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HII REGIONS IN HYDROSTATIC BALANCE BETWEEN GAS PRESSURE, RADIATION PRESSURE AND GRAVITY

A. C. Raga,[1,4] J. Cantó,[2] G. Mellema,[3] A. Rodríguez-González,[1] and A. Esquivel,[1,4]

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RESUMEN

Estudiamos las soluciones de una versión modificada de la ecuación de Lane-Emden isotérmica, la cual incorpora el efecto de la presión de radiación (dirigida hacia afuera) asociada a las fotoionizaciones. Estas soluciones son relevantes para regiones HII alrededor de un cúmulo con ≈ 500 estrellas O, que puede fotoionizar el gas hasta distancias de $\approx 200\sqrt{10\,\mathrm{cm}^{-3}/n_0}$ pc (siendo n_0 la densidad central del gas), donde son importantes los efectos tanto de la autogravedad como de la presión de radiación. Encontramos que las soluciones tienen una transición de un régimen "dominado por gravedad" (en el que las soluciones convergen a radios grandes a la solución de la esfera isotérmica autogravitante no singular) a uno "dominado por presión de radiación" (en el que la densidad diverge a un radio finito) para regiones HII con densidades centrales mayores que $n_{crit} = 100$ cm^{-3}. Argumentamos que las soluciones con densidades centrales altas, dominadas por presión de radiación, no ocurrirán en muchas de las situaciones astrofísicamente relevantes, dada la ausencia de un posible medio confinador de presión suficientemente alta.

ABSTRACT

We study the solutions of a modified version of the isothermal Lane-Emden equation, which incorporates the effect of the (owtwards directed) radiation pressure resulting from photoionizations. These solutions are relevant for HII regions around a cluster with over $\approx$500 O stars, which can photoionize gas out to $\approx 200\sqrt{10\,\mathrm{cm}^{-3}/n_0}$ pc (where n_0 is the central gas density), where the effects of the self-gravity and the radiation pressure become important. We find that the solutions have a transition from a "gravity dominated" regime (in which the solutions converge at large radii to the non-singular, isothermal sphere solution) to a "radiation pressure dominated" regime (in which the density diverges at a finite radius) for central HII region densities above $n_{crit} = 100$ cm^{-3}. We argue that the high central density, radiation pressure dominated solutions will not occur in most astrophysically relevant situations, because of the absence of a possible confining environment with a high enough pressure.

Key Words: hydrodynamics — ISM: evolution — ISM: kinematics and dynamics — ISM: HII regions — stars: formation

1. INTRODUCTION

This paper presents solutions to a modified isothermal, Lane-Emden equation with an extra term that models the radiation pressure associated with photoionization processes in the gas. This equation models an extended HII region (ionized by one or more centrally concentrated stellar sources), in which both the self-gravity of the photoionized gas and the radiation pressure are important.

Some previous work on the relevance of radiation pressure in photoionized region has been done. (Haehnelt 1995) suggested that the radiation pressure associated with the capture of photons through photoionization processes might have an important

[1]Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Mexico.

[2]Instituto de Astronomía, Universidad Nacional Autónoma de México, Mexico.

[3]Department of Astronomy and Oskar Klein Centre, Stockholm University, Sweden.

[4]Nordita.

effect on the formation of concentrations of gas during galaxy formation. This scenario was explored numerically by (Wise et al. 2012), who presented numerical simulations with HII regions expanding under the effect of the radiation pressure from the ionizing sources.

(Krumholz & Matzner 2009) studied an analytic model of an expanding neutral shell surrounding an HII region, pushed out by the gas pressure of the photoionized gas and by the radiation pressure. These authors also discuss the conditions necessary for the expansion to stall. Our present work describes a later evolutionary state, in which such an expanding HII region has reached a hydrostatic configuration.

The paper is organized as follows. In § 2 we develop the model, discussing the form of the radiation pressure force term, the resulting modified Lane-Emden equation, and analytic and numerical solutions of this equation. In § 3 we discuss the application of the results to HII regions. Finally, the results are summarized in § 4.

2. THE HYDROSTATIC GRAVITY/GAS+RADIATION PRESSURE CONFIGURATION

2.1. *The radiation pressure force*

In order to obtain a simplified model, we only consider the photoionization of H. The photoionization equilibrium condition within an HII region is:

$$n_{HI}\phi_H = n_e n_{HII}\alpha_H(T)\,, \tag{1}$$

where n_{HI} is the neutral H number density, n_{HII} and n_e are the HII and electron number densities (respectively), $\alpha_H(T)$ is the recombination coefficient and

$$\phi_H = \int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu}\sigma_\nu\, d\nu \tag{2}$$

is the H photoionization rate, in which J_ν is the angularly averaged intensity of the ionizing radiative field, ν_0 is the Lyman limit frequency, h is Planck's constant and σ_ν is the photoionization cross section of H.

