# ANALYSIS OF CHROMATIC EFFECTS IN THE SHIFT-AND-ADD METHOD

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### RESUMEN

Se estudian los efectos cromáticos en sistemas de óptica adaptativa de "desplazar y sumar" ("Shift-And Add", SAA) mediante simulaciones numéricas. Los efectos cromáticos que investigamos surgen cuando se efectuan medidas y correcciones a distintas longitudes de ondas. El estudio de las propiedades cromáticas de la técnica SAA es útil para el diseño de sistemas de óptica adaptativa SAA. También es importante para la reducción de datos de observación obtenidos mediante espectroscopía de motas. Se presenta la comparación de dos técnicas: SAA y "tip-tilt", y se señalan las ventajas de los sistemas adaptativos SAA.

#### ABSTRACT

Chromatic effects in shift-and-add (SAA) adaptive systems are considered by means of computer simulations. Chromatic effects under investigations arise when measurements and corrections are performed in different wavelengths. A study of chromatic properties of SAA technique is useful for designing of SAA adaptive systems. Also it is important for data reduction of speckle-spectroscopic observations. The comparison of two approaches of low order adaptive corrections (SAA and tip-tilt) is presented. The advantages of SAA adaptive systems are outlined.

 $\begin{array}{c} \textit{Key Words: instrumentation: adaptive optics - techniques: high angular resolution - techniques: interferometric - techniques: speckle \end{array}$ 

## 1. INTRODUCTION

There are different methods in speckle interferometry (Labeyrie 1970) which allow one to recover the diffraction-limited information related to astronomical objects. One of them is the shift-and-add (SAA) approach which was introduced by Bates and Cady (Bates & Cady 1980) for the a posteriori data reduction of the sets of short-exposure images (specklograms). The method finds the brightest point in each image, shifts the image in such a way that this point is moved to the image center, and then averages all the shifted images together. The resulting SAA image contains a telescope diffraction-limited image sitting on top of a seeing-limited "pedestal".

However, the SAA technique can be applied not only for the a posteriori data reduction, but also for the real-time adaptive correction. Comparing to an advanced adaptive system which assumes a complicated wavefront sensor and a flexible mirror, a SAA one is very simple and inexpensive, because it consists of just two main elements: a CCD camera and a tip-tilt corrector.

The comparisons of the SAA approach to the modal compensation have been published by many authors (Christou 1991; Roddier, Northcott, & Graves 1991) which show that the SAA technique is always more efficient than the tip-tilt correction. Also for point sources there is evidence for an improvement of the image quality with SAA methods over those with the bispectrum (Klückers et al. 1996).

Now, to avoid future confusions, we have to clarify one terminological point. There are many investigations related to the real-time SAA correction (Weintraub et al. 1996). However, in this paper we are talking about SAA adaptive correction rather than about a real-time SAA correction. The difference between them is as follows: the real-time SAA correction is performed shifting and adding digitalized images, while for the SAA adaptive correction a mechanical image shifting is carried out, using a tiptilt corrector. In what follows we call such a system as SAA adaptive system.

In the traditional application of SAA (data reduction a posteriori) there are certain restrictions of the method, and two of them are quite serious. The first one is that the technique can be applied for bright objects only; and the second one is that it can not be used for spectroscopy. However as soon as the method is implemented for the real-time adaptive correction (i.e. shifting images mechanically with a tip-tilt corrector), both restrictions mentioned above can be overcome. For example, weak stars can be observed with the SAA adaptive system using bright reference stars.

It is well known that the SAA technique allows one to reach the diffraction-limited resolution and it gives better results than the tip-tilt. In other words, the adaptive SAA is a low-order adaptive optics (AO) system with high resolution. The detailed comparison of an image quality for tip-tilt and SAA techniques was done by Christou (1991). Also SAA is better than the autocorrelation for determining magnitude differences (Bagnuolo 1983).

Another attractive feature of the SAA technique is that the SAA image always contains a significant diffraction-limited component. This means that the diffraction-limited image can be obtained using a simple deconvolution procedure. Also, due to the presence of the diffraction-limited peak, SAA adaptive systems can be successfully used for spectroscopy.

The chromatic difference problem can appear in two-wavelength SAA observations (Tomono, Doi, & Nishimura 2000). The two-wavelength SAA technique is a solution for the case when the object has low flux in the waveband of interest A but the flux in the other waveband B is sufficient for using SAA. In this case short exposure images in the A band can be corrected utilizing the B band images taken simultaneously as position references. In addition, the problem can appear in spectroscopic observations such as speckle-spectroscopic (Baba 1989) and spectroscopy with adaptive SAA.

