THE SURFACE GRAVITATIONAL REDSHIFT OF THE MASSIVE NEUTRON STAR PSR J0348+0432

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RESUMEN

Se calcula el corrimiento al rojo gravitatorio en la superficie de la estrella neutrón masiva PSR J0348+0432 en el marco de la teoria relativista de campo medio, escogiendo adecuadamente las constantes de acoplamiento para los hiperones. Se encuentra que el intervalo de valores para el radio y el corrimiento al rojo gravitatorio en la superficie de PSR J0348+0432 es de 12.062–12.957 km y 0.3473–0.04064, respectivamente. Según nuestros cálculos, los valores para el radio y el corrimiento al rojo gravitatorio para una estrella neutrón de masa canónica son de 13.245 km y 0.2226, respectivamente. Así, el corrimiento al rojo gravitatorio de PSR J0348+0432 resulta ser 1.56–1.8 veces mayor que el de la estrella neutrón de masa canónica.

ABSTRACT

The surface gravitational redshift of the massive neutron star PSR J0348+0432 is calculated in the framework of the relativistic mean field theory by choosing the suitable hyperon coupling constants. It is found that the value ranges of the radius and the surface gravitational redshift of the massive neutron star PSR J0348+0432 are determined as 12.062–12.957 km and 0.3473–0.4064, respectively. In our calculations, the radius and the surface gravitational redshift of the canonical mass neutron star are, respectively 13.245 km and 0.2226. So, the surface gravitational redshift of the massive neutron star PSR J0348+0432 is about 1.56–1.8 times larger than that of the canonical mass neutron star.

1. INTRODUCTION

The surface gravitational redshift of a neutron star is closely connected to the value of M/R, with M being the mass and R the corresponding radius. If either the mass M or the radius R is known from other measurements then the other quantity is known (Glendenning 1997).

The value of the neutron star mass will constrain the radius by a causality equation, through which the mass of the neutron star PSR J1614-2230 was constrained in the range of 8.3–12 km (Özel et al. 2010). So the observation of a new neutron star mass would provide a new constraint to the radius and, furthermore, would give a further constraint to the surface gravitational redshift. Conversely, the surface gravitational redshift also will constrain the mass and the radius of neutron stars.

The massive neutron star PSR J0348+0432, whose mass is $2.01 \pm 0.04\ M_{\odot}$, the largest by far, was observed in 2013 (Antoniadis et al. 2013). This massive mass of the neutron star must restrict its surface gravitational redshift. For a neutron star whose mass is less than that of the massive neutron star PSR J0348+0432, the surface gravitational redshift is determined in the range of 0.25–0.35 (Liang 1986). But what about that of this massive neutron star?

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In order to determine the surface gravitational redshift of the massive neutron star PSR J0348+0432, its mass together with the radius must be obtained theoretically at first.

Considering the hyperon degree of freedom, the mass of a neutron star was determined in the range of 1.5–1.97 $M_\odot$, but only considering nucleons the neutron star mass can reach 2.36 $M_\odot$ (Glendenning 1985; Glendenning & Moszkowski 1991; Zhao & Jia 2012). The emergence of the hyperon will reduce the mass of the neutron star.

Neutron stars are compact stars and therefore the hyperon degree of freedom should be considered (Glendenning 1985). The theoretical results show that the neutron star mass is sensitive to the hyperon coupling constants (Zhao & Jia 2012). One of the methods to select the hyperon coupling constants is the coupling constants of ρ, ω mesons chosen by SU(6) symmetry and those of σ meson chosen by fitting the Λ, Σ and Ξ well depth in nuclear matter.

It can be expected that the mass of the massive neutron star PSR J0348+0432 may be obtained if the hyperon coupling constants is suitably chosen.

In this paper, we use the relativistic mean field (RMF) theory to calculate the surface gravitational redshift of the massive neutron star PSR J0348+0432.