The radiation pressure force per unit volume resulting from the photoionization processes is:

$$f_H = n_{HI}\int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu}\frac{h\nu}{c}\sigma_\nu\, d\nu\,, \tag{3}$$

where c is the speed of light.

We now use the standard ISM "grey" approximation, in which we assume that all of the ionizing photons are close to the Lyman limit, and we take out of the integrals (in equations 2 and 3) all of the factors which are multiplying $J_\nu/(h\nu)$. In this approximation, we then have that:

$$f_H = n_{HI}\phi_H\frac{h\nu_0}{c}\,. \tag{4}$$

Finally, we consider that within the HII region we have $n_{HII} \approx n_e \approx n_H$ (where $n_H = n_{HI} + n_{HII}$ is the total H density), and then combine equations (1) and (4) to obtain:

$$f_H = n_H^2\alpha_H\frac{h\nu_0}{c}\,. \tag{5}$$

This equation is the condition that every photoionization event (balanced by a corresponding recombination) gives an $h\nu_0/c$ outwards directed mometum to the gas. Clearly, there are other processes not considered here which could also contribute to the radiation pressure. Examples of this could be the presence of dust within the HII region (absorbing stellar photons at all wavelengths), or the multiple scattering of Lyα photons.

2.2. *The model equation*

We now consider a uniform temperature, spherical HII region in which the gas is subjected to the gravitational force of the centrally located ionizing photon source (or sources), the gas pressure force, and the radiation pressure force derived in § 2.1. The equation describing the resulting hydrostatic configuration is:

$$\frac{d\rho}{dR} = -\frac{G\rho}{a^2R^2}\left[4\pi\int_0^R R'^2\rho(R')\,dR' + M_*\right] + \frac{n_H^2\alpha_H h\nu_0}{a^2c}\,, \tag{6}$$

where M_* is the mass of the central source (or sources), R is the spherical radius, ρ the mass density, G the gravitational constant and a ($\approx$10 km s^{-1}) is the isothermal sound speed of the ionized gas. The last term on the right of equation (6) is the radiation pressure force derived in § 2.1 (equation 5).

If we multiply equation (6) by R^2/ρ and take the derivative with respect to R we obtain:

$$\frac{d}{dR}\left(\frac{R^2}{\rho}\frac{d\rho}{dR}\right) = -\frac{4\pi G}{a^2}\rho R^2 + \frac{\alpha_H h\nu_0}{\mu^2 m_H^2 a^2 c}\frac{d}{dR}\left(\rho R^2\right)\,, \tag{7}$$

where m_H is the mass of the hydrogen atom and μ is the mean molecular mass of the atoms+ions.

Note that in taking the derivative of equation (6) after multiplying by R^2/ρ, the term with the central mass M_* automatically disappears. In the resulting second order differential equation (see equation 7) the effect of a central mass has to be introduced in the form of a boundary condition.

This absence of an explicit term corresponding to a central mass is of course also present in the standard, isothermal Lane-Emden equation (without a radiation pressure term). In this equation, if one introduces the effect of a central mass (in the form of a boundary condition for the second order differential equation), one can show that solutions exist only for zero or negative (i.e., repulsive) values of M_* (see, e.g., the book of Chandrasekhar 1967). As will be shown in § 2.4, the modified isothermal Lane-Emden equation with radiation pressure (equation 7) does allow solutions with $M_* > 0$.

This is the traditional, isothermal Lane-Emden equation (hereafter, the L-E equation) with an extra term representing the radiation pressure force. As is usually done for the Lane-Emden equation, we write equation (7) in dimensionless form by defining

$$r = \frac{R}{R_0}\,; \qquad \eta = \frac{\rho}{\rho_0}\,, \tag{8}$$

where the core radius R_0 and the central density ρ_0 obey the relation

$$R_0 = \sqrt{\frac{3a^2}{2\pi G\rho_0}}\,. \tag{9}$$

In terms of these dimensionless variables, equation (7) takes the form:

$$\frac{d}{dr}\left(\frac{r^2}{\eta}\frac{d\eta}{dr}\right) = -6\eta r^2 + \lambda\frac{d}{dr}\left(\eta r^2\right)\,, \tag{10}$$

where λ is a dimensionless parameter defined as:

$$\lambda = \sqrt{\frac{3\rho_0}{2\pi G}}\frac{\alpha_H h\nu_0}{\mu^2 m_H^2 ac}\,. \tag{11}$$

