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A NEW ORBIT FOR COMET C/1861 J1 (GREAT COMET OF 1861)

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RESUMEN

Se ha calculado una órbita nueva para el Cometa C/1861 J1 (Gran cometa de 1861) que reemplaza la órbita de Kreutz calculada en 1880. La órbita usa 2,362 observaciones, 1,198 en ascensión recta y 1,164 en declinación, hechas entre mayo de 1861 y marzo de 1862. El período del cometa de 364.96±0.51 años es bastante diferente del período de 409 años calculado por Kreutz. Se enlaza esta órbita con una calculada con 5 observaciones hechas en el oriente en 1500, quizás observaciones del Gran cometa, mediante el uso de condiciones de contorno para las ecuaciones diferenciales de movimiento. La órbita enlazada posee un período de 361.3 años, y aumenta la probabilidad que las observaciones de 1500 sean del Gran cometa. Aunque los residuos sean relativamente aleatorios (16.0% probabilidad de aleatoridad), los residuos pos-perihelio son menos aleatorios, indicando posibles desvíos del movimiento kepleriano, tales como errores sistemáticos o fuerzas no-gravitatorias.

ABSTRACT

A new orbit is calculated for Comet C/1861 J1 (Great comet of 1861) to replace Kreutz's orbit of 1880. The orbit is based upon 2,362 observations, 1,198 in right ascension and 1,164 in declination, made between May of 1861 and March of 1862. The comet's period of 364.96 ± 0.51 yr differs significantly from Kreutz's period of 409 yr. This orbit is linked with one calculated from 5 observations made in the Far East in 1500, possibly of the Great comet, by imposing boundary conditions on the differential equations of motion. The linked orbit indicates a period of 361.3 yr. This increases the probability that the Great comet is indeed the comet observed in 1500. Although the residuals are relatively random (16.0% probability of randomness), post-perihelion residuals are less random indicating possible deviations from Keplerian motion such as systematics error or nongravitational forces.

Key Words: celestial mechanics — comets: individual (C/1861 J1) — methods: data analysis

1. INTRODUCTION

Although there is no precise definition of a "Great comet" one can say that a comet becomes great when it increases markedly in brightness and is noticed with the naked eye by many. According to the Marsden & Willliams catalog (2003) during the 19th century seventeen comets were considered great comets. The number would be eighteen should Donati's comet (C/1858 L1), the second brightest comet of the 19th century, be explicitly classified as a great comet. Most of these comets have orbits calculated before the advent of modern computers. This is certainly true of the Great Comet of 1861 (C/1861 J1) with an orbit that Kreutz (1880) calculated in 1880. Such orbits are excellent candidates for improvement, and I have already done so for the Great June comet of 1845, the Great comet of 1854, Donati's comet, and the Great comet of 1860. Modern orbits eschew the use of normal places, a computational expedient neither necessary nor desired today because they degrade, if only slightly, the solution. Modern orbits also efficiently implement techniques such as robust

TABLE 1

ELLIPTIC ORBITAL ELEMENTS EPOCH JD 2399400.5 AND EQUINOX J2000

Unknown	Value
Perihelion passage	JD 2398391.99353(1861 June 12.0068)
a	55.082652
e	0.985070
q	0.822384
Ω	280.°9099
i	85.°4424
ω	330.°0841
P(year)	409

estimation for processing the observations, and allow one to include all perturbing planets, not just the main contributors. Because most 19th century observations are differential, measuring with a micrometer the comet's position with respect to a reference star, one can re-reduce the observation taking the reference star from a modern catalog. One thus obtains a more precise observation than that published by the observer.

Not only should these improvements be applied to Comet C/1861 J1, but a further consideration becomes germane. Hasegawa & Nakano (1995) find 5 observations in both coordinates from 1500 made in China, Korea, and Japan apparently corresponding to the comet. If this is correct, and the evidence they present looks strong, then the period should be ≈361 yr. whereas Kruetz finds 409 yr.

To permit comparison with my orbit, given in a later section, Table 1 shows Kreutz's orbit as published in Marsden & Williams (2003): the time of perihelion passage, T_0; the eccentricity, e; the semi-major axis, a; the perihelion distance, q; the inclination, i; the node, Ω; the argument of perihelion, ω, and the period, P. The semi-major axis, not given in the catalog, is calculated from $a = q/(1-e)$.

2. THE OBSERVATIONS AND THEIR TREATMENT

John Tebutt in Australia discovered the Great comet of 1861, from now on simply "the comet", on 13 May 1861. During May and June the comet was observed in Australia, Brazil, and at the Cape Observatory. The comet became visible in the northern hemisphere in June and was widely observed in 1861 and part of 1862. The Marsden & Williams catalog (2003) states that Kreutz used 1,159 observations to calculate his orbit. I was able to collect 2,362 using the ADS database[1] and the *Comptes rendus hebdomadaires des séances de l'Académie des Sciences*[2]. The observations were made by equatorial telescopes with ring or filar micrometer, the heliometer, and a few with meridian instruments: transit circle, transit instrument, or vertical circle. Discriminated by coordinate, right ascension (α) and declination (δ), the total becomes 1,198 in α and 1,164 in δ.

