2003, 2004).

The work of Pauldrach et al. (2003, 2004) is based on hydrodynamically consistent, spherically symmetric model atmospheres which are able to reproduce the observed ultraviolet spectra of hot, mas-
TABLE 1

DATA FOR THE CENTRAL STARS

<table>
<thead>
<tr>
<th>Name</th>
<th>(\epsilon(O))</th>
<th>(\epsilon(N))</th>
<th>(\log(N/O))</th>
<th>(M_*(M_\odot))</th>
<th>(\log(L_*/L_\odot))</th>
<th>(\log p_w)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 2392</td>
<td>8.50</td>
<td>8.38</td>
<td>-0.12</td>
<td>0.73</td>
<td>4.08</td>
<td>26.63</td>
</tr>
<tr>
<td>NGC 3242</td>
<td>8.66</td>
<td>7.91</td>
<td>-0.75</td>
<td>0.59</td>
<td>3.85</td>
<td>26.44</td>
</tr>
<tr>
<td>IC 4637</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IC 4593</td>
<td>8.32</td>
<td>7.24</td>
<td>-1.08</td>
<td>0.54</td>
<td>3.77</td>
<td>25.98</td>
</tr>
<tr>
<td>He-108</td>
<td>8.22</td>
<td>7.85</td>
<td>-0.37</td>
<td>0.66</td>
<td>3.96</td>
<td>26.17</td>
</tr>
<tr>
<td>IC 418</td>
<td>8.54</td>
<td>7.82</td>
<td>-0.72</td>
<td>0.60</td>
<td>3.87</td>
<td>26.35</td>
</tr>
<tr>
<td>Te 1</td>
<td>8.71</td>
<td>7.67</td>
<td>-1.04</td>
<td>0.54</td>
<td>3.77</td>
<td>26.37</td>
</tr>
<tr>
<td>He-131</td>
<td>8.67</td>
<td>8.23</td>
<td>-0.44</td>
<td>0.65</td>
<td>3.95</td>
<td>26.60</td>
</tr>
<tr>
<td>NGC 6826</td>
<td>8.43</td>
<td>7.50</td>
<td>-0.93</td>
<td>0.56</td>
<td>3.80</td>
<td>26.14</td>
</tr>
</tbody>
</table>

On the basis of the observed wind characteristics, namely the terminal velocity and the mass loss rate, the main stellar parameters can be derived. In this case, the obtained stellar mass is independent of any adopted mass-luminosity relation, which in principle may be an advantage. On the other hand, the results by Kudritzki et al. (1997) are based on a core mass-luminosity relationship for post-AGB stars, in the framework of a modelling of stellar H and He line profiles. Although this method may be affected by complications such as contamination by nebular lines, it generally provides accurate determinations of the wind properties as well as of the stellar gravity and mass. Therefore, we need further information in order to clarify the discrepancy in the stellar masses observed in Figure 1.

2.2. The N/O masses

This problem can be investigated taking into account the CSPN masses derived from observed nebular abundances of N/O, as discussed in Cazetta & Maciel (2000) and Maciel (2000). From AGB evolution theory, there is a general correlation between the N/O abundance ratio and the core mass, that is, the CSPN mass (see for example Marigo 2001). Theoretical models for AGB stars predict that higher stellar masses are associated with larger N/O abundances, which are enhanced by the dredge-up episodes that occur in the CSPN, especially the second one. Cazetta & Maciel (2000) and Maciel (2000) presented a detailed discussion of the core mass-N/O abundance relation based both on theoretical models of AGB stars and on empirical mass determinations. This relation takes into account several initial mass-final mass relations available in the literature, as well as different individual mass determinations, so that it can be considered as independent of a single mass-luminosity relation, such as the one adopted by Kudritzki et al. (1997). As a result, two calibrations were determined, which were referred to as the high-mass and the low-mass calibration (cf. Maciel 2000 for details). The latter is considered as more accurate, as it produces core masses \(M_* \geq 0.55 M_\odot\) and main sequence masses \(M_{MS} \geq 1 M_\odot\), in agreement with the detailed CSPN masses of Stasińska, Górny, & Tylenda (1997), as well as with the masses of Type II PN proposed by Peimbert (1978). The adopted initial mass-final mass relation was based on the gravity distance work of Maciel & Cazetta (1997).