2.3. *The radiation pressure dominated case*

Neglecting the gravitational term in equation (6), one obtains the trivial integral:

$$\rho_{rad}(R) = \frac{\rho_0}{1 - R/R_{rad}}\,, \tag{12}$$

with

$$R_{rad} \equiv \frac{(\mu m_H a)^2 c}{\alpha_H h\nu_0\rho_0}\,, \tag{13}$$

and where ρ_0 is the central density. This solution has a flat inner region, and a divergence in the density at $R \to R_{rad}$.

In the dimensionless variables of the previous section, equation (12) takes the form

$$\eta_{rad} = \frac{1}{1 - \lambda r}\,, \tag{14}$$

with

$$\lambda = \frac{R_0}{R_{rad}}\,, \tag{15}$$

which coincides with the definition of λ given in equation (11). The dimensionless density stratification given by equation (14) diverges at

$$r_{rad} = 1/\lambda\,. \tag{16}$$

2.4. *The singular solution*

It is clear that equation (7) shares the singular solution

$$\rho_s(R) = \frac{a^2}{2\pi G R^2}\,, \tag{17}$$

of the (radiation pressure-less) isothermal L-E equation, as the second term on the right hand side of equation (7) is equal to zero for this radial density dependence.

However, if we insert solution (17) into the integro-differential equation (6), we see that this singular solution requires the presence of a central mass

$$\begin{aligned} M_{*,s} &= \frac{\alpha_H h\nu_0 a^2}{2\pi G^2\mu^2 m_H^2 c} \\ &= 7.17\times 10^5\, M_\odot \left(\frac{a}{10\,\mathrm{km\,s^{-1}}}\right)^2\,, \end{aligned} \tag{18}$$

where we have set $\alpha_H = 2.60\times 10^{-13}\mathrm{cm^3\,s^{-1}}$ (the H case B recombination coeficient at 10^4K) and $\mu = 1.3$. This assumption for the value of μ amounts to assuming that the gas is 90% H and 10% He, and treating the HeI to HeII photoionization and recombination as if it had the same properties as the HI/II ionization.

$M_{*,s}$ (equation 18) is the central mass that is necessary to provide a gravitational force that exactly balances the radiation pressure term (last term on the right of equation 7) at all points when $\rho = \rho_s$. Therefore, the singular solution of equation (7) is only valid for a precise choice of central mass of the modeled HII region.

Furthermore, the singular solution of course has an infinite number of recombinations for volumes with $R \to 0$, so that a source with an ionizing photon rate $S_* \to \infty$ would be necessary to photoionize this solution. This problem could in principle be avoided by requiring the existence of an empty inner sphere of radius R_w (which could correspond to a region occupied by a hot stellar wind bubble), and a $\rho \propto R^{-2}$ solution for $R > R_w$. However, the requirement of a precise central mass (necessary for balancing the radiation pressure force) will still remain, so that the singular solution described above remains only as a mathematical curiosity. Of course, other non-singular solutions (corresponding to different values of M_*) of equation (7) probably exist, and deserve further study.

2.5. *The non-singular solution: second order in r*

Since more than a century of research has not lead to the discovery of an analytic non-singular solution to the isothermal, L-E equation, we can safely assume that we will not find such a solution for its extended version (equation 10, including the radiation pressure term). Therefore, we propose a Taylor series expansion to second order in dimensionless radius r (of the form $\eta = 1 + a_1 r + a_2 r^2$), insert this expansion into the right and left hand terms of equation (10) and equate the resulting coefficients multiplying r and r^2. In this standard way, we obtain the second order solution

$$\eta_2(r) = 1 + \lambda r + (\lambda^2 - 1)r^2 \,. \qquad (19)$$

For $\lambda = 0$, this solution correctly coincides with the $\eta = 1 - r^2$ solution of the isothermal L-E equation, and deviates substantially for $\lambda \approx 1$ and above.

Inserting the second order solution (equation 19) into equation (6), it is clear that it corresponds to the $M_* = 0$ case. Even though the L-E equation does not have solutions for central masses $M_* > 0$ (though it does have solutions with unphysical $M_* < 0$ values), we see that equation (10) has at least one solution with $M_* = M_{*,s} > 0$ (i.e., the singular solution described in § 2.4, see equations 17-18), and might also have (singular) solutions for central masses in the $0 < M_* < M_{*,s}$ range.