The total would have been even higher, but unfortunately not all observations could be used. The Bonn Observatory published 26 heliometer observations of the comet (Krüger 1879). These, however, had not been converted to α and δ, but left in instrumental coordinates s and p. See Chauvenet (1960, pp. 407–423) for a description of the functioning of the heliometer. In theory one could convert s and p to α and δ by solving a nonlinear equation. But a glance at the value of $\log 2 \sin s/2$ for 5 July 1861, 3.39559, shows that something is wrong. Neither natural nor common logarithms permit a value of this magnitude because it implies that $\sin s/2$ is greater than 1. The same is true for the remaining observations.

The mural circle observers at the Brussels Observatory published 8 observations of the "Comète" made between 1–3 July 1861 (Brussels Obs. 1866). Whatever comet was observed, if indeed it was a comet, it fails to be the Great comet because on those dates the zenith distance should have been close to 0° whereas the actual zenith distances vary from ≈73°to ≈63°.

A Dr. Mackay published two orbits for the comet based on his own and some French observations (1862). Mackay's observations, using "a sextant and an old ship's chronometer" were published in the *Friend of India*,

[1] `http://adswww.harvard.edu/`.
[2] `http://gallica.bnf.fr/`.

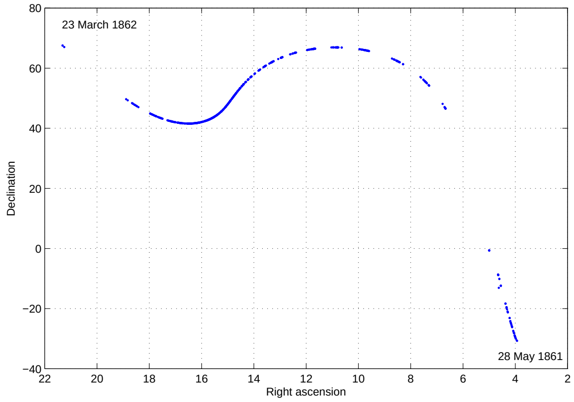

Fig. 1. The observations.

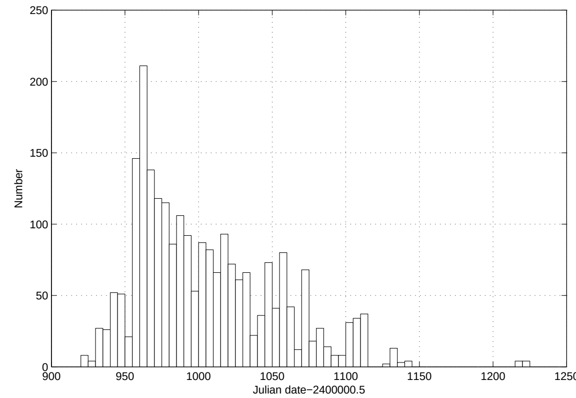

Fig. 2. Histogram of observations.

a newspaper, and the French observations in the *Journal des débats*, also a newspaper. Newspapers are hardly a standard publishing medium for astrometric observations nor are sextant observations of much use, as I found when calculating the orbit of the Great comet of 1854 (Branham 2005a). Because the *Journal des débats* is available on-line at the *Gallica* web site, it was possible to attempt to find the observations. But I found no trace of the 30 June 1861 observation that Mackay mentions. Mackay may have been confused about the date or where the observation was published. Regarding sextant observations I only include the four made in Williamstown, Australia, on June 6–19 (Ellery 1862), which the observer, White, says were "very careful" and accurate to $10''$, and also seven observations made by Commander Mansell, RN, (1861) near Sidon, Lebanon, which were also claimed to be carefully reduced.

It was my intention at first to include the Far Eastern observations given in Hasegawa & Nakano (1995). Difficulties might present themselves in linking the 1861-62 observation with those from 1500, but these can be overcome by treating the best fit 1861–62 orbit with the corresponding 1500 orbit as a boundary value problem. I have done just this with Comet 122 P (de Vico) when linking the less precise 1846 observations with the more precise 1995-96 observations (Branham 2005b). See that publication for details. The overall mean error for the 1500 observations, however, is so high, $\approx 3.^\circ 1$ according to Hasegawa & Nagano, that they would substantially degrade the final solution and cannot be used directly. They do, however, permit one to question once again the comet's period as given in Table 1. With perihelion passage in 1861 a period of 409 years would have made the comet visible in 1454, not 1500. If the comet of 1500 is indeed the Great comet of 1861 then the period of Table 1 cannot be correct. This matter will be discussed later.