2. THE RMF THEORY AND THE MASS OF A NEUTRON STAR

The Lagrangian density of hadron matter reads as follows (Glendenning 1997)

$$
\begin{aligned}
\mathcal{L} = &\sum_B \overline{\Psi}_B(i\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu - \frac{1}{2}g_{\rho B}\gamma_\mu\tau\cdot\rho^\mu)\Psi_B + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} \\
&+ \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu\cdot\rho^\mu - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 + \sum_{\lambda=e,\mu}\overline{\Psi}_\lambda\left(i\gamma_\mu\partial^\mu - m_\lambda\right)\Psi_\lambda .
\end{aligned}
\tag{1}
$$

The energy density and pressure of a neutron star are given by

$$
\begin{aligned}
\varepsilon = &\frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 + \sum_B\frac{2J_B+1}{2\pi^2}\int_0^{\kappa_B}\kappa^2 d\kappa\sqrt{\kappa^2+m^{*2}} \\
&+ \frac{1}{3}\sum_{\lambda=e,\mu}\frac{1}{\pi^2}\int_0^{\kappa_\lambda}\kappa^2 d\kappa\sqrt{\kappa^2+m_\lambda^{*2}} ,
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
p = &-\frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 + \frac{1}{3}\sum_B\frac{2J_B+1}{2\pi^2}\int_0^{\kappa_B}\frac{\kappa^4}{\sqrt{\kappa^2+m^{*2}}}d\kappa \\
&+ \frac{1}{3}\sum_{\lambda=e,\mu}\frac{1}{\pi^2}\int_0^{\kappa_\lambda}\frac{\kappa^4}{\sqrt{\kappa^2+m_\lambda^{*2}}}d\kappa ,
\end{aligned}
\tag{3}
$$

where, m^* is the effective mass of baryons

$$
m^* = m_B - g_{\sigma B}\sigma .
\tag{4}
$$

We use the O-V equation to obtain the mass and the radius of neutron stars

$$
\frac{dp}{dr} = -\frac{(p+\varepsilon)\left(M+4\pi r^3 p\right)}{r\left(r-2M\right)} ,
\tag{5}
$$

$$
M = 4\pi\int_0^r \varepsilon r^2 dr .
\tag{6}
$$

The gravitational redshift of a neutron star is given by

$$
z = (1-\frac{2M}{R})^{-1/2} - 1 .
\tag{7}
$$

TABLE 1

GL85 NUCLEON COUPLING CONSTANT

m MeV	m_σ MeV	m_ω MeV	m_ρ MeV	g_σ	g_ω	g_ρ	g_2	g_3	C_3	ρ_0 fm^{-3}	B/A MeV	K MeV	a_{sym} MeV	m^*/m
939	500	782	770	7.9955	9.1698	9.7163	10.07	29.262	0	0.145	15.95	285	36.8	0.77

TABLE 2

HYPERON COUPLING CONSTANTS FITTED TO EXPERIMENTAL WELL DEPTH DATA
$U_\Lambda^N = -30$ MeV, $U_\Sigma^N = +40$ MeV AND $U_\Xi^N = -28$ MeV

$x_{\sigma\Lambda}$	$x_{\omega\Lambda}$	$U_\Lambda^{(N)}$	$x_{\sigma\Sigma}$	$x_{\omega\Sigma}$	$U_\Sigma^{(N)}$	$x_{\sigma\Xi}$	$x_{\omega\Xi}$	$U_\Xi^{(N)}$
0.4	0.3679	-30.0300	0.4	0.8250	40.0005	0.4	0.3811	-28.0041
0.5	0.5090	-30.0100	0.5	0.9660	40.0044	0.5	0.5221	-28.0002
0.6	0.6500	-30.0032				0.6	0.6630	-28.0116
0.7	0.7909	-30.0146				0.7	0.8040	-28.0076
0.8	0.9319	-30.0106				0.8	0.9450	-28.0037

3. PARAMETERS

In this work, we choose the nucleon coupling constant GL85 set (Glendenning 1985) listed in Table 1. For the hyperon coupling constant, we define the ratios:

$$x_{\sigma h} = \frac{g_{\sigma h}}{g_\sigma} = x_\sigma, \tag{8}$$

$$x_{\omega h} = \frac{g_{\omega h}}{g_\omega} = x_\omega, \tag{9}$$

$$x_{\rho h} = \frac{g_{\rho h}}{g_\rho}. \tag{10}$$

Here, h denotes the hyperons Λ, Σ and Ξ.

We choose $x_{\rho\Lambda} = 0$, $x_{\rho\Sigma} = 2$, $x_{\rho\Xi} = 1$ by SU(6) symmetry (Schaffner & Mishustin 1996). The experiments show $U_\Lambda^{(N)} = -30$ MeV (Batty, Friedman, & Gal 1997), $U_\Sigma^{(N)} = 10 - 40$ MeV (Kohno et al. 2006; Harada & Hirabayashi 2005, 2006; Friedman & Gal 2007) and $U_\Xi^{(N)} = -28$ MeV (Schaffner-Bielich & Gal 2000). We choose $U_\Lambda^{(N)} = -30$ MeV, $U_\Sigma^{(N)} = 40$ MeV and $U_\Xi^{(N)} = -28$ MeV.

