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THE KINEMATICS AND VELOCITY ELLIPSOID OF THE G III STARS

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RESUMEN

Para estudiar la cinemática de las estrellas G gigante (clase de luminosidad III) se usan movimientos propios y paralajes de 3,075 estrellas, de las cuales 658 tienen velocidades radiales. Estas estrellas provienen de la reducción nueva hecha por van Leeuwen del catálogo Hipparcos. La solución da para la velocidad solar 16.72 ± 0.41 km s^{-1}; para las constantes de Oort, en unidades de km s^{-1} kpc^{-1}, $A = 14.05 \pm 3.28$ y $B = -9.30 \pm 2.87$, valores que representan una velocidad local de rotación de 198.48 ± 26.95 km s^{-1} si suponemos una distancia al centro Galáctico de 8.2 ± 1.1 kpc. Para las dispersiones de velocidades obtenemos, en unidades de km s^{-1}: $\sigma_x = 51.78 \pm 0.55$, $\sigma_y = 42.81 \pm 0.32$, $\sigma_z = 28.45 \pm 0.22$ con una desviación del vértice de $3.°88 \pm 6.°62$. Una comparasión de esta dispersión con las obtenidas de otras clases espectrales indica que la discontinuidad de Parengo existe también para las estrellas gigantes.

ABSTRACT

To study the kinematics of the G giant stars (luminosity class III) use is made of proper motions and parallaxes taken from van Leeuwen's new reduction of the Hipparcos catalog. 3,075 stars, of which 658 have radial velocities, were used in the final study. The solution gives: solar velocity of 16.72 ± 0.41 km s^{-1}; Oort's constant's, in units of km s^{-1} kpc^{-1}, $A = 14.05 \pm 3.28$ and $B = -9.30 \pm 2.87$, implying a rotational velocity of 198.48 ± 26.95 km s^{-1} if we take the distance to the Galactic center as 8.2 ± 1.1 kpc; velocity dispersions, in units of km s^{-1}, of: $\sigma_x = 51.78 \pm 0.55$, $\sigma_y = 42.81 \pm 0.32$, $\sigma_z = 28.45 \pm 0.22$ with a vertex deviation of $3.°88 \pm 6.°62$. A comparison of the velocity dispersions with those given by other spectral types shows that Parenago's discontinuity also exists for the giant stars.

Key Words: Galaxy: kinematics and dynamics — methods: numerical

1. INTRODUCTION

This paper continues a series on the kinematics and velocity ellipsoids of the giant stars (luminosity class III). Previously studied were the O-B5 giants (Branham 2006), the M giants (Branham 2008), the B6-9 and A giants (Branham 2009a), the K giants (Branham 2009b), and the F giants (Branham 2010). The G giants fill the remaining lacuna and complete the investigation of all of the giant stars. A search of the ADS data base[1] shows that the G giants as a group have not been studied since the research of Parenago in 1951 (Delhaye 1965), which adds impetus to this current study.

The methodology remains similar to that for the previous studies, so similar that I will eschew presentation of the mathematical development and refer the reader to the relevant previous publications where the necessary equations can be found. As in my investigation of the F giants the velocity ellipsoid calculation uses the singular value decomposition (SVD) to include stars for which only tangential velocities, but no radial velocities, are available. The section on the equations of condition briefly discusses this matter.

[1] `http://adswww.harvard.edu/`.

Little evidence exists to suggest that G stars form part of the Gould belt. There appears to be a clean break between the O-B stars and the later spectral types regarding participation in the Gould belt; O-B stars have definite Gould belt members whereas the others do not. Nevertheless, a plane will be fit to the G giants in a later section, but to examine the randomness of the data, not to infer that some G giants actually belong to the Gould belt.

To summarize briefly the mathematical procedure used, one solves for the kinematics and velocity ellipsoid of the G giants by use of semi-definite programming (SDP), which forces the solar motion calculated from the velocity ellipsoid to be the same as that calculated from the kinematical parameters. Nor is it necessary to use the same adjustment criterion for the two sets of calculations: the kinematical parameters may be reduced by use of a least squares criterion whereas the velocity ellipsoid may be calculated with the robust L_1 criterion (minimize the sum of the absolute values of the residuals), or with the same L_1 criterion for both. For a readable discussion of SDP see Vandenberge & Boyd (1996).

In their classical work *Statistical Astronomy*, Trumpler & Weaver (1962) refer to two incompleteness factors, K_1, which compensates for the deficiency of proper motions in a parallax catalog compared with a proper motion catalog, and K_2, which corrects for the absence of proper motions nearly in the line of sight and thus not detectable in either a proper motion or a parallax catalog. This study, however, shows that the calculation of incompleteness factors for the G giants is unnecessary or counterproductive.

2. THE OBSERVATIONAL DATA

The proper motions and parallaxes used in this study were taken from van Leeuwen's version of the Hipparcos catalog (van Leeuwen 2007), henceforth called simply the Hipparcos catalog, the radial velocities from the Wilson (Nagy 1991) and Strasbourg Data Centre (Barbier-Brossat & Figon 2000) catalogs. van Leeuwen's catalog (2007) omits a few stars contained in the original catalog (ESA 1997). For those few stars the relevant data were taken directly from the original catalog. The equinox of the Hipparcos catalog is J2000 and the catalog epoch is J1991.25. Stars listed as spectral class G, luminosity class III were extracted from the catalog. This resulted in a total of 3,075 G giants, of which 658 have radial velocities. The G giants are skewed towards the later giants; over 94% fall between G5 and G9, and the G8 stars alone account for 64.9% of the total.

The star's HD number determined if either of the two radial velocity catalogs contained an entry for that particular star. Not all of the data could be accepted. Negative parallaxes were excluded as were parallaxes smaller than 1 mas because the Ogorodnikov-Milne (OM) model was used for the equations of condition (Ogorodnikov 1965, pp. 61–63). This model, valid out to about 1 kpc, should be adequate because the minimum parallax used in this study, 1 mas, corresponds to a distance of 1 kpc. For a justification of this distance limit see Smart (1968, p. 285). Parallaxes smaller than 1 mas have such large mean errors that their inclusion seems unwarranted because of the uncertainty in their distances. Known multiple stars, flagged in the catalog, contaminate the proper motion by confusing orbital motion with genuine proper motion and were also excluded. And some of the solutions for the astrometric data in the catalog, also flagged, are substandard and were likewise excluded. Smith & Eichhorn (1996) have derived a procedure to correct the observed parallaxes, and this procedure was used to transform all of the parallaxes used in this study. In my study of the M giants (Branham 2008) I show that the Smith-Eichhorn correction seems to leave little residual parallax error.