2.6. *Numerical solution*

It is trivial to integrate equation (10) numerically. We first choose a value for the dimensionless parameter λ, and use the second order solution (19) to compute $\eta_i = \eta(r_i)$, where we take $r_i = 10^{-5}$ (or any $r \ll 1$ value such that the second order solution is accurate). These (r_i, η_i) values, together with the value of the

$$I_i = \int_0^{r_i} r^2 \eta_2(r) dr \qquad (20)$$

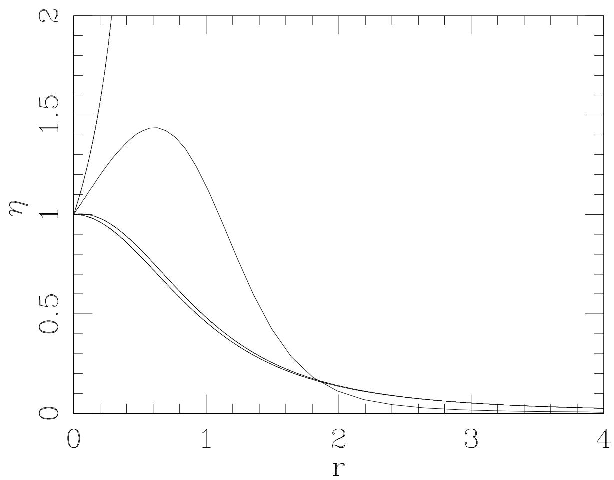

Fig. 1. Dimensionless density $\eta = \rho/\rho_0$ (where ρ is the density and ρ_0 its central value) as a function of dimensionless radius $r = R/R_0$ (where R_0 is the core radius) obtained from numerical integrations of equation (10) with different values of the dimensionless parameter λ (see equation 11). The lower curve in the $r < 1$ region corresponds to $\lambda = 0$ (i.e., the nonsingular solution of the L-E equation), and going upwards in density we have the $\lambda = 0.1$, 1.0 and 2.0 solutions. The $\lambda = 2.0$ solution diverges at a finite radius.

integral are then used to initialize the numerical integration of equation (10). We have checked that for high values of λ, the numerical integration follows the "gravity less" solution (equation 14) and for $\lambda \ll 1$ it follows the non-singular solution of the isothermal L-E equation (see, e.g., Raga et al. 2013).

Figure 1 shows the $\eta(r)$ solutions obtained for different values of the dimensionless parameter λ (see equation 11). It is clear that for $\lambda = 0.1$ we obtain a solution similar to the $\lambda = 0$, radiation pressure-less solution. For $\lambda = 1$, we obtain an $\eta(r)$ solution with a density hump at $r \approx 0.6$. For $\lambda = 2$, we obtain a solution in which the density diverges at $r \approx 0.5$, in a qualitatively similar way to the gravity-less solution (see equation 14).

Therefore, we find that the numerical solutions fall into two categories:

- $\lambda < \lambda_c$ → solutions that for $r \to \infty$ approach the $\lambda = 0$ solution (i.e., the non-singular solution of the isothermal L-E equation),
- $\lambda > \lambda_c$ → solutions in which the density diverges at a finite value r_{max} of the dimensionless radius.

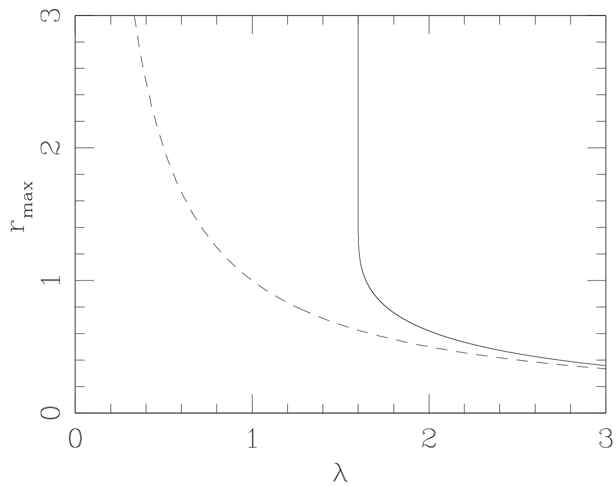

Fig. 2. Radius of density divergence r_{max} (solid line) as a function of the dimensionless parameter λ obtained from the numerical integrations of equation (10). Also shown is the radius of density divergence r_{rad} (dashed line) deduced from the "no self-gravity" model (see equation 16).