It is then interesting to investigate whether the masses obtained from the analysis of nebular abundances can be used to distinguish between the masses derived from model atmospheres and stellar wind analysis.

Following Cazetta & Maciel (2000) and Maciel (2000), the core mass (in solar masses) can be written as

\[
M_* = 0.7242 + 0.1742 \log (N/O) ,
\]

for \(-1.2 \leq \log (N/O) < -0.26\), and

\[
M_* = 0.825 + 0.936 \log (N/O) + 1.439 \, \left[ \log (N/O) \right]^2 ,
\]

for \(-0.26 \leq \log (N/O) < 0.20\), where \((N/O)\) is the nitrogen over oxygen abundance ratio by number of atoms. Columns 3 to 6 of Table 1 show the abundances of oxygen, nitrogen, and N/O adopted from our IAG/USP database, and the calculated masses for the sample of CSPN shown in Figure 1. The object IC 4637 is not in our database, so that we have adopted an average abundance \(\log(N/O) = -0.56\), which is typical for elliptical planetary nebulae, according to the discussion of Górny, Stasińska, & Tylenda (1997).
Fig. 2. Masses of CSPN as a function of the N/O masses given in Table 1. Empty circles: Kudritzki et al. (1997), filled circles: Pauldrach et al. (2004).

Figure 2 shows the masses as given by Pauldrach et al. (2004, solid dots) and Kudritzki et al. (1997, empty circles) as a function of the masses given in Table 1, which we may call N/O masses. It can be seen that the N/O masses are generally closer to the masses derived by Kudritzki et al. (1997) and Kudritzki, Urbaneja, & Puls (2006) than to the values by Pauldrach et al. (2004), which show a much higher mass dispersion. In view of the similarity between our N/O masses and most of the masses obtained by Kudritzki et al. (1997, it is tempting to calibrate the luminosities using the mass-luminosity relation adopted in that work, which can be approximately written as

$$\log \left( \frac{L}{L_\odot} \right) = 2.90 + 1.61 \left( \frac{M}{M_\odot} \right).$$

The derived luminosities are given in Column 7 of Table 1 and the N/O masses and corresponding luminosities are also included in Figure 1 as black stars. It can be seen that these masses have a much more limited range, roughly $0.5 < M(M_\odot) < 0.8$, than obtained by Pauldrach et al. (2004), and are also somewhat more restricted than the values by Kudritzki et al. (1997).

### 2.3. Discussion

A comparison of the CSPN masses discussed in the previous subsections can be made both on the basis of the known mass distributions of these objects and their white dwarf descendants as well as of the rather limited individual mass determinations for the central stars.

A set of central star masses was obtained by Górný et al. (1997), based on measured nebular expansion velocities and central star parameters, within the framework of a simple evolutionary model for planetary nebulae. A comparison of our derived masses given in Table 1 with their results shows a good agreement, as all objects have a mass difference under 0.10 $M_\odot$, and five of the stars have even smaller differences, under 0.03 $M_\odot$. The only exception is NGC 2392, for which our derived mass is 0.12 $M_\odot$ larger than the mass given by them, a difference of about 17%. However, this object has a very high nitrogen (and He, see § 3) abundance, which is consistent with a larger mass than derived by Górný et al. (1997).