What about the quality of the data? I have already commented on the high quality of the Hipparcos proper motions (Branham 2009b). Tangential velocities calculated from proper motions, therefore, should also be high quality. The radial velocities, however, come from disparate sources incorporated into the Wilson and the Strasbourg Data Center catalogs. My study of the F giants showed lower homogeneity in the radial velocities. One could perform a similar analysis with the G giants, but this seems otiose. The data are what they are and must be used as they are. The radial velocities, moreover, are not used alone but multiplied by the parallax in the equations of condition. A runs test shows that the radial velocities are of somewhat lower quality than the tangential velocities, but of acceptable quality. See the § 6. The runs test measures how often a variable, distributed about the mean, changes sign from plus to negative or negative to positive. The changes of sign, the runs, have a mean for n data points of $n/2 + 1$ and a variance of $n(n-2)/4(n-1)$. An advantage of the runs test over other tests for randomness resides in its being nonparametric, making no assumption about the normality of the data, although to actually calculate probabilities for the observed runs one does assume approximate normality. For a detailed description of the runs test see Wonnacott & Wonnacott (1972, pp. 409–411).

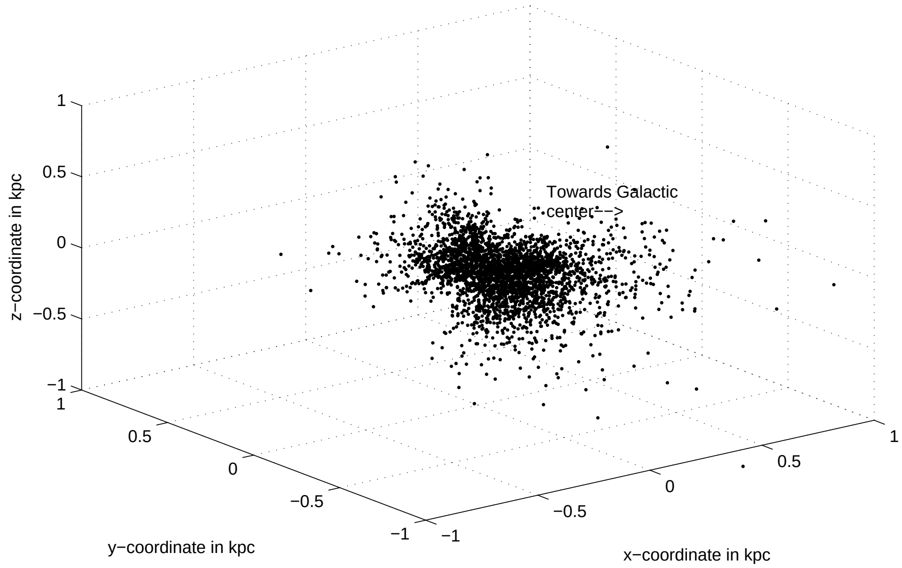

Fig. 1. Space distribution of G giants.

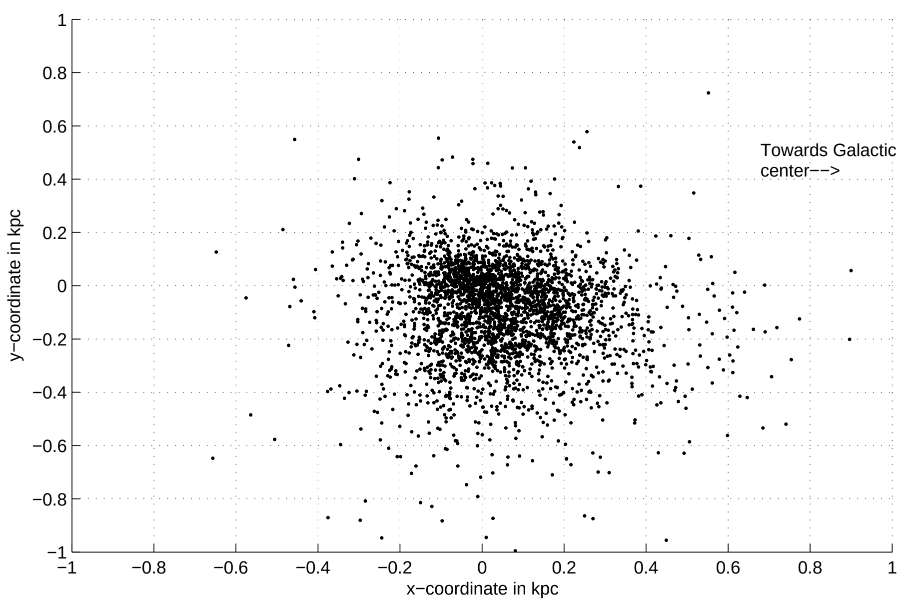

Fig. 2. Distribution in $x - y$ plane.

3. THE SPACE DISTRIBUTIONS

Let x, y, z be rectangular coordinates with origin at the Sun: x points towards the Galactic centre, y is perpendicular to x in the direction if increasing l, and z is positive for positive Galactic latitude. From ϖ, the star's parallax, l, its Galactic longitude, and b, its Galactic latitude, we calculate

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{\varpi} \begin{pmatrix} \cos l \cos b \\ \sin l \cos b \\ \sin b \end{pmatrix} . \tag{1}$$

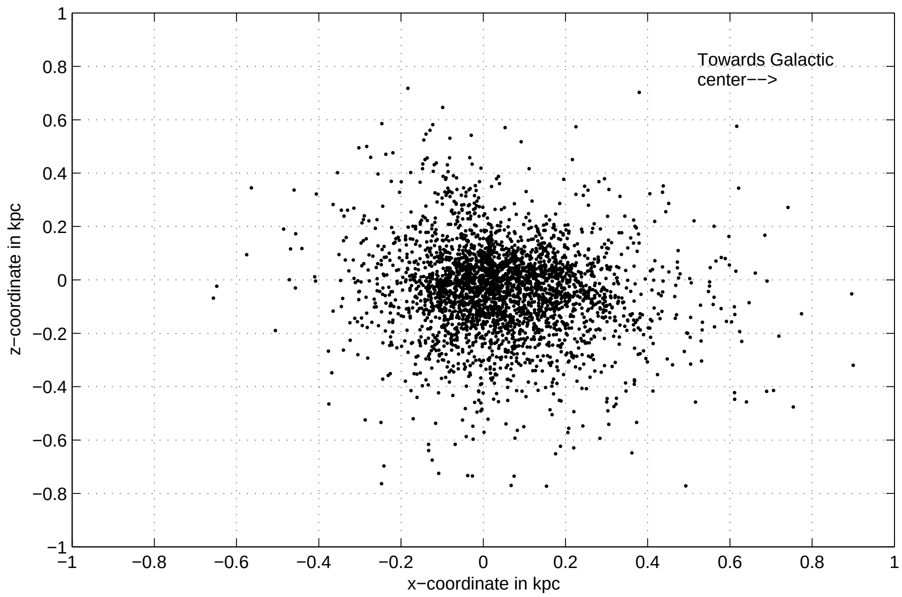

Fig. 3. Distribution in $x - z$ plane.