Figure 1 graphs the observations, and Table 2 shows their distribution among observatories. The distribution of the observations with respect to time is hardly uniform. Figure 2 shows a histogram of observations versus Julian date, with an evident concentration near JD 2400965.5 (9 July 1861). The Far Eastern observations are excluded because they would distort excessively the histogram. The first astrometric observation occurs on 28 May 1861 and the last on 23 March 1862. These are genuine astrometric observations rather than mere descriptions of the comet, some of which go beyond March of 1862; see G.W. Kronk's web site *cometography.com.* The Marsden and Williams catalog gives the first observation as 27 May, but when it is corrected for the difference between the astronomical and the civil day the date becomes 28 May. The catalog also lists the last observation as 1 May 1862, but this must refer to a description rather than astrometric observation. I can find nowhere in the literature between 1861 and 1879 a genuine astrometric observation for this date. Kronk merely says that the comet was "detected" then. Struve (1862) makes a similar statement; observations were made on 20, 21, 22, 25, and 28 March 1862, those on 20 March by Dr. Winnecke, and a "determination" on 16 April. How a determination differs from an observation remains unspecified. A search of the literature, however, shows that Winnecke observed the comet on 20 and 22 March whereas nothing can be found for the other dates. Struve seems to have left these observations unpublished. The matter might be settled by examining Kreutz's dissertation, but this is hard to come by and not available on internet. Publications available on internet give the last astrometric observation as 23 March 1862.

TABLE 2

OBSERVATIONS OF COMET C/1861 J1 (GREAT COMET)

Observatory	Obsns. in α	Obsns. in δ	Reference[1]
Sydney, Australia	70	68	MN, 1861, 21, 254
Williamstown, Australia	4	4	AN, 1861,56, 53/54
Kremsmünster, Austria	74	74	AN, 1862, 57, 145
Vienna, Austria	140	136	Annalen Wien, 1861, 12, 78
Rio de Janeiro, Brazil	4	4	AN, 1861, 55, 365
Cambridge, England	68	68	AN, 1862, 57, 235
Greenwich, England	1	1	Green. Obs. 1861, 1863, 23, xciv
Liverpool, England	20	20	AN, 1862, 56, 365
Oxford, England	116	114	MN, 1861, 22, 50; AN, 1862, 57, 235
Paris, France	117	100	Annal. Paris, 1863,17, 156
Toulouse, France	23	23	AN, 1861, 56, 235
Altona, Germany	42	43	AN, 1862, 57, 375
Berlin, Germany	49	49	AN, 1862, 57, 177
Bilk, Germany	3	3	AN, 1861, 56, 43
Bonn, Germany	4	4	AN, 1861, 55, 309
Breslau, Germany	1	1	AN, 1861, 55, 305
Göttingen, Germany	5	5	AN, 1854, 38, 353; AJ, 1854, 4, 5
Hamburg, Germany	4	4	AJ, 1854, 4, 46
Köingsberg, Germany	2	2	AN, 1861, 56, 77
Leipzig, Germany	3	3	AN, 1863, 60, 87
Athens, Greece	68	66	AN, 1862, 57, 23
Armagh, Ireland	13	13	MN, 1861, 22, 17
Florence, Italy	4	4	AN, 1861, 55, 377
Padua, Italy	53	53	AN, 1861, 56, 91; AN, 1861, 55, 305
Rome, Italy	7	7	CR,1861, 53, 87
Sidon, Lebanon	7	7	MN, 1861, 22, 15
Leiden, Netherlands	67	63	AN, 1863, 61, 33
Christiana, Norway	21	22	AN, 1861, 56, 137
Moscow, Russia	9	9	AN, 1862, 58, 195
St. Petersburg, Russia	5	5	AN, 1861, 55, 307; AN, 1862, 57, 203
Cape, South Africa	19	16	MemRAS, 1864, 22, 5
Geneva, Switzerland	18	18	AN, 1861, 56, 53
Cambrige, Mass., USA	48	48	AN, 1862, 57, 353
Albany, NY, USA	4	2	MN, 1861, 22, 18
Clinton, NY, USA	52	52	AN, 1863, 60, 117
Washington, D.C., USA	45	45	AN, 1862, 56, 373; MN,1861,21, 258
China, Japan, Korea	5	5	PASJ, 1995, 47, 699
Total	1203	1169	

[1]AN: *Astron. Nachr.*; MN: *Monthly Notices RAS*; AJ: *Astron. J.*; CR: *Comptes rendux*; MemRAS: *Memoirs RAS*.

Processing 19th century observations presents difficulties and becomes far from trivial. See Branham (2011) for details, although two should be mentioned here. Because the observations are 19th century those published as mean positions were corrected for the E-terms of the aberration. See Scott (1964) for a discussion of the E-terms. Meridian observations were not corrected for geocentric parallax because such observations have traditionally been reduced to the geocenter. All observations were reduced to the common format of: Julian Day (JD), terrestrial time (TT), right ascension, and declination.