The ratio of hyperon coupling constant to nucleon coupling constant is determined in the range of ∼1/3 to 1 (Glendenning & Moszkowski 1991). For x_σ, we choose x_σ=0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0. For each x_σ, the x_ω first is chosen as 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, respectively. Then, with the restriction of the hyperon well depth (Glendenning 1997)

$$U_h^{(N)} = m_B \left(\frac{m_n^*}{m_n} - 1 \right) x_{\sigma h} + \left(\frac{g_{\omega N}}{m_\omega} \right)^2 \rho_0 x_{\omega h}, \tag{11}$$

the hyperon coupling constants x_ω will be slightly adjusted. The parameters that fit to the experimental data of the hyperon well depth are listed in Table 2.

As $U_\Sigma^{(N)} = 40$ MeV > 0, the Σ hyperon cannot produce (Zhao 2011). So we can only choose $x_{\sigma\Sigma} = 0.4$ and $x_{\omega\Sigma} = 0.825$ while $x_{\sigma\Sigma} = 0.5$ and $x_{\omega\Sigma} = 0.9660$ can be deleted. Herein, from Table 2 we can make up 25 sets of suitable parameters, for which we calculate the mass of the neutron star. We see that only parameters No.24 ($x_{\sigma\Lambda} = 0.8$, $x_{\omega\Lambda} = 0.9319$; $x_{\sigma\Sigma} = 0.4$, $x_{\omega\Sigma} = 0.825$; $x_{\sigma\Xi} = 0.7$, $x_{\omega\Xi} = 0.804$) and No.25 ($x_{\sigma\Lambda} = 0.8$, $x_{\omega\Lambda} = 0.9319$; $x_{\sigma\Sigma} = 0.4$, $x_{\omega\Sigma} = 0.825$; $x_{\sigma\Xi} = 0.8$, $x_{\omega\Xi} = 0.945$) can give the mass of the massive neutron star
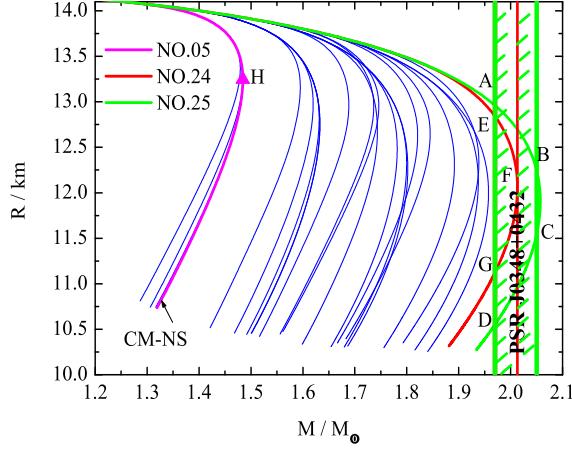
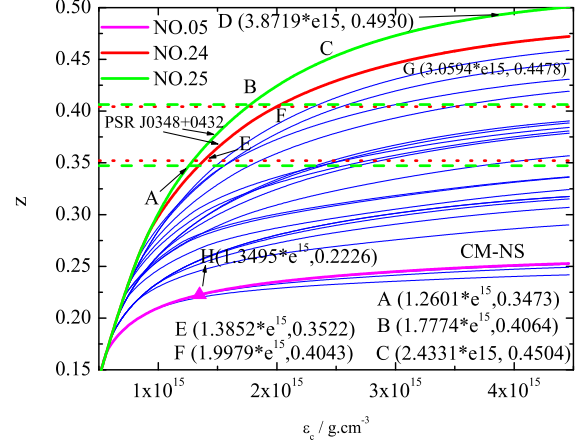Fig. 1. The radius as a function of the mass of the neutron star.

Fig. 2. The surface gravitational redshift as a function of the central energy density.

PSR J0348+0432. So we can use them to describe the surface gravitational redshift of the massive neutron star PSR J0348+0432. In addition, the parameter No.05 ($x_{\sigma\Lambda} = 0.4$, $x_{\omega\Lambda} = 0.3679$; $x_{\sigma\Sigma} = 0.4$, $x_{\omega\Sigma} = 0.825$; $x_{\sigma\Xi} = 0.8$, $x_{\omega\Xi} = 0.945$) gives the neutron star mass 1.4843 $M_\odot$, which is close to the mass of the canonical neutron star (named as CM-NS), and we choose it as a comparison.