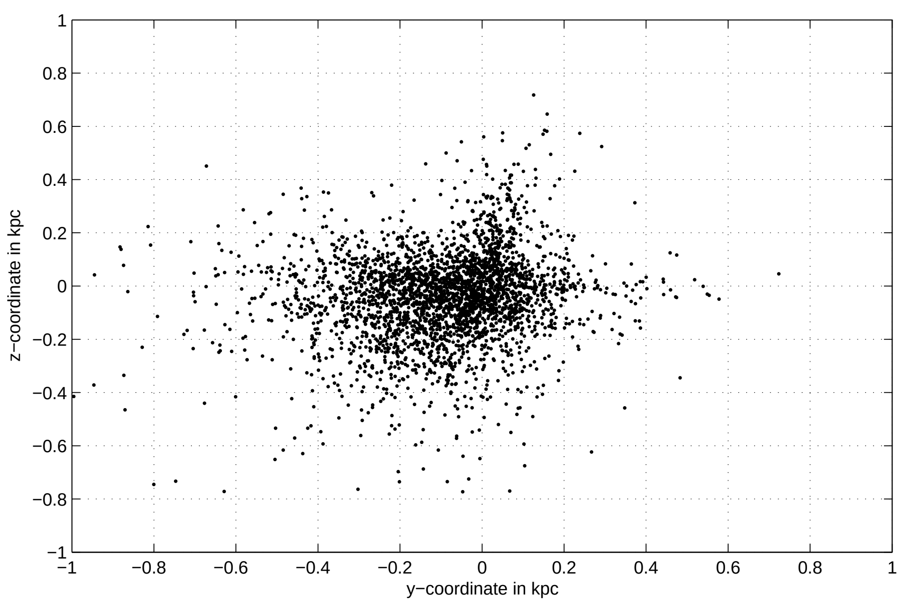

Fig. 4. Distribution in $y - z$ plane.

Figure 1 shows the distribution of the G giants in space, Figures 2, 3, and 4 the distributions in the $x - y$, $x - z$, and $y - z$ planes. Define a moment matrix, referred to the centroid of the distances, $\bar{x}, \bar{y}, \bar{z}$, from the x, y, z:

$$\begin{pmatrix} \sum_i (x_i - \bar{x})^2 & \sum_i (x_i - \bar{x})(y_i - \bar{y}) & \sum_i (x_i - \bar{x})(z_i - \bar{z}) \\ \sum_i (y_i - \bar{y})(x_i - \bar{x}) & \sum_i (y_i - \bar{y})^2 & \sum_i (y_i - \bar{y})(z_i - \bar{z}) \\ \sum_i (z_i - \bar{z})(x_i - \bar{x}) & \sum_i (z_i - \bar{z})(y_i - \bar{y}) & \sum_i (z_i - \bar{z})^2 \end{pmatrix} . \tag{2}$$

Before use of the moment matrix outliers should be eliminated from the distances. The criterion selected for the cutoff was five times the median of the distances to the stars. This cutoff results in only a sparse trim of the data. Stigler (1977) has shown that modest trimming works better than extreme trimming. Although the eigenvalues of the moment matrix indicate little concentration towards the Galactic plane, 129.7, 94.0, and 80.9 pc^2, the normalized eigenvector associated with the z-axis points towards $b_g = 55.^\circ 671$, a significant tilt with respect to the Galactic plane. This tilt, however, seems to be a selection effect associated with the giants rather than a tilt associated with the Gould belt. Look at the G stars as a whole without discrimination as to luminosity class. There are 82 supergiants, 184 bright giants, 3,075 giants, 1,152 subgiants, 4,656 main sequence, and 9,332 unspecified luminosity class G stars. With this heterogeneous group the tilt becomes $79.^\circ 494$. Further evidence that the tilt seems not associated with the Gould belt arises from an attempt to fit a plane to the G giants. Branham (2003) outlines the procedure for doing this. 1,389 stars are classified as "Gould belt", but the tilt of the plane of the remaining, supposedly "Galactic belt", stars becomes even worse, $35.^\circ 58$, and with high correlations, up to 40%, among the x, y, z coordinates.. Thus, the procedure that works so well with the O-B5 stars to discriminate between Gould belt and Galactic belt stars fails completely for the G giants.

That the tilt should not unduly bias the solution can be inferred from a look at the randomness of the rectangular coordinates. The correlation between x and y is -10.1%, between x and z -12.9%, and between y and z 11.3%. Regarding the randomness in distance a runs test yields, after elimination of discordant distances, 1,487 runs out of an expected 1531 implying an 11.3% chance that the distances are random. The statistics for the G giants, therefore, indicate *relative* randomness.

4. EQUATION OF CONDITION FOR KINEMATICS AND THE VELOCITY ELLIPSOID

The equations of condition, given in detail in Branham (2009a) and which come from Ogordnikov (1965, pp. 74–75), involve twelve unknowns for the kinematical parameters: the components of the reflex solar motion X, Y, Z and the components of the displacement tensor $u_x, u_y, u_z, v_x, v_y, v_z, w_x, w_y, w_z$. All of these quantities are referred to Galactic latitude and longitude, l and b, rather than right ascension α and declination δ. Proper motions in α and δ are converted to proper motions in l and b expressed in milli-arc-seconds (mas) per year; radial velocity is expressed in km s^{-1}. Parallax ϖ is also expressed in mas. These kinematical parameters are calculated from the least squares criterion because there are fewer discordant observations, handled by a 2.5% trim of the data, than with the velocity ellipsoid calculation, for which the robust L_1 criterion is indicated. As with linear programming, one does this by minimization of an objective function. If r_i is one of m residuals from a solution for the kinematical parameters and r is the m-vector of the residuals, then we impose the condition $r^T \cdot r - \tau = 0$, where τ is an arbitrary parameter, and minimize τ in the objective function.