3. EPHEMERIDES AND DIFFERENTIAL CORRECTIONS

The procedure for calculating coordinates, velocities, and partial derivatives for differential corrections has been given in previous publication of mine. See Branham (2005b), for example.

TABLE 3

ERRORS AND MISSING INFORMATION FOR THE GREAT COMET OF 1861

Observatory	Reference[1]	Date or ref. star no.	Error or missing data
Sydney, Australia	MN, 1861, Vol. 21, p. 254	8 June; all obs.	Ref. star Tycho 6461 00634 1
"	MN, 1861, Vol. 21, p. 255	11 June; all obs.	Ref. star Tycho 6459 00242 1
"	MN, 1861, Vol. 21, p. 256	20 June; all obs.	Ref. star Tycho 5894 00382 1
"	"	23 June; all obs	Ref. star Tycho 5321 01823 1
Vienna, Austria	Ann. Wien, 1863, Vol. 12, p. 78	4 July	Last $\Delta\delta - 21'32.''8$
"	Ann. Wien, 1863, Vol. 12, p. 80	14 July	Ref. star wrong; unidentifiable
"	Ann. Wien, 1863, Vol. 12, p.87	9 Oct.	Ref. star wrong; unidentifiable
Rio de Janeiro, Brazil	AN, 1861, No. 55, pp. 365-366	15 July	Date really 14 July 1861
Cambridge, England	MN, 1861, Vol 22, p. 50	5 July, all obsns.	Ref. star ambiguous; use observer α, δ
Liverpool, England	AN, 1862, Vol.56, pp. 365-366	5 July, $11^h08^m34.^s1$	Should be $11^h28^m34.^s1$
Oxford, England	MN, 1861, Vol. 22, p. 51	9 July	Ref. star wrong; unidentifiable
"	MN, 1861, Vol. 22, p. 52	15 July, $11^h08^m42.^s2$	Ref. star NPD $+44°32'50.''0$
Toulouse, France	AN, 1861, Vol. 56, pp. 231-232	1 July, 2nd obs.	Ref. star unidentifiable
"	"	5 Julu, 1st obs-	"
Athens, Greece	AN, 1861, Vol. 57, pp. 27-28	6 Nov.	δ ref. star $+42°16'14.''8$
Paris, France	Ann. Paris, 1863, Vol. 17, p. 157	2 July, $17^h41^m51.^s7$	$\Delta\alpha$ probably $-8^m47.^s19$
"	Ann. Paris, 1863, Vol. 17, p. 160	21 Nov., $22^h53^m18^s$	$\Delta\alpha$ probably $+4^m50.^s04$
"	Ann. Paris, 1863, Vol. 17, p. 162	1, 2 Dec.	Ref. star unidentifiable
Berlin, Germany	AN, 1862, No. 57, pp. 179-180	4 July	Star b unidentifiable
Leiden, Netherlands	AN, 1863, No. 61, pp. 85–86	17 Sept.	$\Delta\alpha$ probably $-0^m32.^s52$
Padua, Italy	AN, 1861, Vol. 56, pp. 91-92	12,15,18,21 Sept.	Ref. stars unidentifiable
"	"	8 Aug.	Change signs $\Delta\alpha, \Delta\delta$
Rome, Italy	CR, 1861, Vol. 53, p. 87	7 July	δ probably $+63°39'05.''6$
Christiana, Norway	AN, 1861, Vol. 56, pp. 139–140	13 July, $11^h10^m22.^s8$	Ref. star unidentifiable
Geneva, Switzerland	AN, 1861, Vol. 56, pp. 55–56	21 July	Ref. star unidentifiable
"	"	22 July, $09^h51^m25.^s6$	Star i Tycho 3480 01077 1
"	"	25 July, $09^h23^m24.^s1$	Star m Tycho 3477 00469 1
Cambridge, Mass., USA	AN, 1862, Vol. 57, pp. 355–356	17 July	Ref. star unidentifiable
"	AN, 1862, Vol. 57, pp. 357-358	23 Sept.	Ref. star unidentifiable
Clinto, NY, USA	AN, 1863, Vol. 60, pp. 117-118	16 Sept.	$\Delta\alpha$ probably $-3'14.''3$
(Old) U.S. Naval, USA	MN, 1861, Vol. 21, p. 258	1 Aug. 1861	$\Delta\alpha$ probably $+1^m04.^s48$
"	AN 1862, Vol. 56, pp. 373–374	6 Sept. 1861	Ref. star Tycho 3059 00777 1

[1]AJ: *Astronomical Journal*; AN: *Astronomische Nachrichten*; CR: *Comptes rendus*; MN: *Monthly Notices RAS*; PASJ: *Publ. Astron. Soc. Japan.*

The first two differential corrections were based on the robust L_1 criterion, minimize the sum of the absolute values of the residuals, and insensitive to discordant data. The far eastern observations were excluded leaving 2,362 equations of condition. Having the initial observed minus calculated position, (O-C)'s, one can search for discordant observations, many of which could be corrected. Table 3 lists some errors for the comet that could be identified and corrected along with reference stars previously unidentified that could be identified. The corrections are based on the precepts given for processing 19th century observations in Branham (2011). Many observations could not be corrected and the large (O-C)'s were accepted as genuine outliers.