Summing up the above considerations, we can conclude that SAA adaptive systems have many attractive features which can be used for astronomical observations. However, a successful development of such systems needs certain preliminary investigations. In this paper we investigate chromatic effects in SAA adaptive systems by means of computer simulations. Such a study is important for the usage of SAA adaptive systems with reference stars, for the two-wavelength shift-and-add method (Tomono et al. 2000), for spectroscopy, and for speckle spectroscopy (Afanasev et al. 1988).

#### 2. SIMULATIONS

The main goal of our simulations is to study chromatic effects in SAA adaptive systems for the case of the correction of atmospherically-induced degradations of astronomical images. Additionally, in order to show the advantages of the SAA adaptive system, we also compare its efficiency to the efficiency of tiptilt corrections.

The simulations are performed for D = 2.1 m (obscuration ratio is equal to 0.237) that corresponds to the existing telescope at the San Pedro Mártir (SPM) observatory. Atmosphericallydistorted wavefront samples at the telescope aperture are generated using the random wave vectors (RWV) method (Voitsekhovich et al. 1999). The following RWV parameters are used in simulations:  $L_0 = 10$  m,  $K_1 = 0$ ,  $K_2 = 10^4$ , M = 100, where  $L_0$  denotes the outer scale of the turbulence,  $K_1$  and  $K_2$  are lower and upper boundaries of the spatial frequencies, and M is the number of harmonics. All the samples are simulated for a seeing equal to 0.7 arcsec, which is typical for the SPM observatory.

The image samples at the telescope focal plane (specklegrams) are calculated by applying FFT to the samples of atmospherically-distorted fields at the aperture. The spatial scale of specklegrams is chosen equal to 0.03 arcsec/pixel, which corresponds to the good speckle-imaging sampling for the 2.1 m telescope in the visible and to the perfect one in the infrared wavebands.

To take into account chromatic effects we assume that measurements and corrections are performed in different wavelengths. From a mathematical viewpoint it can be considered as follows.

Let us assume that we have a set of N images  $I_k(x, y)$ , where x, y stand for the spatial coordinates, and k denotes the image number. Because in what follows we will compare SAA adaptive correction to the tip-tilt one, the corresponding expressions for both types of correction are given below. The resulting (long-exposure) tip-tilt corrected image (tit-tilt correction) is expressed as:

$$I_{c}(x,y) = \sum_{k=1}^{N} I_{c}(x - x_{k}^{c}, y - y_{k}^{c}) , \qquad (1)$$

where

$$\begin{aligned} x_k^c &= \int x I_k(x, y) dx dy \Big/ \int I_k(x, y) dx dy , \\ y_k^c &= \int y I_k(x, y) dx dy \Big/ \int I_k(x, y) dx dy . \end{aligned}$$
(2)

The resulting SAA image  $I_s$  is calculated as:

$$I_{s}(x,y) = \sum_{k=1}^{N} I_{k}(x - x_{k}^{s}, y - y_{k}^{s}) , \qquad (3)$$

where  $x_k^s, y_k^s$  are the coordinates of the brightest point in the k-st specklegram.

To study chromatic effects, it is enough to assume that  $x_k^s, y_k^s$  are measured at the wavelength  $\lambda_2$  and then the results are applied to shift an image measured at the wavelength  $\lambda_1$ . In this case the resulting SAA image  $I_s^{\lambda_1,\lambda_2}$  is expressed as:

$$I_{s}^{\lambda_{1},\lambda_{2}}(x,y) = \sum_{k=1}^{N} I_{k}^{\lambda_{1}}\left(x - x_{k}^{s,\lambda_{2}}, y - y_{k}^{s,\lambda_{2}}\right),$$
(4)

where  $x_k^{s,\lambda_2}, y_k^{s,\lambda_2}$  are the coordinates of the brightest point in the k-st specklegram which is formed in the  $\lambda_2$  wavelength.

In the next section we present and analyze  $I_s^{\lambda_1,\lambda_2}(x,y)$  as a function of two parameters  $\lambda_1$  and  $\lambda_2$ . To get the final results, 5000 samples for 8 wavelengths ( $\lambda = 517, 582, 655, 776, 873, 982, 1035,$  and 1164 nm) are used. All resulting images were normalized to the theoretical maximum peak intensity of a perfect telescope working at the diffraction limit.

#### **3. QUALITATIVE RESULTS**

At the beginning of this section we compare qualitatively the efficiency of the SAA adaptive correction to the tip-tilt one, assuming that measurements and corrections are performed at the same wavelength. In Figures 1 and 2 we plot long-exposure images corresponding to the two types of correction. Also as a reference we show a long-exposure image obtained without any correction. Figure 1 presents the results for the visible waveband ( $\lambda = 517$  nm), while Figure 2 shows the images for the infrared case ( $\lambda = 1164$  nm). Comparing the curves presented in Figures 1 and 2, one can clearly see the advantages of SAA technique over the tip-tilt one. Also SAA technique allows one to get a diffraction-limited peak that is not reachable with tip-tilt correction.