A sparse trim of the data seems indicated not only by what Stigler (1977) has found, light trimming or even no trimming works better than extreme trimming, but by what a General Colby reported to the Astronomer Royal, Sir George Airy (Airy 1854). In a geodetic adjustment for the determination of the scale of longitudes for England, inclusion of all data rather than just the most concordant data gave results that went from, to use Airy's words, "considerably in error" to "perfectly good."

The equations as derived by Ogorodnikov actually use the distance $1/\varpi$ rather than the parallax ϖ itself, but it is important to recast the equations to remove the parallax error from the denominator and thus ameliorate any possible Lutz-Kelker bias. The equations, therefore, are multiplied by ϖ, which places the parallax in the numerator.

To calculate the velocity ellipsoid use $\dot{x}, \dot{y}, \dot{z}$, the space velocities of a star, found from the proper motions and radial velocity and expressed in km s^{-1} are:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} -\sin l & -\cos l \sin b & \cos l \cos b \\ \cos l & -\sin l \sin b & \sin l \cos b \\ 0 & \cos b & \sin b \end{pmatrix} \cdot \begin{pmatrix} \kappa \mu_l \cos b/\varpi \\ \kappa \mu_b/\varpi \\ \dot{r} \end{pmatrix}. \tag{3}$$

If radial velocities are unavailable, then we seem unable to calculate the space velocities. This, however, can be done if we use the SVD to employ only the tangential velocities. See Branham (2010) for details. There are other possibilities. Fuchs et al. (2009) use a deprojection formulation for the proper motions that assumes the lines of sight towards the stars are statistically uncorrelated with the velocities of the stars. The Section 7 shows that the SVD approach satisfies the assumption of statistical randomness and seems, therefore, satisfactory.

One fits a quadric surface with ten coefficients, $a, b, c, d, e, f, g, h, k, l$, to these velocities. To assure that the equation indeed corresponds to an ellipsoid one must impose the condition that the matrix of the a, b, c coefficients

$$A = \begin{pmatrix} a & c/2 & d/2 \\ c/2 & b & e/2 \\ d/2 & e/2 & c \end{pmatrix},$$

in fact be positive-definite and symmetric. To avoid the trivial solution $a = b = \cdots = q = 0$ another condition must be imposed. The one I use is that the volume of the ellipsoid must be a maximum. Because the volume is proportional to the determinant of A, the condition becomes $\det(A) = \max$. An eigenvalue-eigenvector decomposition of the matrix A yields the axes of the velocity ellipsoid and their orientation with respect to the Galactic coordinate system. Because of generally greater error in the data, the coefficients $a, \ldots, l$ are calculated by use of the robust L_1 criterion. SDP allows one to combine without difficulty the least squares criterion for the kinematical parameters and the L_1 criterion for the velocity ellipsoid. For the latter if there are n space velocities with n corresponding residuals $r_{sv,i}$, let $\gamma_i, i = 1, \ldots, n$, be n *positive* parameters. Then minimize the γ in the objective function subject to the conditions

$$\mathrm{diag}(\gamma + r_{sv}) > 0; \qquad \mathrm{diag}(\gamma - r_{sv}) > 0; \qquad \mathrm{diag}(\gamma) > 0.$$

The SDP formulation of the velocity ellipsoid calculation along with the L_1 criterion for the minimization of the residuals offers advantages over competing methods. The ellipsoid calculated is unique and represents a global minimum of the reduction criterion when that criterion is the L_1 (Calafiore 2002). Bochanski, Hawley, & West (2011) prefer a geometric simplex minimization with the least squares criterion. Although simplex minimization can be used with the L_1 criterion, thus making the procedure robust-see the algorithm in Branham (1990, pp. 191–197), the method can converge to a local rather than to a global minimum, particularly when many parameters are being fit, unless a good starting approximation is available. Pourbaix (1998) , using the simplex algorithm to calculate orbits of double stars, estimates that when n parameters are being fit there are $\approx \mathrm{O}(e^n)$ local minima. He implements a simulated annealing modification of the simplex algorithm to reject the local minima. With SDP and the L_1 criterion such a strategy becomes unnecessary because the minimum *is* global.

The solar velocity, $S_0 = \sqrt{X^2 + Y^2 + Z^2}$, calculated from both the solution for the kinematical parameters $X, Y, \ldots, w_y, w_z$ and the coefficients of the velocity ellipsoid must be the same. This condition can be imposed when one uses the SDP formulation of the problem.

5. SOME CORRECTIONS TO THE OBSERVATIONS AND COVARIANCE MATRICES

The total space motions needed in the velocity ellipsoid calculation should be corrected for the effects of Galactic rotation by modifying the proper motions and radial velocities used in the calculations to remove the rotation. This was done by the same procedure used in Branham (2009a).

In theory one should also apply a correction for the incompleteness of the sample of the G giant stars taken from the Hipparcos catalogue. Trumpler & Weaver (1962, p. 374) define a factor of incompleteness K_1 as

$$K_1 = \frac{N(m, \mu)}{N_\varpi(m, \mu)}, \tag{4}$$

where $N(m, \mu)$ is the number of stars in the sky for magnitude interval $m \pm \Delta m/2$ and proper motion interval $\mu \pm \Delta\mu/2$ and $N_\varpi(m, \mu)$ is the number of stars in the parallax catalogue for the same intervals. Equation (4) is difficult to apply if there is insufficient overlap between the proper motion catalog and the parallax catalog. For the Hipparcos parallaxes a logical proper motion catalog would be the Tycho II catalog (Høg et al. 2000). But for K_1 the sparse overlap between the two catalogs assures that the factor becomes large with large mean errors. One must question whether such corrections are realistic and should be applied. I feel they should not and that the randomness of the data is more important.