Various weighting schemes are possible once one has post-fit residuals from a differential correction. One of the oldest weightings, Pierce's criterion (Branham 1990, pp. 79-80), establishes a cutoff for an acceptable residual. All residuals within the cutoff receive equal weight. More modern schemes usually assign higher weight to smaller residuals and zero weight to large residuals, recognizing them as errors rather than genuine but improbable residuals. Among these robust weightings are the biweight, the Talwar, and the Welsch. Branham (1990, § 5.5) discusses all of these weighting possibilities. Let $\mathbf{A}$ represent the matrix of the equations of condition, here of size 2,362×6, $\mathbf{d}$ the right-hand-side, $\Delta\mathbf{x}$ the solution for correction to the osculating

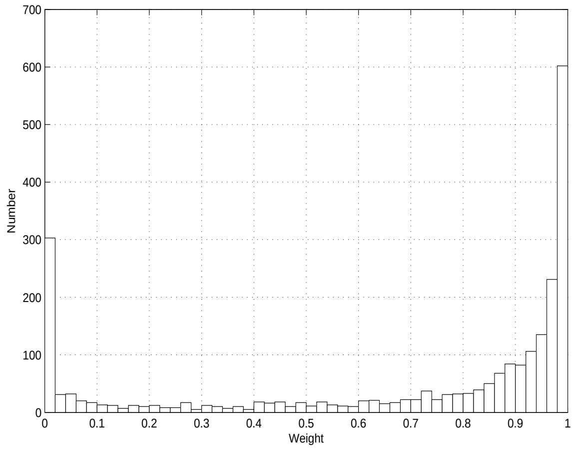

Fig. 3. Histogram of Welsch weights.

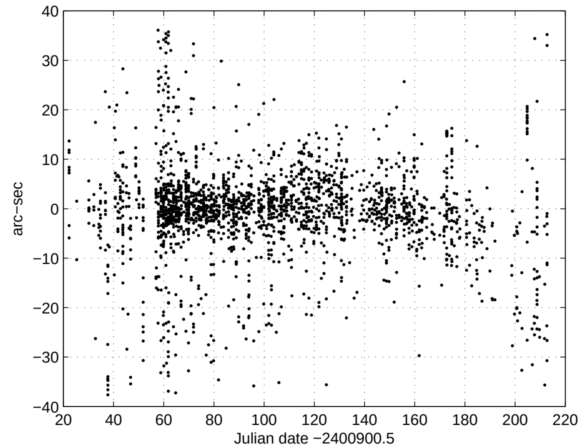

Fig. 4. The trimmed residuals.

rectangular coordinates and velocities $\mathbf{x}$, and $\mathbf{r}$ the vector of the residuals, $\mathbf{r} = \mathbf{A}\cdot\Delta\mathbf{x} - \mathbf{d}$. After a solution has been calculated one should check for the randomness of the residuals.

To use the biweight, a weighting scheme I have used many times when working with comet orbits, double star orbits, and Galactic kinematics, one scales an individual post-fit residual r_i by the median of the absolute values of the residuals and assigns a weight wt as

$$\begin{aligned} wt &= [1-(r_i/4.685)^2]^2; |r_i| \le 4.685, \\ wt &= 0; |r_i| > 4.685. \end{aligned} \tag{1}$$

Talwar weighting is similar to Pierce's criterion, but the acceptance criterion for a residual becomes more stringent. Take once again the post-fit residual r_i. Talwar's criterion is

$$\begin{aligned} wt &= 1; |r_i| \le 2.795, \\ wt &= 0; |r_i| > 2.795. \end{aligned} \tag{2}$$

Welsch weighting accepts all residuals, but assigns low weight to large residuals, so low as to become less than the machine ϵ for extremely large residuals,

$$wt = \exp(-r_i/2.985)^2; |r_i| < \infty. \tag{3}$$

Both the biweight and the Talwar schemes should be excluded because they reject too many residuals, 16.6% for the biweight and 23.8% for the Talwar. It seems difficult to justify such a high rejection criterion particularly when Stigler (1977) has shown, from a study of historical examples, that a 5%−10% rejection works best. Welsch weighting rejects only 4.95% of the residuals less than the machine ϵ, 2.2×10^{-16} for the Intel processor used for the computations, although 17.1% receive a weight less than 0.1 and 26.7% weight less than 0.5. Figure 3 shows a histogram of the weights. The mean error of unit weight, $\sigma(1)$, becomes $3.''85$. Pierce's criterion, applied after the second differential correction, imposes a cutoff of 3.75 times the penultimate $\sigma(1)$. This corresponds to elimination of 227 residuals, a 9.6% trim, and a final mean error of $9.''71$.

