SOLID STATE PHYSICS OF IMPACT CRATER FORMATION: FURTHER CONSIDERATIONS

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RESUMEN

Los cráteres de impacto existen en los planetas con superficies sólidas, en sus satélites y en muchos asteroides. El objeto de este artículo es proponer una expresión teórica para el producto $\rho r^3 v_1^2$, donde los tres símbolos denotan la densidad de masa, el radio y la velocidad del impacto, respectivamente. Se deriva esta expresión utilizando resultados conocidos de la física del estado sólido. La fórmula puede usarse para estimar los parámetros de los impactadores que han causado la formación de cráteres en diversos cuerpos sólidos del Sistema Solar.

ABSTRACT

Impact craters exist on solid surface planets, their satellites and many asteroids. The aim of this paper is to propose a theoretical expression for the product $\rho r^3 v_1^2$, where the three symbols denote the mass density, radius and speed of the impactor. The expression is derived using well known results of solid state physics, and it can be used in estimating parameters of impactors which have led to formation of craters on various solid bodies in the Solar System.

Key Words: planets and satellites: interiors — planets and satellites: surfaces

1. INTRODUCTION

Craters of various sizes have been observed on the terrestrial planets, their satellites and the major part of the asteroids. The study of these craters, and the resulting constraints on the related impactors has become a separate field of research in planetary science (see for example Melosh 1989). At the time of this writing, the latest example testifying about the importance of impacts into the Earth and their consequences is the small asteroid which entered the atmosphere over the city of Chelyabinsk in Russia on February 15, 2013. One of the fundamental questions concerning the impactors is what can be concluded about them by combining astronomical data with results of solid state physics.

Recent theoretical work for example (Celebonovic & Souchay 2010) has shown that the application of elementary principles of solid state physics to this problem gives physically plausible results in reasonable agreement with those obtained by celestial mechanics. It was assumed there that the material of the target was a crystal lattice, and the calculations were performed *per unit volume*. It was assumed in that paper that the condition for the formation of a crater is that the kinetic energy of a unit volume of the impactor has to be equal to the internal energy of a unit volume of the material of the target. The result was an expression for the minimal speed which an impactor of given parameters must have when hitting a target with a predefined set of parameters, in order to form a crater. Obviously, a certain part of surfaces of planets, satellites, asteroids... in which impacts occur are granular. For recent theoretical work on "granular impact" see, for example Clark & Behringer (2013) or Ringl, Bringa, & Urbassek (2012).

The aim of the calculation reported here is to take into account the dimensional effects – to consider both the impactor and the crater it forms as objects of finite dimensions. The novelty of the approach discussed in the next section, compared to existing work such as Holsapple & Housen (2007), is its generality – it is based on principles of solid state physics, and it can be applied to any solid material. The formation of impact craters is

here discussed from the viewpoint of pure solid state physics as the following analogous problem: what kinetic energy of an impactor is needed to produce a hole of given dimensions in a material with a predefined set of parameters? The calculations will be performed under the assumptions that the material of the target is a crystal, that one of the five usual bonding types exists in it and that in the impact the target does not heat up to its melting temperature, so that solid state physics can be applied. For recent work on the problem of heating in impacts, see Celebonovic (2012). The expression for the cohesion energy used in the present work is one of the theoretically possible expressions. More details are avaliable, for example, in Landau & Lifchitz (1976).

2. CALCULATIONS

The physical keypoint of the calculation to be discussed in this section is the idea that the kinetic energy of the impactor must be greater than or equal to the internal energy of some volume, V_2 of the material of the target. The kinetic energy E_k of the impactor of mass m_1 and speed v_1 is obviously:

$$E_k = \frac{1}{2}m_1 v_1^2. \tag{1}$$

The internal energy E_I of a volume V_2 of the target material is equal to the sum of the following three "contributions": the cohesion energy E_C, the thermal energy E_T and the energy $E_H(T)$ required for heating the specimen by an amount ΔT in the moment of impact:

$$E_I = E_C + E_T + E_H(T). \tag{2}$$

The condition for the formation of a crater as a consequence of impact of a projectile into a target is, in general terms, given by

$$E_k = E_I. \tag{3}$$

In order to ensure the stability of matter, the interatomic interaction energy must have the form

$$E = \frac{-K}{R^p} + \frac{C}{R^n}, \tag{4}$$

where R is the inetartomic distance, the first term on the right hand side is attractive and the second one repulsive. It can be shown (Dolocan, Dolocan, & Dolocan 2008) that the cohesion energy is given by

$$E_C = -\frac{9B_0\Omega_m}{pn}, \tag{5}$$

where $B_0 = -V(\partial P/\partial V)_T = \rho(\partial P/\partial \rho)_T$ is the bulk modulus of a material and Ω_m the volume per pair (Dolocan et al. 2008). The speed of sound waves in a material with pressure P and density ρ is given by

$$\bar{u}^2 = \frac{\partial P}{\partial \rho}, \tag{6}$$

and therefore

$$E_C = -9\frac{\Omega_m B_0}{pn} = -9(\frac{\Omega_m}{pn})\rho\frac{\partial P}{\partial \rho} = -9\frac{\Omega_m \rho \bar{u}^2}{pn}. \tag{7}$$