4. THE SURFACE GRAVITATIONAL REDSHIFT OF THE MASSIVE NEUTRON STAR PSR J0348+0432

Figure 1 shows the radius as a function of the mass of the neutron star. The mass of the massive neutron star PSR J0348+0432 is 1.97 $M_\odot < M < 2.05$ $M_\odot$. We see only parameters No.24 and No.25 can give a mass greater than 1.97 $M_\odot$ and therefore they can represent the mass of the massive neutron star PSR J0348+0432. In Figure 1, sections AB and EF indicate stable neutron stars while sections BC and FG express unstable ones. We are mainly concerned with the stable neutron stars. We also see that parameter No.05 gives the canonical mass neutron star.

The surface gravitational redshift as a function of the central energy density is given in Figure 2. The sections AB and EF indicate the surface gravitational redshift of the massive neutron star PSR J0348+0432. Section AB indicates the value range of the surface gravitational redshift $0.3473 < z < 0.4064$ and section EF gives $0.3522 < z < 0.4043$. Combining these two cases, the surface gravitational redshift of the massive neutron star PSR J0348+0432 is determined in the range of $z = 0.3473 - 0.4064$. For the canonical mass neutron star (No.05), the surface gravitational redshift corresponding to the maximum mass is $z = 0.2226$. The surface gravitational redshift of the massive neutron star PSR J0348+0432 is about 1.56–1.8 times larger than that of the canonical mass neutron star.

Figure 3 displays the surface gravitational redshift corresponding to the maximum mass as a function of the parameter group number. We see that for different groups of parameters the surface gravitational redshift corresponding to the maximum mass is different. For parameter No.05, the $x_{\sigma\Lambda}$ and $x_{\omega\Lambda}$ is less than those for No.24 and No.25. From Figure 1 we see that the mass for No.05 is less than that for No.24 and No.25 while the radius for No.05 is greater than that for No.24 and No.25. Therefore, the surface gravitational redshift for No.05 is less than that for No.24 and No.25 (Figure 3). It indicates that greater $x_{\sigma\Lambda}$ and $x_{\omega\Lambda}$ correspond to greater surface gravitational redshifts.

The surface gravitational redshift as a function of the radius is shown in Figure 4. We see that the surface gravitational redshift decreases with increasing radius. For No.24 the value of the radius is increasing in the range 12.062–12.840 km and that for No.25 in the range 12.246–12.957 km. In other words, the radius of the massive neutron star PSR J0348+0432 is in the range 12.062–12.957 km. As for the canonical mass neutron star (No.05), the radius is larger, 13.245 km. But all these results cannot explain the numerical size of the surface gravitational redshift.
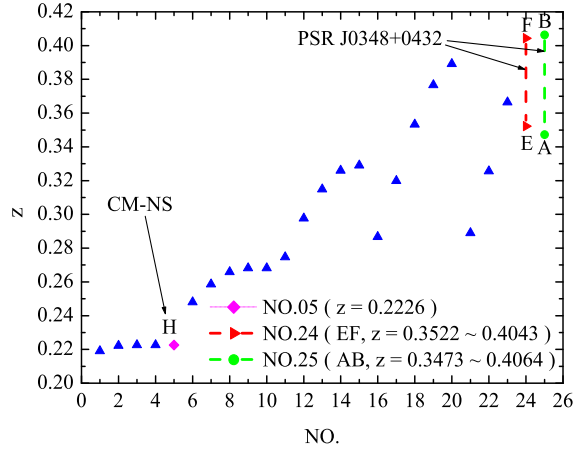
Fig. 3. The surface gravitational redshift corresponding to the maximum mass as a function of the parameter group number.
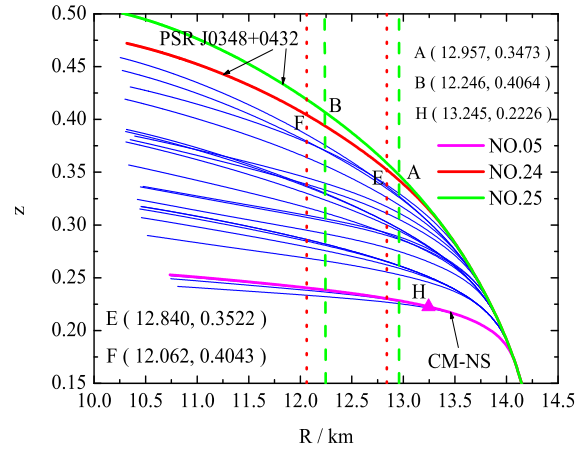