For a number of reasons, discussed later, I take the Pierce solution as the better of the two and base the final orbit on it. But in reality neither of the solutions leads to significant changes in the osculating rectangular coordinates and velocities and either one of them could be taken as the "final" solution.

4. THE SOLUTION

Table 4 shows the final solution for the rectangular coordinates, x_0, y_0, z_0, and velocities, $\dot{x}_0$, $\dot{y}_0$, $\dot{z}_0$, along with their mean errors and also the repeated mean error of unit weight $\sigma(1)$ for the comet. Table 5 gives the

TABLE 4

SOLUTION FOR RECTANGULAR COORDINATES AND VELOCITIES FOR THE GREAT COMET OF 1861: EPOCH JD 2401000.5 (13 AUG. 1861), EQUINOX J2000

Unknown	Value	Mean Error
x_0 (au)	2.5158068e−001	4.9517341e−006
y_0 (au)	−1.2080907e+000	4.1330995e−006
z_0 (au)	6.0434275e−001	4.8853049e−006
$\dot{x}_0$ (au day^{-1})	9.1453274e−004	1.1528306e−007
$\dot{y}_0$ (au day^{-1})	−4.5583132e−003	1.2184842e−007
$\dot{z}_0$ (au day^{-1})	2.0082508e−002	1.0259700e−007
$\sigma(1)$	9.″71	

TABLE 5

COVARIANCE (DIAGONAL AND LOWER TRIANGLE) AND CORRELATION (UPPER TRIANGLE) MATRICES FOR THE GREAT COMET OF 1861

0.010272	0.0014599	0.0082711	0.00023014	1.4575e−005	0.0001622
0.17028	0.0071563	0.0024734	1.2197e−005	0.00020634	2.6752e−005
0.81615	0.29241	0.00017558	5.5676e−006	−3.4729e−007	3.8714e−006
0.96232	0.061102	0.74419	5.5676e−006	−3.4729e−007	3.8714e−006
0.057662	0.97804	0.18627	−0.059015	6.2198e−006	2.1565e−007
0.76213	0.15059	0.8913	0.78131	0.041177	4.4097e−006

corresponding covariance and correlation matrices. The highest correlation, 97.8% between y_0 and $\dot{y}_0$, although significant along with certain other correlations nevertheless does not imply an unstable solution because the condition number of 3.9×10^2 for the data matrix shows that the solution is stable. Table 6 converts the rectangular coordinates to elliptical orbital elements using the well known expressions linking orbital elements with their rectangular counterparts. The orbit represents a high eccentricity ellipse and differs, significantly in some instances such as the period, from the orbit in Table 1.

The calculation of the mean errors of the orbital elements proceeds via a modernized version of Rice's procedure (1902). See Branham (2005b).

5. DISCUSSION

Which of the two solution, that given by Welsch weighting or that by Pierce's criterion is better, in some sense of the term "better"? Given the large number of what seem to be mediocre observations I consider that the mean error given by Pierce's criterion is more realistic. The comet aroused much popular interest. In the United States its appearance coincided with the beginning of the Civil War, where it was known as the "great war comet". Observations of the Great comet were published in nonstandard media, such as newspapers. The number of rejected residuals, moreover, is higher than what I usually find with 19th century comets. For Donati's comet, for example, also a great comet although not called one, Pierce's criterion rejected 3.5% of the observations (Branham 2014). The mean error given by Welsch weighting, therefore, seems far too optimistic.

Statistics on the residuals also are better for Pierce's criterion. A runs test measures how often a variable, distributed about the mean, changes sign from plus to negative or negative to positive, the runs, which have a mean for n data points of $n/2 + 1$ and a variance of $n(n-2)/4(n-1)$ (Wonnacott & Wonnacott 1972, pp. 409–411). An advantage of the runs test over other tests for randomness resides in its being nonparametric, making no assumption about the normality of the data, although to actually calculate probabilities for the observed runs one does assume approximate normality. The trimmed residuals from Pierce's criterion evince 1,035 runs out of an expected 1,067.5, or a 16.0% chance of being random. With the Welsch weighting, on the

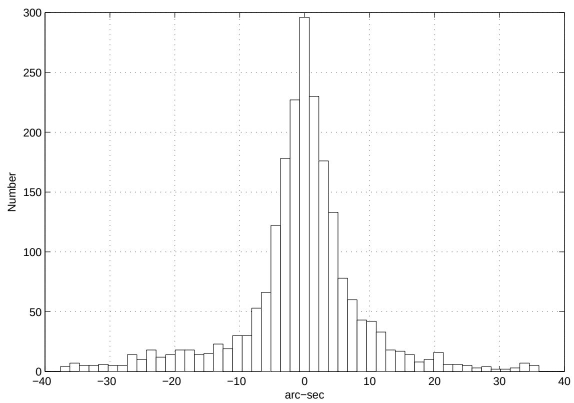

Fig. 5. Histogram of trimmed residuals.