The analytical form of the function E_T depends on the temperature of the solid. It can be shown (for example Landau & Lifchitz 1976) that for temperatures

$$k_B T_0 \gg \frac{\hbar \bar{u}}{a}, \tag{8}$$

($\hbar$ is Planck's constant divided by 2π, $\bar{u}$ is the mean speed of sound in the solid and a is the lattice constant), one should use the "high temperature" limit for the thermal energy. Inserting for example $\bar{u} = 5$ km s^{-1} and $a = 5 \times 10^{-6}$ m it follows that $T_0 \gg 77$ K.

The thermal energy E_T is given by

$$E_T = 3N\nu k_B T D_3(x), \tag{9}$$

where $x = T_D/T$, T_D is the Debye temperature, ν is the number of particles in the elementary crystal cell, N is the number of elementary crystal cells in the specimen, and k_B is Boltzmann's constant. The symbol $D_3(x)$ denotes the $n = 3$ case of the Debye function, which is given by

$$D_n(x) = \frac{n}{x^n} \int_0^x \frac{t^n dt}{\exp[t] - 1} = (n/x^n)x^n \left[\frac{1}{n} - \frac{x}{2(n+1)} + \sum_{k=1}^{\infty} \frac{B_{2k} x^{2k}}{(2k+n)(2k)!} \right], \tag{10}$$

and B_{2k} denote Bernoulli's numbers. Integrating and retaining the terms up to and including second order leads to:

$$E_T \cong 3N\nu k_B T \left[1 - \frac{3}{8} \frac{T_D}{T} + \left(\frac{1}{20} \right) \left(\frac{T_D}{T} \right)^2 \right]. \tag{11}$$

The specific heat C_V is given by

$$C_V = \frac{\partial E_T}{\partial T} = 3N\nu k_B \left[D_3(T_D/T) - (T_D/T)D_3'(T_D/T) \right], \tag{12}$$

which leads to the following approximate result:

$$C_V \cong 3N\nu k_B \left[1 - \left(\frac{1}{20} \right) \left(\frac{T_D}{T} \right)^2 + \left(\frac{1}{560} \right) \left(\frac{T_D}{T} \right)^4 \right]. \tag{13}$$

The energy $E_H(T)$ is obviously given by $E_H = C_V(T_1 - T)$, where T_1 is the temperature to which the material of the target heats up in the moment of impact and T is the initial temperature.

One of the problems consists in knowing the volume V_2, which is, in first approximation, equal to the volume of the crater. It is known from theoretical work (Holsapple & Housen 2007, and references therein) that the form of a crater is determined by a combination of "gravity scaling" and "strength scaling", where the term "strength" refers to the material strength of the target. Explicit expressions for the form of craters exist in Holsapple & Housen (2007) but they are applicable to four material types. In order to simplify somewhat the calculations, and to get slightly more general results, it was assumed in the present work that craters have the shape of a half of a rotating elipsoid, with distinct semi axes, denoted by a,b and c. Physically, a and b denote the semi axes of the "opening" of the crater, while c is the depth. In that approximation, the volume of the crater is obviously given by

$$V_2 = \frac{2}{3} \pi abc. \tag{14}$$

Inserting the definition of the mass density ($\rho_1 = m_1/V_1$) in equation (1) and assuming further that the impactor has the form of a sphere of radius r_1, the following expression for the kinetic energy of the impactor is obtained:

$$E_k = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} \times \frac{4}{3} \pi r_1^3 \rho_1 v_1^2 = \frac{2}{3} \pi \rho_1 r_1^3 v_1^2. \tag{15}$$

Inserting equations (7), (11), (13) and (15) into equation (3) one gets the final form of the energy condition which has to be satisfied for the formation of an impact crater:

$$3k_B T_1 N\nu \left[1 - \frac{3}{8} \frac{T_D}{T_1} - \frac{1}{20} \left(\frac{T_D}{T} \right)^2 + \frac{1}{10} \left(\frac{T_D^2}{T T_1} \right) + \left(\frac{1}{560} \right) \left(\frac{T_D}{T} \right)^4 - \frac{1}{560} \frac{T_D^4}{T^3 T_1} - \frac{3\bar{u}^2 \rho \Omega_m}{np\nu k_B T_1} \right] = \frac{2\pi \rho_1}{3} r_1^3 v_1^2. \tag{16}$$