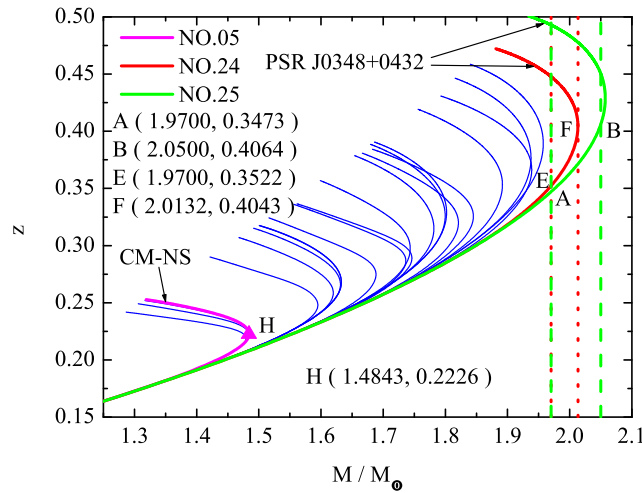Fig. 4. The surface gravitational redshift as a function of the radius.

Fig. 5. The surface gravitational redshift as a function of the mass.

Figure 5 gives the surface gravitational redshift as a function of the mass. We see, for the sections AB and EF corresponding to the stable neutron star, the surface gravitational redshift increases with increasing mass. For No.24 the mass of the neutron star is in the range $1.97 - 2.0132 \ M_\odot$ and that for No.25 is in the range $1.97 - 2.05 \ M_\odot$. That is to say, the mass of the massive neutron star PSR J0348+0432 is in the range $1.97 - 2.05 \ M_\odot$. As for the canonical mass neutron star (No.05), the maximum mass is smaller, namely $1.4843 \ M_\odot$. It can be seen that the mass of the massive neutron star PSR J0348+0432 is greater than that of the canonical neutron star mass while the radius of the massive neutron star PSR J0348+0432 is smaller than that of the canonical neutron star, i.e., the ratio M/R of the massive neutron star PSR J0348+0432 is greater that that of the canonical neutron star. By the formula (7), the surface gravitational redshift is positively correlated with the ratio M/R. So, the surface gravitational redshift of the massive neutron star PSR J0348+0432 is greater than that of the canonical mass neutron star.

5. SUMMARY

In this paper, the surface gravitational redshift of the massive neutron star PSR J0348+0432 is calculated in the framework of the RMF theory by choosing suitable hyperon coupling constants. It is found that if we choose $U_\Lambda^{(N)} = -30$ MeV, $U_\Sigma^{(N)} = 40$ MeV and $U_\Xi^{(N)} = -28$ MeV companied with $x_{\rho\Lambda} = 0, x_{\rho\Sigma} = 2, x_{\rho\Xi} = 1$ and the nucleon coupling constant GL85, 25 sets of suitable parameters can be made up. Two of them can describe the mass of the massive neutron star PSR J0348+0432. We see the value ranges of the radius and the surface gravitational redshift of the massive neutron star PSR J0348+0432 are 12.062–12.957 km and 0.3473–0.4064, respectively. However, the radius and the surface gravitational redshift of the canonical mass neutron star are respectively 13.245 km and 0.2226. The surface gravitational redshift of the massive neutron star PSR J0348+0432 is about 1.56–1.8 times larger than that of the canonical mass neutron star.

In our work, the neutron star is assumed to be a sphere. But a real neutron star is a rotating configuration with high speed and strong magnetic fields. We know that the rotation and strong magnetic fields of the neutron star will affect its mass and radius and thus also its surface gravitational redshift.

In addition, Astashenok, Capozziello, & Odintsov (2013) considered neutron star models in perturbative $f(R)$ gravity with realistic equations of state. The results showed that this effect gives rise to more compact stars than in General Relativity (Astashenok et al. 2013) and it will inevitably affect the surface gravitational redshift of the neutron star.

From the above we see that the technique adopted in this paper is not accurate enough, and some features, such as the rotation, strong magnetic fields and perturbative $f(R)$ gravity of the neutron star, escaped our attention.

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