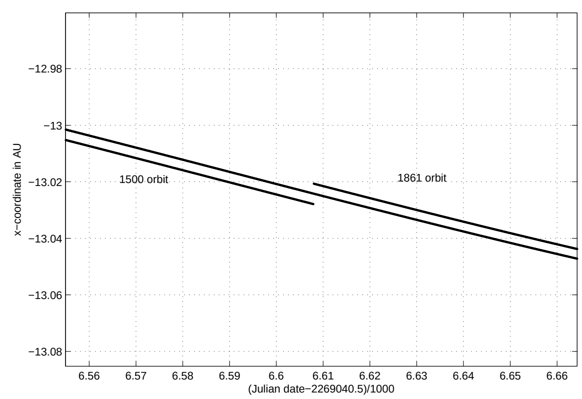

Fig. 6. The two orbits and joined orbit.

other hand, there is only a 0.3% chance of the residuals being random. Therefore, despite the higher mean error the Pierce criterion solution becomes better and is the one adopted.

The randomness of the few, 48, pre-perihelion residuals, 24 runs, becomes 100%, indicating that any non-randomness arises post-perihelion. This *may* be a sign of nongravitational forces, but also of systematic error in the more extensive series of observations. The evidence remains inconclusive.

Figure 4 graphs the residuals and Figure 5 shows their histogram. Statistics on the trimmed residuals indicate that they are somewhat skewed (factor of skewness −0.199 versus 0 for a normal distribution), leptokurtic (kurtosis of 1.53 versus 3 for a normal distribution), and lighter tailed as measured by Hogg's Q factor of 0.39 versus 2.58 for the normal.

The five observations made in China, Japan, and Korea in 1500 should somehow be reflected in the solution. The initial orbit that Hasegawa & Nakano (1995) calculate is so similar to that of the Great comet, see their Table 5, that to identify the Far Eastern observations as pertaining to the Great comet seems reasonable. The two authors calculate an orbit for these observations and link these 1500 observations with an 1861 orbit with 87 observations, although give few details of how the linking was done nor if the 1500 observations are referred to the Julian or to the Gregorian calendar. Given the high mean error of the 1500 observations, $3.^\circ1$, they cannot be included directly in the solution because the entire solution would become degraded. They can, however, be included indirectly by our linking the best fit orbit of Table 4 with the orbit of Hasegawa and Nagano.

I therefore took their orbit as the best fit to the 1500 observations and Table 4 or 6 as the best fit 1861–1862 orbit. Neither orbit will represent well the other orbit as can be seen in Figure 6, which shows the two orbits near JD 2335120.5, close to their mid-point. The smooth orbit interpolating the two best fit orbits comes from solution of a boundary value problem constraining the two to agree at JD 2335120.5 (29 March 1681). See Branham (2005b) for the details of how to do this. The first approximation to the solution comes from the rectangular coordinates from the 1500 orbit up to JD 2335120.5 and Table 4's orbit from JD 2335120.5 onwards. The solution is iterated until a convergence criterion is satisfied. Velocities are also needed to calculate orbital elements, but these are not included because they double the size of the linear system, which becomes large even for a band matrix. The velocities, however, can be found by numerical differentiation of the coordinates. Because it is difficult to encounter 10th order formulas for numerical differentiation, I will show how such formulas can be obtained.

Let f_i be the value whose derivative we want. Taylor expansions up to the 10-th order for an integration interval h are:

$$\begin{aligned}
f_{i-5} &= f_i - 5hf_i^{'} + (5h)^2/2!f_i^{''} + \cdots + (5h)^{10}/10!f_i^{10} + O(h^{11}),\\
f_{i-4} &= f_i - 4hf_i^{'} + (4h)^2/2!f_i^{''} + \cdots + (4h)^{10}/10!f_i^{10} + O(h^{11}),\\
&\vdots\\
f_{i+4} &= f_i + 4hf_i^{'} + (4h)^2/2!f_i^{''} + \cdots + (4h)^{10}/10!f_i^{10} + O(h^{11}),\\
f_{i+5} &= f_i + 5hf_i^{'} + (5h)^2/2!f_i^{''} + \cdots + (10h)^{10}/10!f_i^{10} + O(h^{11}).
\end{aligned} \tag{4}$$