Trumpler & Weaver (1962, p. 375) also define a second incompleteness factor, K_2, to correct for the absence of proper motions in the parallax catalog nearly along the line of sight and hence undetectable. K_2 depends

TABLE 1

SOLUTION FOR KINEMATIC PARAMETERS FOR THE G III STARS

Quantity	Value	Mean Error
$\sigma(1)$ (mean error of unit weight in mas km s^{-1})	112.44	...
u_x (in mas km s^{-1})	-7.63	6.13
u_y (in mas km s^{-1})	23.35	4.06
u_z (in mas km s^{-1})	-3.59	5.23
v_x (in mas km s^{-1})	4.75	4.63
v_y (in mas km s^{-1})	-8.08	6.00
v_z (in mas km s^{-1})	-6.87	5.21
w_x (in mas km s^{-1})	1.57	4.15
w_y (in mas km s^{-1})	-7.68	3.74
w_z (in mas km s^{-1})	-11.39	5.86
S_0 (solar velocity in km s^{-1})	16.72	0.41
A (Oort constant in km s^{-1} kpc^{-1})	14.05	3.28
B (Oort constant in km s^{-1} kpc^{-1})	-9.30	2.87
$V0$ (Circular velocity in km s^{-1})	198.48	26.95
l_1 (longitude displacement)	$-26.^\circ 45$	$7.^\circ 05$
K (K term in km s^{-1})	-7.85	4.99

TABLE 2

VELOCITY DISPERSION AND VERTEX DEVIATION OF THE G III STARS

Quantity	Value	Mean Error
Mean absolute deviation of residuals in km s^{-1}	11.00	...
S_0 (solar velocity in km s^{-1})	16.72	1.05
σ_1 (velocity dispersion in x in km s^{-1})	51.78	0.55
σ_2 (velocity dispersion in y in km s^{-1})	42.81	0.32
σ_3 (velocity dispersion in z in km s^{-1})	28.45	0.22
l_1 (longitude of σ_1)	$3.^\circ 88$	$6.^\circ 62$
b_1 (latitude of σ_1)	$0.^\circ 28$	$0.^\circ 61$
l_2 (longitude of σ_2)	$93.^\circ 93$	$1.^\circ 90$
b_2 (latitude of σ_2)	$9.^\circ 86$	$0.^\circ 65$
l_3 (longitude of σ_3)	$-87.^\circ 72$	$1.^\circ 62$
b_3 (latitude of σ_3)	$80.^\circ 14$	$0.^\circ 49$

on the velocity ellipsoid. Branham (2009b) shows how this incompleteness factor can be evaluated. For the G giants this factor becomes an insignificant $5.6 \cdot 10^{-7}$. Therefore, neither the K_1 nor the K_2 incompleteness factor need be applied.

The covariance matrix to calculate mean errors is given in equation (25) of Branham (2006), and equation (26) of that publication shows how to calculate the errors for quantities, such as the Oort constants, derived from the displacement tensor.

6. RESULTS

After the equations of condition for the kinematical parameters had been formed, I applied two checks for the adequacy of the reduction model. The first check simply calculates the singular values of the matrix of

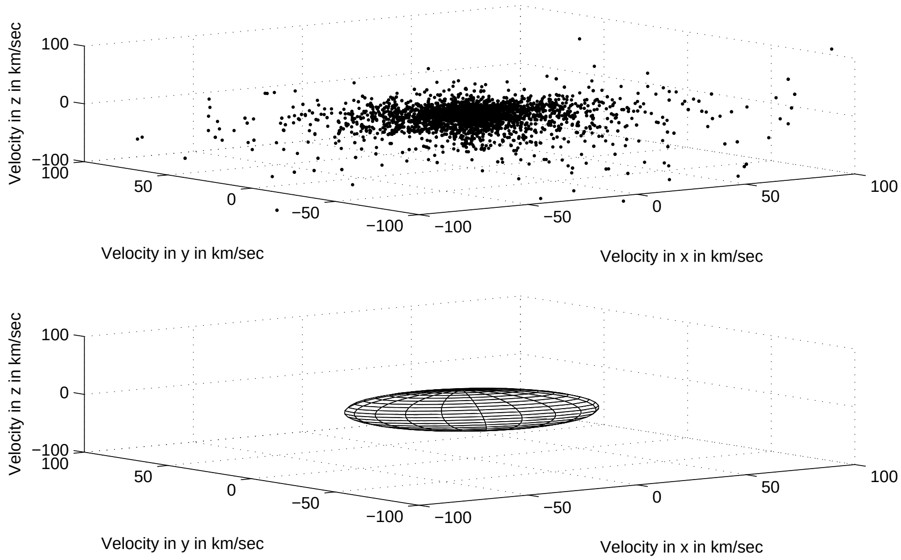

Fig. 5. '.'=rectangular velocity of star (upper); velocity ellipsoid (lower).

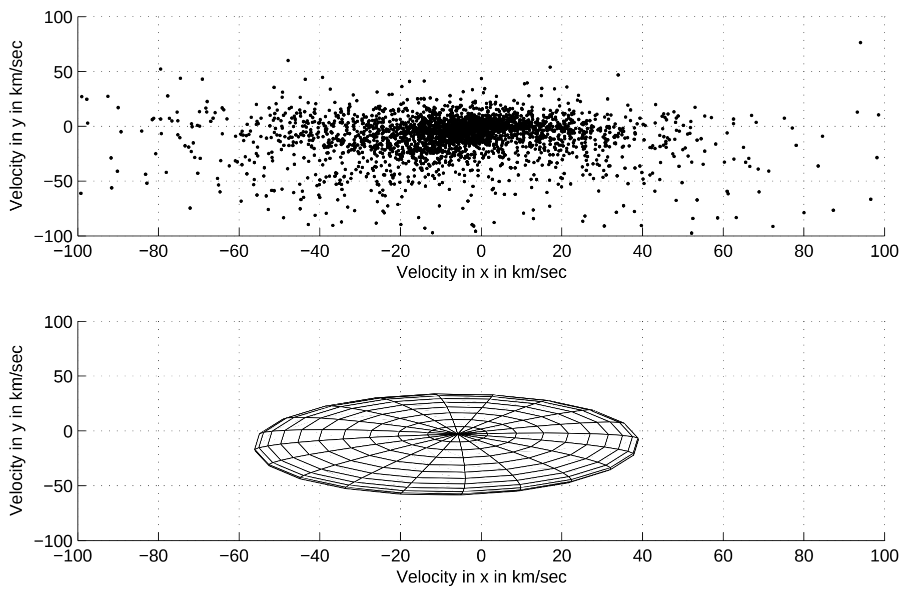

Fig. 6. Ellipsoid in $x - y$ plane.

the equations of condition. An inadequate reduction model, for example one in which some unknowns are strongly correlated, results in a high condition number for the matrix because of small singular values. The condition number of the matrix of the equations of condition for the G giants, however, is low, 22.8. The second check calculates Eichhorn's efficiency (Eichhorn 1990), a parameter that varies from 0 to 1 with 0 indicating redundancy in the parameters and 1 that all parameters are necessary. The efficiency of 0.94 indicates that *all* of the variables in the model are necessary and with little correlation among themselves.