Recently, Zijlstra, van Hoof, & Perley (2008) presented the results of a 25-year monitoring program of radio observations of the planetary nebula NGC 7027, possibly the best studied object of its kind. From the evolution of the radio flux during this period, an improved expansion distance was obtained. Taking into account theoretical tracks for post-AGB stars, a distance-independent mass of $0.655 \pm 0.01 M_\odot$ was derived. This result agrees very well with our N/O mass for this object, which is $0.66 M_\odot$, adopting abundances from our database (see for example Maciel & Quireza 1999 and Maciel & Chiappini 1994). Using alternative models, Zijlstra et al. (2008) find a slightly larger mass, still in good agreement with our result.

Also, in a recent work Traulsen et al. (2005) analyzed high resolution ultraviolet spectra of a few hydrogen-rich CSPN obtained with HST and FUSE, and were able to locate these objects in a log $g$ vs. log $T_{\text{eff}}$ plane using theoretical evolutionary tracks. For the three objects in our abundance dataset (NGC 4361, NGC 6853, and NGC 7293, see also Perinotto, Morbidelli, & Scatarzi 2004 and Henry, Kwitter, & Balick 2004), the central star masses obtained by Traulsen et al. (2005) cluster around 0.6 $M_\odot$, within about 15% of the N/O masses derived for these objects.

Stasińska et al. (1997) applied the method described by Górný et al. (1997) and derived a mass distribution of CSPN with a very restricted range, in which more than 80% of the objects have masses between 0.55 and 0.65 $M_\odot$. This distribution peaks around 0.6 $M_\odot$ with some dispersion that depends on the adopted nebular mass, and falls steeply towards large masses. Therefore, these results fit well the lower masses found by the N/O method, but the large masses found in a few cases by Kudritzki et al. (1997) and especially by Pauldrach et al. (2004) seem rather unlikely. A similar conclusion has been recently reached by Napiwotzki (2006), who argued that the high CSPN masses close to the Chan-
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drassékhar limit derived by Pauldrach et al. (2004) are physically implausible on the basis of kinematical parameters extracted from galactic orbits and average abundances for the different PN types.

Recent work on the much better known mass distribution of white dwarfs fully supports this conclusion, as can be seen from the white dwarf mass distribution of Madej, Należyty, & Althaus (2004), especially regarding the largest mass observed in the sample, which in our case is under \(0.8 \, M_\odot\). According to the investigation by Madej et al. (2004), which was based on a large sample of about 1200 white dwarfs from the Sloan Digital Sky Survey, their mass distribution peaks around 0.56 \(M_\odot\), with very few objects having masses larger than about 0.8 \(M_\odot\), in excellent agreement with our results. According to this work, less than 5% of the central stars are expected to have masses larger than 0.8 \(M_\odot\), a much lower fraction than observed either in the samples by Kudritzki et al. (1997) and Pauldrach et al. (2004).

The mass distributions of CSPN and white dwarfs have also been recently considered by Gesicki & Zijlstra (2007), based on a dynamical method which allows mass determinations within 0.2 \(M_\odot\). It results that both the CSPN and white dwarf distributions peak around 0.6 \(M_\odot\) as in Madej et al. (2004) and Stasiński et al. (1997), although the white dwarf distribution shows a broader mass range. The CSPN distribution has essentially no objects with masses higher than 0.7 \(M_\odot\) in their sample, while for the DA white dwarfs – presumably the progeny of H-rich CSPN – a small fraction of objects have masses larger than 0.8 \(M_\odot\), similar to the results of Madej et al. (2004). Although there may be some differences between the recently obtained white dwarf mass distributions, as discussed by Gesicki & Zijlstra (2007), they all agree in the sense that any sizable sample of CSPN is expected to have masses close to 0.6 \(M_\odot\), a result that is reproduced by our N/O masses.

A further support to the N/O masses comes from the revision by Tinkler & Lamers (2002) of the Kudritzki et al. (1997) masses based on a homogeneous set of parameters derived from Zanstra temperatures, dynamical ages of the nebulae and evolutionary tracks. According to these results, an average mass of 0.60 \(M_\odot\) is obtained, in excellent agreement with the results of Table 1, and showing no objects with masses larger than 0.8 \(M_\odot\).