The coefficients of the various terms in equation (4) can be represented by the matrix

$$\mathbf{A} = \begin{pmatrix} -5 & -4 & \cdots & 4 & 5 \\ 5^2/2! & 4^2/2! & \cdots & 4^2/2! & 5^2/2! \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ -5^9/9! & -4^9/9! & \cdots & 4^9/9! & 5^9/9! \\ 5^{10}/10! & 4^{10}/10! & \cdots & 4^{10}/10! & 5^{10}/10! \end{pmatrix}. \tag{5}$$

We wish to find coefficients $c_1, c_2, \cdots, c_{10}$ that will eliminate all derivatives but the first in equation (4). These come from the solution of the liner system

$$\mathbf{A} \cdot \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_9 \\ c_{10} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}, \tag{6}$$

which leads to the values

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{pmatrix} = \begin{pmatrix} -0.000793650793651 \\ 0.009920634920635 \\ -0.059523809523808 \\ 0.238095238095233 \\ -0.833333333333324 \\ 0.833333333333324 \\ -0.238095238095233 \\ 0.059523809523808 \\ -0.009920634920635 \\ 0.000793650793651 \end{pmatrix}. \tag{7}$$

The first derivative becomes

$$f_i^{'} = \sum_{i=-5}^{-1} c_{i+6} f_i + \sum_{i=1}^{5} c_{i+5} f_i. \tag{8}$$

The same scheme can be used to estimate second and higher order derivatives. It becomes a question of where to put the 1 in the right-hand-side of equation (6). Table 7 shows the interpolated coordinates for JD 2335120.5 (29 March 1681).

These coordinates should not be considered a genuine orbit because given $\sigma(1) \approx 0.^\circ 21$ for the combined 1500 and 1861–1862 observations their mean errors would be large. They also represent poorly the 1861-1862 observations, as one can see by integrating forwards and finding typical errors of $\approx 1^\circ$. They do, however, provide a good handle on the comet's period. We derive a semi-major axis of a =50.726411 au and a subsequent period of $P = 361.3$ yr. Although a full covariance matrix has not been derived for these coordinates, an estimate of the error in the period can nevertheless be given by our taking differentials of the relationship between period and semi-major axis:

$$\begin{aligned} P &= 2\pi a^{1.5}/k; \\ dP &= 3\pi\sqrt{a} da/k, \end{aligned} \tag{9}$$

where k is the Gaussian gravitational constant. If da is the difference in the $a's$ between Table 6 and Table 7, then $dP = 2.4$ yr. Although the two periods would be considered distinct by a classical comparison of means test, see Wonnacott & Wonnacott (1972, p. 164), their relative closeness precludes such a large period as that Kreutz calculates. This also greatly enhances the probability that the 1500 observations are indeed of the Great comet.

TABLE 6

ELLIPTIC ORBITAL ELEMENTS AND MEAN ERRORS FOR THE GREAT COMET OF 1861: EPOCH JD 2401000.5 (13 AUG. 1861), EQUINOX J2000

Unknown	Value	Mean Error
T_0	JD2400938.48914 12.48914 June 1861	$0.^d00038$
P (yr)	364.96	0.51
a (au)	51.0682	0.0232
e	0.983896	0.105076e−005
q (au)	0.822383	0.105076e−004
Ω	$281.^\circ 81709$	$0.^\circ 48879$e−002
i	$90.^\circ 10938$	$0.^\circ 98605$e−002
ω	$26.^\circ 09288$	$0.^\circ 20072$e−001

TABLE 7

JOINED COORDINATES FOR JD 2335120.5 (29 MARCH 1681)

x_0	−1.3024209e+001
y_0	5.9283262e+001
z_0	8.0245124e+001
$\dot{x}_0$	−4.2657212e−005
$\dot{y}_0$	1.6911650e−004
$\dot{z}_0$	−1.3470286e−004
$\sigma(1)$	$\approx 0.^\circ 21$

6. CONCLUSIONS

The orbit of the Great comet of 1861 (C/1861 J1) has been calculated by use of 1,198 observations in α and 1,164 in δ made between May 1861 and March 1862. No evidence for astrometric observations after 22 March 1862 can be found in the literature. Perturbation by nine planets were taken into account. The post-fit residuals are relatively random, although the possibility of nongravitational forces acting after perihelion passage, or systematic error, cannot be definitively ruled out. Although this new orbit calculates a period of 364.96 ± 0.51 yr, linking this orbit with Hasegawa and Nakano's orbit for 10 observations made in 1500 by means of constraining the two orbits to agree at JD 2335120.5 (29 March 1681) indicates a period of 361.3 yr. Kreutz's period of 409 yr, therefore, seems unsustainable. It is, moreover, highly probable that the Far Eastern observations of 1500 indeed pertain to the Great comet of 1861.

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