The number N is equal to the ratio of the volume of the crater, approximated by equation (14), and the volume of the elementary crystal cell, v_e: $N = V/v_e$. Equation (16) groups known or measurable parameters on the left hand side, while on the right hand side it contains parameters of the impactor. Applying this equation demands the knowledge of the equation of state (EOS) of the material of the target. Generally speaking, regardless of its detailed form, an EOS can be expressed in the following analytical form

$$P(\rho) = \sum_{i=0}^{\infty} a_i \left(\frac{\rho}{\rho_0} \right)^i, \tag{17}$$

where a_i are some coefficients, ρ_0 is the density at some pressure P_0 and ρ is the density at pressure P. It follows that

$$\left(\frac{\partial P}{\partial \rho}\right) = \sum_{i=0}^{\infty} (i+1)a_{i+1}\left(\frac{\rho}{\rho_0}\right)^i . \tag{18}$$

A well known example of the EOS of a solid is the Birch-Murnaghan EOS (Stacey 2005)

$$P(\rho) = \frac{3B_0}{2}\left[\left(\frac{\rho}{\rho_0}\right)^{7/3} - \left(\frac{\rho}{\rho_0}\right)^{5/3}\right] \times \left\{1 + (3/4)(B_0' - 4)\left[\left(\frac{\rho}{\rho_0}\right)^{2/3} - 1\right]\right\} . \tag{19}$$

The symbols ρ_0 and ρ denote the mass density of a specimen under consideration at the initial value of the pressure P_0 and at some arbitrary value P.

More refined results could be obtained by using analytic approximations to the Helmholtz free energy, from which all the thermodynamic potentials can be derived. This is, for example, the approach used in the ANEOS EOS (Thompson 1990). However, ANEOS uses a file of up to 40 parameters for the characterization of materials, which complicates its applications.

3. A TEST EXAMPLE

As a test of the applicability of the procedure discussed in the preceeding paragraph, it was applied to the Barringer crater in Arizona. One of the minerals found in and around the crater is Forsterite – Mg_2SiO_4. The dimensions of the crater are $a = b = 1.186$ km and $c = 0.17$ km. Take that the exponents in the expression for the interparticle potential are $p = 1$ and $n = 9$. Assume furher that $\bar{V} = 5$ km s^{-1}, which is the mean value of the measured velocity of seismic waves in the Earth, that the temperature before impact is $T = 300$ K and that the target heats up to $T_1 = 550$ K at the point of impact. This is safely far from the melting temperature of Forsterite (which is around $T_M = 1900$ C) so that solid state physics can be applied. All the parameters of Forsterite exist in the literature, such as Pandey & Srivastava (2011) or http://webmineral.com. Taking the necessary values, and assuming that the impactor had a radius of $r_1 = 65$ m and density of $\rho_1 = 8500$ kg m^{-3} one finally gets that $v_1 \cong 41$ km s^{-1}.

This value is larger than existing results, such as for example Melosh & Collins (2005) or Kring (2007). The discrepancy can be traced to the assumption, made in the calculation reported here, that the target material is only Forsterite. In reality, this is certainly not the case. Taking that 10 percent of the target is Forsterite, and that the object had a radius of $r_1 = 60$ m would give $v_1 \cong 15$ km s^{-1}, which is much closer to the results of celestial mechanics.

4. DISCUSSION

In this paper we have presented a simple procedure which gives the possibility of estimating the value of the product of the density and radius of the impactor and the speed of impact: $\rho_1 r_1^3 v^2$ in terms of various material parameters of the target and the impactor. The expression for this product has been derived using basic principles of solid state physics, without any special assumption(s) about the materials. In (at least some of the) terrestrial applications the density of the impactor and the inclination of the trajectory can (in principle) be measured or estimated, which gives the possibility of estimating the impact speed if the value of r_1 can somehow be estimated.

The shape of the craters has been approximated as a half of a rotating elipsoid, regardless of the composition of the material of the target. As a first approximation, the speed of the seismic waves has been "put in" instead of being calculated from the equation of state of the material of the target. Calculating this speed would have demanded the precise knowledge of the chemical composition of the target materials, and of B_0 and B_0' for them. The impact of a projectile into a target leads to heating, and possibly melting and even vaporisation of the target around the impact point. The thermodynamic result of an impact depends on the heat capacity of the material of the target and on the kinetic energy of the impactor. The heat capacity can be measured, or theoretically estimated, assuming prior knowledge of the chemical composition of the target. Knowledge of the heat capacity is vital for estimates of the temperature to which the target heats in the impact. Therefore, the question is which impacts lead to melting and vaporisation of the material of the target, and which just

provoke heating of the target. Even if the material of the target gets partially melted and as such flows away, the dimensions of the crater formed by the impact are determined by the volume of the material pushed away in the impact. This volume is, in turn, determined by the ratio of the kinetic energy of the impactor to the internal energy of some volume of the target. Some details of this problem have been discussed in (Celebonovic 2012).

The general conclusion of the work reported here is that by using simple well known results of solid state physics and a given appoximation of the shape of impact craters, it becomes possible to estimate the value of the product $\rho_1 r_1^3 v_1^2$. This in turn can be used to draw conclusions about the impactors which made various craters on the solid surfaces of objects in the planetary system.

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