Finally, it could be mentioned that 5 central stars of PN in the galactic bulge have been recently studied by Hultsch et al. (2007) based on high-resolution Keck spectra and NLTE modelling. In view of the fact that the distance to the galactic bulge is well known, the stellar parameters can be determined with better accuracy than in the case of field PN, although there is some uncertainty due to the adopted extinction law. The masses derived from their “Method 1”, which seems to be more reliable, come from evolutionary tracks on the log \(T_{\text{eff}} - \log g\) diagram, and are lower than about 0.80 \(M_\odot\) in all cases. Also, the estimated luminosities are close to the values obtained by equation (3), supporting our results given in Table 1.

3. THE MODIFIED WIND MOMENTUM OF CSPN

For a star with radius \(R_\star\) having a wind with terminal velocity \(v_\infty\) and a mass loss rate \(dM/dt\), the modified wind momentum is defined as

\[
p_w = \frac{dM}{dt} \, v_\infty \sqrt{R_\star}.
\]

(4)

It is well known from radiation-driven wind theory that the mass loss rate is a function of the stellar luminosity \(L_\star\) and the effective mass \(M_{\text{eff}}\), which can be written as

\[
\frac{dM}{dt} \propto L_\star^{1/\alpha} \, M_{\text{eff}}^{(\alpha-1)/\alpha},
\]

(5)

where \(\alpha\) is the so-called force multiplier parameter, which is about \(\alpha \approx 2/3\) for hot stars (see for example Lamers & Cassinelli 1999; Pauldrach et al. 2003). The exponent \(\alpha\) is the power-law exponent of the line strength distribution function. The effective mass \(M_{\text{eff}}\) is simply the stellar mass \(M_\star\) corrected for the electron scattering radiative force, \(M_{\text{eff}} = M_\star (1 - \Gamma_e)\), where \(\Gamma_e\) is the electron scattering efficiency.

As a first approximation, \(v_\infty\) can be assumed to be proportional to the stellar escape velocity \(v_{\text{esc}} = (2GM_{\text{eff}}/R_\star)^{1/2}\), so that \(v_\infty \propto \sqrt{M_{\text{eff}}/R_\star}\). For example, for a large sample of galactic CSPN from a variety of sources (Perinotto 1993; Kudritzki et al. 1997; Malkov 1997; Pauldrach et al. 2004), we obtain a scaling factor between the terminal and escape velocity of about 2 to 4. Combining this equation with equations (4) and (5), it is easy to see that the modified wind momentum depends weakly on the stellar mass, so that a relation of the form \(p_w \propto L_\star^{1/\alpha}\) is obtained. For instance, Pauldrach et al. (2003, 2004) obtained a relation that produces a very good fit to the properties of hot massive galactic stars given by

\[
\log p_w = 20.479 + 1.507 \log(L_\star/L_\odot),
\]

(6)

where \(p_w\) is in g cm s\(^{-2}\). Figure 3 shows the modified wind momentum with CSPN data by Kudritzki et al.
(1997) (empty circles) and Pauldrach et al. (2004) (filled circles), using the same symbols as before, as a function of the stellar luminosity. The solid line represents the fit to hot, massive stars by Pauldrach et al. (2004) given by equation 6. These stars are not shown in Figure 3, and occupy the upper right corner of the figure. Note that for NGC 2392, NGC 3242, IC 4637 and Tc 1 the mass loss rates from Kudritzki et al. (1997) are upper limits.