The first solution for the G giants was calculated from all of the equations of condition. These solutions calculated residuals needed to find discordant data. For the G stars the criterion was five times the mean absolute deviation (MAD) of the residuals. This eliminated 168 of the 6,808 equations of condition, a 2.5%

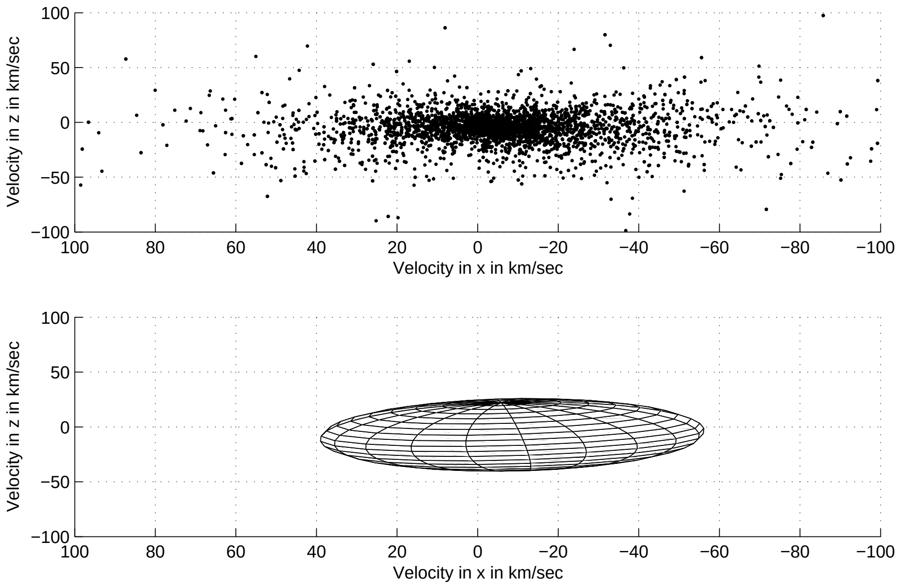

Fig. 7. Ellipsoid in $x - z$ plane.

trim. I have used this elimination criterion in the past, generally with good results, but a further reason exists to justify its use. For the G giants there is a 19.0% chance that the original residuals are random, as calculated by a runs test, but a 78.7% chance that the trimmed residuals are random. Eliminating some of the residuals, therefore, increases the randomness of the sample.

Table 1 shows the solution for the kinematical unknowns and Table 2 for the coefficients and orientation of the velocity ellipsoid for the G stars. For convenience the components for the displacement tensor are converted to the more familiar form of the solar motion, Oort constants, the deviation l_1 between the longitude of the geometric center and the kinematic center of the Galaxy, and K term. Also shown is the circular velocity V_0, found from the relation $V_0 = (A - B)R_0$, where R_0 is the distance to the centre of the Galaxy. Kerr & Lynden-Bell (1986) determine a value of 8.5 ± 1.1 kpc for R_0. Perryman (2008, App. A), however, after a survey of recent determinations feels that 8.2 kpc is a better determination. The mean error for V_0 comes from the procedure given in Branham (2008) and uses 8.2 kpc for R_0 with the same mean error as given by Kerr & Lynden-Bell (1986).

The orientation of the velocity ellipsoid in space and in the $x - y$, $x - z$, and $y - z$ planes is shown in Figures 5–8 for the G giants. (Because of the density of data points, it proved impossible to plot both the stars and the velocity ellipsoid on one graph because the stars blotted out the ellipsoid; therefore a subgraph was used for the stars and another for the ellipsoid.)

7. DISCUSSION

The distribution of the residuals from the kinematical solution, after eliminating discordant residuals, is seen in the histogram of Figure 9. As mentioned in the previous section, 168 of the residuals were eliminated, a 2.5% trim. The distribution is somewhat skewed, coefficient of skewness 0.09, more platykurtic, kurtosis of 1.42, than the normal distribution, kurtosis of 3, and more lighter tailed, Hogg's Q factor of 0.36, than a normal distribution, Q=2.58. The Q factor is defined as

$$Q = \frac{(U_{0.05} - L_{0.05})}{(U_{0.5} - L_{0.5})}, \tag{5}$$

where U_α and L_α are averages of the respective upper and lower 100α of the data (Stigler 1977). A runs test, however, reveals 3,350 runs out of an expected 3,404. The residuals, therefore, can be considered random. To be specific, there is a 19.0% chance that the residuals are taken randomly from a normal distribution.

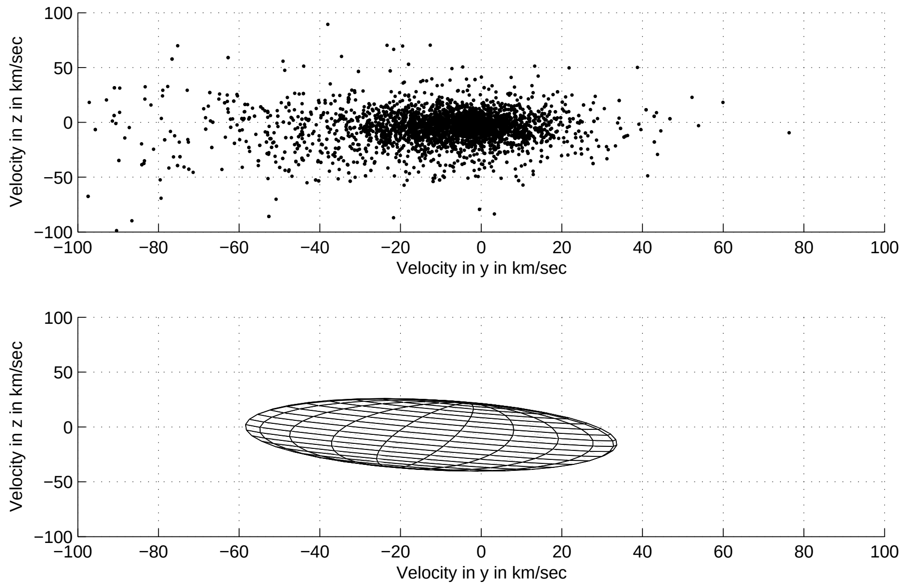

Fig. 8. Ellipsoid in $y - z$ plane.