Now let us consider how chromatic effects affect the quality of the SAA correction. From the physical point of view this corresponds to the case when we are observing the SAA corrected image in a certain wavelength, while the measurements of brightest points needed for the correction are performed at another wavelength. The results corresponding to the above case are shown in Figures 3–7.

Figure 3 illustrates the case when we are observing the SAA corrected image in the visible ( $\lambda =$  517 nm) but the measurements of the brightest



Fig. 1. Comparison of long-exposure images for the visual waveband ( $\lambda = 517$  nm). Solid line – SAA correction, dashed line – tip-tilt correction, dotted line – no correction. The intensity is normalized to the theoretical maximum peak intensity of a perfect telescope working at the diffraction limit.



Fig. 2. Comparison of long-exposure images for the infrared waveband ( $\lambda = 1164$  nm). Solid line – SAA correction, dashed line – tip-tilt correction, dotted line – no correction.

points are performed at longer wavelengths:  $\lambda = 517$  nm, 776 nm, 1164 nm.

Analyzing Figures 3–7, one can conclude that the efficiency of the SAA technique decreases with increasing difference between the wavelengths of observed and reference images. However, the chromatic region where the SAA correction gives good results is large enough to be efficient for many practical applications, as is clearly seen from Figure 5.

### 4. QUANTITATIVE RESULTS

In this section we consider how chromatic phenomena affect the quality of a SAA corrected image.



Fig. 3. The SAA image in the visual ( $\lambda = 517$  nm) corrected with the data obtained at longer wavelengths ( $\lambda = 655$  nm and  $\lambda = 1164$  nm).



Fig. 4. The SAA image in the infrared ( $\lambda = 1164$  nm) corrected with the data obtained at shorter wavelengths ( $\lambda = 873$  nm and  $\lambda = 517$  nm).



Fig. 5. Top row: SAA correction for different wavelengths using data obtained at  $\lambda = 873$  nm. Central row: Tip-tilt correction for different wavelengths using data obtained at  $\lambda = 873$  nm. Bottom row: long-exposure images without correction.



Fig. 6. Strehl Ratio for  $\lambda = 776$  nm. Solid line – SAA correction, dashed line – tip-tilt correction, dotted line – no correction.



Fig. 7. Strehl Ratio  $R_{\text{SAA}}, R_{\text{T}-\text{T}}$  as a function of  $\lambda_1$  for different  $\lambda_2$ .

Figure 7 shows the Strehl ratio  $R_{\rm SAA}$  of an image at the wavelength  $\lambda_1$  corrected using the data of a reference image obtained at the wavelength  $\lambda_2$ . As one can see, the Strehl ratio  $R_{\rm SAA}$  of the corrected image almost everywhere is greater than the Strehl ratio  $R_{\rm T-T}$  of the tip-tilt corrected image. However, the Strehl ratio does not give us enough information about SAA-corrected images, because these images are composed of two components: a telescope diffraction-limited image and a seeing-limited "pedestal". One of them is an atmospheric limited component. The second one is a diffraction limited component. In order to analyze the quality of the SAA-corrected images, let us introduce a Diffraction Ratio as:  $DR = R_{\rm SAA}/R_{\rm Long} - 1$ .



Fig. 8. Diffraction ratio DR as a function of  $\lambda_1$  for different  $\lambda_2$ .

The physical meaning of the DR parameter is simple: it shows how high the diffraction component is. Figure 8 shows DR as a function of  $|\lambda_1 - \lambda_2|$ . One can see that the diffraction component in the SAA images is bigger than the atmospheric component for an extended region of differences  $|\lambda_1 - \lambda_2|$ .

To compare the efficiency of the SAA technique to that of the tip-tilt method, we calculate the gain Gm expressed as:  $Gm = 2.5 \log(R_{\text{SAA}}/R_{\text{T-T}})$ . The results are shown in Figure 9.

#### 5. CONCLUSIONS

We have investigated how chromatic phenomena affect a quality of SAA-corrected images. The results obtained allow us to conclude that the "twowavelength" approach (when measurements and corrections are performed in different wavelengths) can successfully be used in practice.

The advantages of the SAA adaptive correction over the tip-tilt one have been considered. The main SAA advantage is the existence of a diffractionlimited peak which can be used for both spectroscopic observations and for the image deconvolution. Also, when using a two-wavelength approach, the efficiency of the tip-tilt technique is decreasing with decreasing wavelength faster than that of SAA. However, from the technical viewpoint, an adaptive SAA system is just a little more complex than a tiptilt one.



Fig. 9. Gain in Strehl Ratioo as a function of  $\lambda_1$  for different  $\lambda_2$ .

Based on the results obtained, we believe that SAA adaptive systems installed on SPM telescopes will give many new and interesting results both for spectroscopic observations and for direct imaging.

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