Adopting the new N/O stellar masses, as shown in the previous section, and the luminosities from the mass-luminosity calibration by Kudritzki et al. (1997), the stellar radii can be obtained, keeping the effective temperatures originally determined. Therefore, the modified wind momentum $p_w$ can be determined using the originally derived $M$ and $v_\infty$ and plotted as a function of the luminosity. This is also shown in Figure 3, where the the open triangles correspond to our correction of the results by Kudritzki et al. (1997) and the filled triangles refer to $M$ and $v_\infty$ data by Pauldrach et al. (2004). It is clear that this is not a strictly consistent procedure, as the main stellar and wind parameters (effective temperature, gravity, abundances, radius, terminal velocity and mass loss rate) should in principle be derived in a consistent way. However, most of these parameters are essentially unchanged in both analyses. In fact, a direct comparison of the Kudritzki et al. (1997) and Pauldrach et al. (2004) results shows that there is a general agreement for $\log g$, $v_\infty$, $dM/dt$, $R_*$, and $T_{\text{eff}}$. Only the stellar mass and luminosity show larger discrepancies: the masses by Pauldrach et al. (2004) are generally larger than those by Kudritzki et al. (1997), while the opposite is true for the luminosities.

From Figure 1 we can see that the new N/O masses of CSPN have a more restricted range, so that the same can be expected for the luminosities, as confirmed by Figure 3. From Figure 3 we can see also that the modified wind momentum has now a less tight correlation with the luminosity, similar to the results by Kudritzki et al. (1997) and in disagreement with the results by Pauldrach et al. (2003, 2004). It can also be seen that the same results are obtained for both sources of data, that is, the empty and filled triangles in Figure 3 show essentially the same dispersion. As mentioned in § 2, these results are in agreement with the results by Napiwotzki (2006), who has argued against the large masses for CSPN found by Pauldrach and co-workers, on the basis of the use of kinematic parameters obtained from galactic orbits and average abundances of the different PN types.

It should be mentioned that the procedure adopted here does not significantly affect the stellar gravity $\log g$, which is a parameter generally well determined from the observations. In fact, a detailed comparison of the resulting $\log g$ values with those originally derived either by Kudritzki et al. (1997) or Pauldrach et al. (2003, 2004) shows an excellent agreement, in the sense that (i) our values lie between the original values determined by these sources, and (ii) the average deviations in $\log g$ amount to 0.2 and 0.1 dex for the Kudritzki et al. (1997) and Pauldrach et al. (2003, 2004) data, respectively. An alternative approach would be to derive the masses from the N/O abundances and the stellar radius from the original $\log g$ values, so that the luminosity would follow from the observed effective temperature. However, we preferred the procedure described earlier, since the mass-luminosity relation is well founded theoretically, and any modifications in the adopted calibration could be easily introduced in this procedure.

The N/O masses depend on the measured relative abundances of nitrogen and oxygen, so that uncertainties in these measurements would affect the derived masses and, consequently, the luminosities. However, N and O abundances relative to hydrogen are among the best determined abundances in photoionized nebulae, with uncertainties generally lower than 0.2 dex. Abundances relative to oxygen, such as the N/O ratio needed by our procedure, are generally
better determined, usually within 0.1 dex. Adopting this as an average, equations (1) and (2) would lead to a maximum uncertainty of about 10% for central star masses in the range $0.6 < M/M_\odot < 0.8$. Clearly, such an uncertainty would have a minor influence in the derived luminosities as plotted in Figure 3.

4. METALLICITY DEPENDENCE OF THE MODIFIED WIND MOMENTUM OF CSPN

4.1. The modified wind momentum

A key point in the analysis of the modified wind momentum is the expected dispersion in the plot of $\log p_w \times \log L/L_\odot$ for a given luminosity. The results by Kudritzki et al. (1997) show a considerable dispersion for CSPN, while the more recent results by Pauldrach et al. (2003, 2004), and Pauldrach (2005) obtain a much lower dispersion and, consequently, a better fit to the line associated with the hot, luminous stars. Our results using the N/O masses indicate a larger dispersion in $\log p_w$ for both sources of data on the stellar winds, which is partly a consequence of our restricted mass – and therefore luminosity – interval. It is then interesting to investigate the expected dispersion at a given luminosity, which we will assume to be caused by the dispersion in the stellar (and nebular) metallicity.