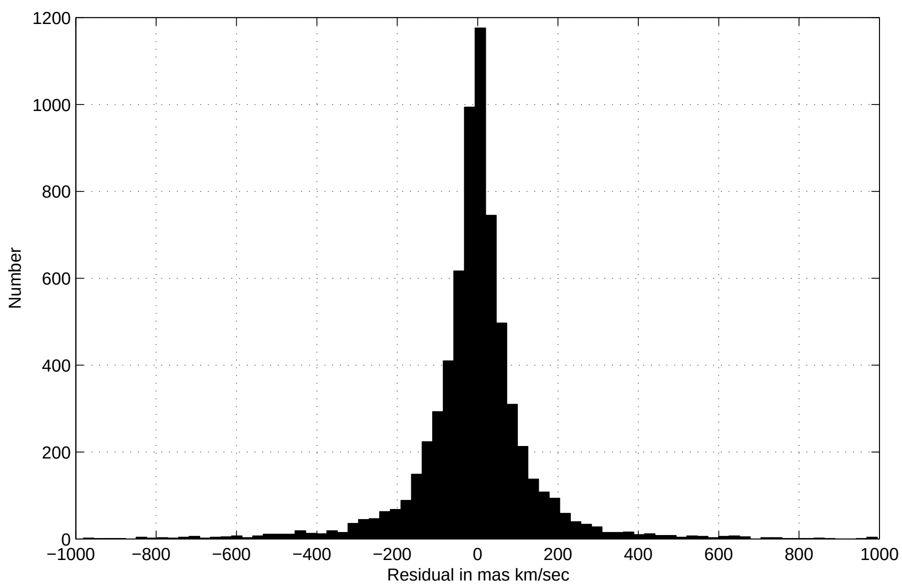

Fig. 9. Histogram of residuals from kinematical solution.

Because we generally use a 5% to 10% limit before rejecting the null hypothesis that a distribution is in fact not random, we infer that although the actual distribution deviates from normality the residuals conform to the null hypothesis of being random. If the runs test is applied separately to the equations of condition arising from the tangential velocities and those from the radial velocities, the former show 3,032 runs out of an expected 3,075 and the latter 303 runs out of an expected 329. This demonstrates that the tangential velocities seem of higher quality than the radial velocities, 31.2% probability of randomness versus 4.6%, but because they are used conjointly the residuals remain relatively random. All of this confirms, along with the singular values and Eichhorn's efficiency, that the reduction model suffers no serious defects and that inclusion of the K_1 incompleteness factor seems unnecessary.

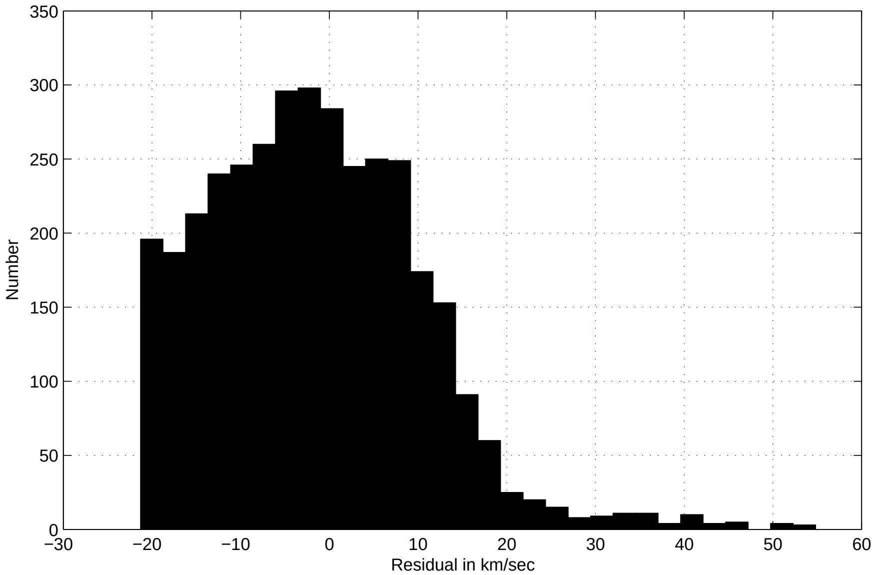

Fig. 10. Residuals in velocity for G giants.

TABLE 3

RECENT DETERMINATIONS OF THE SOLAR VELOCITY

Reference	Class	$-X$ (km s^{-1})	$-Y$ (km s^{-1})	$-Z$ (km s^{-1})	S_0 (km s^{-1})
Branham (2008)	M III	8.99 ± 0.42	20.40 ± 0.40	4.80 ± 0.39	24.20 ± 0.70[a]
Yuan et al. (2008)	O-B5	9.17 ± 0.40	8.66 ± 0.38	5.83 ± 0.34	13.90 ± 0.38[b]
Yuan et al. (2008)	K-M III	18.46 ± 0.32	17.70 ± 0.32	6.32 ± 0.32	20.61 ± 0.32[c]
Aumer & Binney (2009)	mixture IV, V	9.96 ± 0.33	5.25 ± 0.54	7.07 ± 0.34	13.29 ± 0.72
Branham (2009a)	B69 III	9.58 ± 0.39	11.94 ± 0.41	6.03 ± 0.33	16.40 ± 0.40[a]
Branham (2009a)	A III	9.85 ± 0.59	8.00 ± 0.58	5.80 ± 0.52	13.95 ± 0.58[a]
Branham (2009b)	K III	7.53 ± 0.26	19.11 ± 0.26	7.41 ± 0.22	21.83 ± 0.26[a]
Bobylev & Bajkova (2010)	Galactic masers	5.5 ± 2.2	11.0 ± 1.7	8.5 ± 1.2	15.0 ± 3.0
Branham (2010)	F III	10.84 ± 0.49	12.62 ± 0.49	8.14 ± 0.44	16.72 ± 0.41[a]
Schönrich et al. (2010)	F-G V	$11.1^{+0.69}_{-0.75}$	$12.24^{+0.47}_{-0.47}$	$7.25^{+0.37}_{-0.36}$	...
Shen & Zhang (2010)	Galactic Cepheids	12.58 ± 1.09	14.52 ± 1.06	8.98 ± 0.98	21.21 ± 1.81[d]

[a]Mean error calculated from equations (25) and (26) in Branham (2006).

[b]For heliocentric distance 0.2–3 kpc.

[c]For heliocentric distance 0.2–1 kpc.

[d]For heliocentric distance 0.2–3 kpc.

For the residuals from the velocity ellipsoid the situation becomes different, as Figure 10 shows. The residuals deviate even more from a normal distribution, coefficient of skewness 3.62, platykurtic, kurtosis 0.59, and light tailed, Q factor of 0.25. They are, however, even more random than the residuals from the kinematical solution, 1,784 runs out of an expected 1,813. There is thus a 33.7% probability that the residuals are random. This is the principal justification for use of the SVD to calculate the velocity ellipsoid.