The metallicity dependence of winds from red supergiants and AGB stars was recently reviewed by van Loon (2006). The mass ejection process in these stars is assumed to be the action of the stellar radiation pressure on grains. According to van Loon (2006), the dust to gas ratio $\psi$ is proportional to the stellar metallicity, $\psi \propto Z$. Since the grain absorption optical depth $\tau$ is proportional to the dust to gas ratio, $\tau \propto \psi$, we have $\tau \propto Z$. It is reasonable to assume the same is true for CSPN, here attributing the optical depth to the absorption in resonance metallic lines, so that we have that $p_w \propto Mv_\infty \propto \tau \propto Z^k$, where we may assume for simplicity $k \simeq 1$, combining any optical depth effects that either $dM/dt$ or $v_\infty$ might have. In the case of red giants, van Loon (2006) attributes the metallicity dependence on the expansion velocity, leaving the mass loss rate essentially independent of the metallicity. For CSPN, however, this may not apply, since the radiation pressure acts directly on the absorption lines, and is not a continuum-opacity process as in red supergiants and AGB stars.

A similar relation, namely, $p_w \propto Z^{0.9}$ has also been suggested for galactic hot stars with $\log L/L_\odot < 5.7$ by Lamers & Cassinelli (1996) as an upper limit for the correction of the mass loss rates due to metallicity. Such a relation is similar to the metallicity dependence of Wolf-Rayet winds, as recently studied by Vink & de Koter (2005) on the basis of Monte Carlo calculations. As an example, for the less evolved WN stars in which the wind is due to the action of the radiation pressure on many metallic lines, the metallicity dependence is similar as for galactic O stars, namely, $dM/dt \propto Z^m$, where $m \simeq 0.86$. For WC winds they find $m \simeq 0.66$.

Adopting $p_w \propto (L_*/L_\odot)^a$, as discussed in § 3, where $a = 1/\alpha$, we can introduce a dependence on the metallicity $Z$ by writing

$$p_w \propto Z \left( \frac{L_*}{L_\odot} \right)^a. \tag{7}$$

In the case of CSPN, the wind strength is basically related to the abundances of highly ionized heavy elements, which produce P Cyg profiles of optical and ultraviolet lines. Iron is the usual metallicity indicator in stars, but the iron lines are generally weak and the Fe abundance is depleted in photoionized and planetary nebulae due to grain formation (cf. Perinotto et al. 1999; Deetjen et al. 1999; Potash, Bernard-Salas, & Roellig 2008), so that the abundances of this element measured in the nebulae are usually taken as lower limits. However, the metallicity of PN can be accurately determined on the basis of measurements of the oxygen abundance, as well as of other elements such as Ne, S, etc., all of which are strongly correlated, as shown by many recent investigations both in the Milky Way and in other galaxies (cf. Richer & McCall 2006; Maciel, Costa, & Idiart 2006). As recently discussed by Idiart, Maciel, & Costa (2007), average $[O/Fe] \times [Fe/H]$ relationships can be obtained from stellar data or theoretical models, which may provide direct O(Fe) relations. Also, recent work on the [Fe/H] radial gradient in the galactic disk obtained from open cluster stars and cepheid variables (Maciel, Lago, & Costa 2005) shows a strong correlation with the gradients of O, S, etc. determined from photoionized nebulae, thus confirming these elements as representative of the nebular metallicity. Therefore, we can safely assume that in PN and hydrogen-rich CSPN the metal abundance $Z$ is proportional to the oxygen abundance by number $O/H$, that is, $Z = k (O/H)$. For example, for the solar photosphere we have $\epsilon(O) = \log(O/H)_\odot + 12 \simeq 8.66$ and $Z \simeq 0.012$ (Grevesse, Asplund, & Sauval 2007), so that $k \simeq 26$. Therefore, the modified wind momentum can be written as

$$p_w \simeq \beta \left( \frac{L_*}{L_\odot} \right)^a \left( \frac{O}{H} \right), \tag{8}$$
where $\beta$ is a constant to be determined. We have then

$$\log p_w = \log \beta + a \log \left( \frac{L_*}{L_\odot} \right) + \log \left( \frac{O}{H} \right). \quad (9)$$

Using the standard definition $\epsilon(O) = \log(O/H) + 12$, we have

$$\log p_w = \log \beta + a \log \left( \frac{L_*}{L_\odot} \right) + [\epsilon(O) - 12]. \quad (10)$$

According to Méndez (1991), all CSPN considered in this work are H-rich objects, in which stellar H features are clearly distinguished in the CSPN spectrum. Their H and He abundances are normal, with the possible exception of NGC 2392, which shows some He excess. All the other objects have “normal” helium abundances of about 10% by number (for the actual values see Méndez et al. 1988; Méndez, Herrero, & Manchado 1990), again suggesting that these stars have normal Fe abundances, and that their metal abundances are well correlated with the oxygen abundance. Most of these stars are of spectral type Of(H), so that the He II 4686 line appears as a narrow emission line, and two are of type O(H), in which He II 4686 appear in absorption. In the sample of 115 CSPN studied by Méndez (1991), 62% are H-rich and 33% are H-deficient. These objects have probably left the AGB with a H-rich photosphere, and are on their way to become DA white dwarfs. As can be seen from Figure 2 of Méndez (1991), most H-rich CSPN have “normal” He abundances, that is, He/H $\simeq 0.10$, so that their chemical composition includes about 90% of H by number, as in the case of the nebulae themselves. Therefore, we can safely adopt average oxygen abundances in these objects as in the nebulae themselves, that is

$$\epsilon(O) = \bar{\epsilon}(PN) = \bar{\epsilon}(CSPN) \approx 8.50, \quad (11)$$

which is essentially the same average oxygen abundances of galactic HII regions (see for example Costa & Maciel 2006; Henry & Worthey 1999). We can identify equations 10 and 6, from which we have $a = 1.507$, and log $\beta + [\epsilon(O) - 12] = 20.479$, so that log $\beta = 23.98$. Therefore we have

$$\log p_w = 23.98 + 1.507 \log \left( \frac{L_*}{L_\odot} \right) + [\epsilon(O) - 12], \quad (12)$$

again with $p_w$ in cgs units (g cm s$^{-2}$). The new $p_w$ values are shown in the last column of Table 1.

<table>
<thead>
<tr>
<th>$\epsilon(O)$</th>
<th>$\Delta \log p_w$</th>
<th>$\Delta \log p_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$9.20$</td>
<td>$0.70$</td>
<td>$-0.20$</td>
</tr>
<tr>
<td>$9.10$</td>
<td>$0.60$</td>
<td>$-0.30$</td>
</tr>
<tr>
<td>$9.00$</td>
<td>$0.50$</td>
<td>$-0.40$</td>
</tr>
<tr>
<td>$8.90$</td>
<td>$0.40$</td>
<td>$-0.50$</td>
</tr>
<tr>
<td>$8.80$</td>
<td>$0.30$</td>
<td>$-0.60$</td>
</tr>
<tr>
<td>$8.70$</td>
<td>$0.20$</td>
<td>$-0.70$</td>
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<tr>
<td>$8.60$</td>
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<td>$-0.90$</td>
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<tr>
<td>$8.40$</td>
<td>$-0.10$</td>
<td>$-1.00$</td>
</tr>
</tbody>
</table>