Regarding recent determinations of the kinematical parameters Perryman (2008, p. 502) gives his Table 9.3 with pre-2007 results while Table 3 shows some post-2008 values. If we look at the 33 values for the solar velocity in both tables, without discriminating among spectrum-luminosity classes nor weighting by number of stars, there is a range from a minimum of 11.8 km s^{-1} to a maximum of 24.6 with mean 18.29 and standard deviation 3.74. The value in Table 1 falls well within this range. With respect to the Oort constants and

TABLE 4

RECENT DETERMINTIONS OF OORT CONSTANTS AND AUXILIARY QUANTITIES

Reference	Class	A (km s^{-1} kpc^{-1})	B (km s^{-1} kpc^{-1})	$A-B$	$-(A+B)$
Branham (2008)	M III	16.86 ± 2.78	-6.34 ± 2.56	23.20 ± 7.79[a]	-10.52 ± 8.82[a]
Yuan et al. (2008)	O-B5	15.33 ± 0.94	-15.12 ± 0.71	30.45 ± 1.18[b]	-0.21 ± 1.18[b]
Yuan et al. (2008)	K-M III	15.86 ± 1.30	-14.57 ± 1.01	30.44 ± 1.65[c]	1.29 ± 1.65[c]
Bobylev & Bajkova (2010)	Galactic masers	17.8 ± 0.8	-13.2 ± 1.5	31.1 ± 1.7	4.6 ± 1.7
Branham (2009a)	B69 III	11.77 ± 1.66	-9.05 ± 1.38	20.82 ± 3.83[a]	-2.72 ± 1.04[a]
Branham (2009a)	A III	11.48 ± 5.45	-8.29 ± 4.23	19.77 ± 12.09[a]	-3.19 ± 7.44[a]
Branham (2009b)	K III	13.08 ± 1.72	-10.21 ± 1.47	23.29 ± 2.20[a]	2.86 ± 0.82[a]
Branham (2010)	F III	14.85 ± 7.47	-10.85 ± 6.83	23.35 ± 4.07[a]	-4.75 ± 4.63[a]
Shen & Zhang (2010)	Galactic Cepheids	17.42 ± 1.17	-12.46 ± 0.86	29.88 ± 1.45	-4.96 ± 1.45[d]

[a]Mean error calculated from equations (25) and (26) in Branham (2006).
[b]For heliocentric distance 0.2–3 kpc.
[c]For heliocentric distance 0.2–1 kpc.
[d]For heliocentric distance 0.2–3 kpc.

TABLE 5

VELOCITY DISPERSIONS FOR THE GIANT STARS

Spectral type	σ_x (km s^{-1})	σ_y (km s^{-1})	σ_z (km s^{-1})	Number of stars
O-B5	32.44 ± 5.04	26.16 ± 2.75	18.71 ± 2.39	107 total space motion
B6-9	39.25 ± 3.29	10.83 ± 1.16	14.07 ± 0.85	147 total space motion
A	26.95 ± 4.26	23.08 ± 2.14	16.46 ± 0.55	144 total space motion
F	36.89 ± 1.90	24.66 ± 1.16	17.97 ± 0.81	222 total space motion 369 tangential velocity
G	51.78 ± 0.55	42.81 ± 0.32	28.45 ± 0.22	658 total space motion 2417 tangential velocity
K	50.58 ± 0.99	42.42 ± 1.13	32.92 ± 0.56	880 total space motion
M	57.40 ± 1.67	45.86 ± 1.63	33.84 ± 1.02	480 total space motion

associated quantities such as $A - B$ and $-(A + B)$, the former equal to V_0/R_0 and the latter to (dV/dR), Perryman's Table 9.3 shows determinations up to 2007 and Table 4 post-2008 determinations, a total of 28. We see that A ranges from a minimum of 9.6 km s^{-1} kpc^{-1} to a maximum of 19 with a mean of 14.52 and a standard deviation of 2.52, again without discriminating as to number of stars, spectrum or luminosity class, or other indicators. The value given in Table 1 coincides well with this mean. For B the corresponding values are minimum of -24 km s^{-1} kpc^{-1}, maximum of -6.34, mean -12.63, and standard deviation of 3.30. Once again, the value for B given in Table 1 shows no anomaly. The only quantity that shows a possibly discrepant value is the K term, putatively significant only for the early stars, with determinations falling near 5 km s^{-1}, and close to 0 for later spectral types. The value in Table 1, large and moreover negative, seems discrepant. Its mean error, however, is also large and furthermore Branham (2009a) has shown that this term is sensitive to errors in the data; little credence, therefore, should be placed on its value. In a recent paper McMillan & Binney (2010) find that the most probable range for V_0/R_0 falls between 29.9 km s^{-1} kpc^{-1} and 31.6 km s^{-1} kpc^{-1}. Many of the values in Perryman's Table 9.3 and Table 3 fall outside of this range which, however, merely shows that quantities such as the distance to the centre of the Galaxy and the Sun's circular velocity are difficult to determine.

About the velocity ellipsoid little can be said because the only previous study of all of the G giants is that of Parenago (Delhaye 1965, p. 64), which used fewer stars, 345, and a different reduction method, the method of moments (Trumpler & Weaver 1962, pp. 283–286). The dispersions of the velocity ellipsoid are higher than those Parenago found, but this is a consequence of use of the SDP method; see Branham (2004).

Some insight, however, can be gained if we take these results not in isolation but rather conjointly with my previous studies of the giant stars. Because all of these studies use the same reduction method, variations caused by differences in the calculation of the velocity ellipsoid, such as use of the method of moments, will be minimized. Table 5 shows the velocity dispersions for all of the giant stars from O to M.

Two conclusions follow from an examination of Table 5. There is a clear break in all of the velocity dispersions between the F and the G giants. Dehnen & Binney (1998) confirmed this break, known as Parenago's discontinuity, for the main sequence stars, but it appears as if the giant stars also show the discontinuity. Use of the SVD to include stars for which only tangential velocities are available, the second conclusion, seems justified on more than just a statistical basis. Neither the F giants compared with earlier spectral types that use only total space motions nor the G giants with later spectral types exhibit glaring discrepancies, indicating that the tangential velocities are integrated well with the total space motions.

8. CONCLUSIONS

Semi-definite programming proves itself once again a useful tool for problems of Galactic kinematics by allowing one to combine a solution for the kinematical parameters such as the Oort constants with one for the coefficients of the velocity ellipsoid. The singular value decomposition allows one to incorporate stars for which only tangential velocities but no radial velocities are available into the calculation of the velocity ellipsoid. When applied to the G III stars the calculated solutions appear concordant with what others have found. A comparison with giant stars of other spectral types confirms that Parenago's discontinuity exists for the giant stars as well as main sequence stars.

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