show the results for $\epsilon(O) = 7.50$ and $\epsilon(O) = 9.20$, which correspond approximately to the minimum and maximum observed values of the oxygen abundance, respectively. From the metallicity distribution of PN and HII regions (as well as cepheids and CSPN), we know that the large majority of objects have $8.00 < \epsilon(O) < 9.0$, so that the derived dispersion is very well defined. Table 2 shows the dispersion $\Delta \log p_w$ as a function of the adopted value of the oxygen abundance $\epsilon(O)$. It can be seen that, for the average $\epsilon(O) = 8.50$ and $7.5 < \epsilon(O) < 9.2$, the spread is from $\Delta \log p_w = -1.00$ to 0.70, that is, a total spread of 1.70 can be expected, which is similar to the observed scatter in Figure 3 using the N/O masses and both the Kudritzki or Pauldrach databases.

### 4.2. Empirical evidences

Empirical evidences of metallicity effects on the modified wind momentum of CSPN are difficult to obtain for several reasons. First, the modified momentum probably shows a stronger dependence on the stellar luminosity, so that any metallicity variation is expected to become apparent only for stars having similar luminosities. Second, the actual stellar luminosity has some intrinsic uncertainty, which basically derives from uncertainties in the distance scale, which affects particularly planetary nebulae and their central stars. Finally, the abundances themselves have an uncertainty of typically 0.2 dex, depending on the element considered. Despite these shortcomings, data on some of the objects analyzed in this paper show some hints in the sense that an enhanced metallicity is correlated with a higher wind momentum.
Taking into account the results shown in Table 1, we notice that the objects He2-108 and He2-131 have approximately the same luminosity, \( \log L/L_\odot \approx 3.95 \). The first object is clearly more metal-poor than the second, with a difference of about 0.5 dex, or almost a factor 3. According to the results of Malkov (1997, 1998), in a self consistent determination of several parameters of galactic planetary nebulae and their central stars. Among other parameters, Malkov (1997) obtains central star luminosities and their dispersions observed in the nebulae, which are a reflection of the properties of the central stars. We showed that the masses agree very well with the independent estimates of the masses of CSPN, and that the observed mass spread is small, entirely consistent with the expected mass distributions of CSPN and white dwarfs.

In this work we have considered a sample of well-studied CSPN for which the wind properties as well as the basic stellar properties are known, and analyzed two different problems.

First, we have confirmed that there is a large discrepancy between the stellar masses as derived from standard mass-luminosity relation for AGB stars and those derived from stellar atmosphere models affected by winds. We have then determined new values for the central star masses based on a correlation of the core stellar mass and the nebular N/O abundances, and determined the masses of the central stars. We showed that the masses agree very well with the independent estimates of the masses of CSPN, and that the observed mass spread is small, entirely consistent with the expected mass distributions of CSPN and white dwarfs.

Second, we have used the newly derived N/O masses and the standard mass-luminosity relation in order to investigate the dispersion of the modified wind momentum at a given luminosity. We have taken into account two different sets of stellar properties derived from different model atmospheres (Kudritzki et al. 1997 and Pauldrach et al. 2004), and shown that the dispersion on the \( p_w \times L_* \) plane is real, and can be well explained by the metallicity dispersion observed in the nebulae, which are a reflection of the properties of the central stars. Similar results can also be obtained by considering the set of objects analyzed by Perinotto (1993), in which case the mass loss rates have been determined with the SEI (Sobolev plus exact integration) and CFS (co-moving frame) method.

5. SUMMARY AND CONCLUSIONS

In this work we have considered a sample of well-studied CSPN for which the wind properties as well as the basic stellar properties are known, and analyzed two different problems.

First, we have confirmed that there is a large discrepancy between the stellar masses as derived from standard mass-luminosity relation for AGB stars and those derived from stellar atmosphere models affected by winds. We have then determined new values for the central star masses based on a correlation of the core stellar mass and the nebular N/O abundances, and determined the masses of the central stars. We showed that the masses agree very well with the independent estimates of the masses of CSPN, and that the observed mass spread is small, entirely consistent with the expected mass distributions of CSPN and white dwarfs.

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