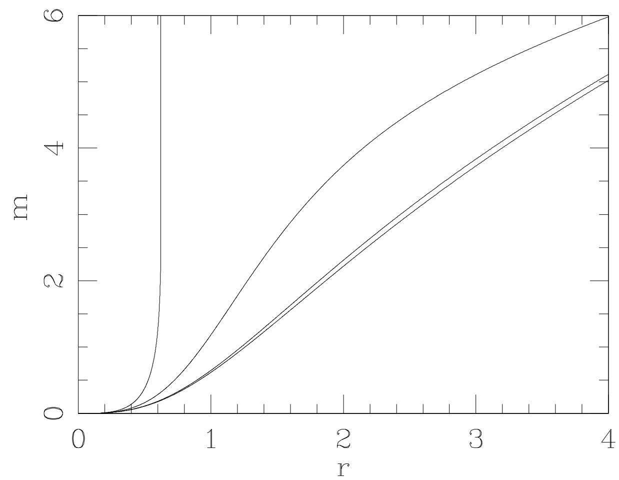

Fig. 3. Dimensionless mass m as a function of dimensionless radius r (see equation 21) for solutions with $\lambda = 0$ (bottom curve), 0.1, 1.0 and 2.0 (top curve).

In Figure 2 we show the maximum (dimensionless) radius r_{max} as a function of λ, as obtained from numerical integrations of equation (10). We see that $r_{max} \to \infty$ for $\lambda \to \lambda_c^+$ with $\lambda_c = 1.60$.

In this figure we also show the radius of density divergence r_{rad} deduced from the "radiation pressure only" model (see equation 16). It is clear that the values of r_{max} obtained from the full model closely approach the values of r_{rad} for $\lambda > 2$.

Finally, with the numerical solutions of equation (10) we have computed the dimensionless mass

$$m(r) = 3 \int_0^r r'^2 \eta(r') dr' \,, \tag{21}$$

(in units of $M_0 = 4\pi R_0^3 \rho_0/3$) and recombination rate

$$q_{rec}(r) = 3 \int_0^r r'^2 \eta^2(r') dr' \,, \tag{22}$$

[in units of $Q_0 = 4\pi\alpha_h R_0^3 \rho_0^2/(3\mu^2 m_H^2)$] within a radius r.

From Figure 3, we see that the $\lambda < \lambda_c$ solutions share the asymptotic $m(r) \propto r$ large r dependendence of the $\lambda = 0$ solution (i.e., the solution of the L-E equation). The $\lambda > \lambda_c$ solutions have masses that strongly diverge as $r \to r_{max}$.

From Figure 4, we see that the recombination rates $q_{rec}(r)$ of the $\lambda < \lambda_c$ solutions reach an asymptotic value $q_\infty(\lambda)$ for $r \to \infty$. On the other hand, the $\lambda > \lambda_c$ solutions have $q_{rec}(r)$ that diverge for $r \to r_{max}$.

Finally, in Figure 5 we show the value of total recombinations q_∞ within the stratified structures as a function of λ. The total number of recombinations diverges for $\lambda \to \lambda_c^-$ (with $\lambda_c = 1.60$).

3. APPLICATION TO HII REGIONS

In § 2.6, we have shown that the hydrostatic structure resutling from the balance between the self-gravity, the gas pressure and the radiation pressure in a photoionized gas has two regimes, depending on whether the dimensionless parameter λ is larger or smaller than the critical $\lambda_c = 1.60$ value. The solutions with $\lambda < \lambda_c$ have densities (and therefore pressures) which asymptotically go to zero for large radii. These hydrostatic solutions could either be fully photoionized (since they have a finite rate of recombinations, see Figure 5), or could have a pressure equilibrium outer boundary with neutral material.

Interestingly, if we use the case B recombination coefficient of H at 10^4K ($\alpha_H = 2.60 \times 10^{-13}\mathrm{cm}^3\,\mathrm{s}^{-1}$) and set $\mu = 1.3$ (i.e., assuming abundances by number of 90% for H and 10% for He, and assuming that the He I/II and ionizations follow H I/II), the dimensionless parameter λ (see equation 11) has the value

$$\lambda = 0.50 \left(\frac{10\,\mathrm{km\,s^{-1}}}{a}\right) \left(\frac{n_0}{10\,\mathrm{cm^{-3}}}\right)^{1/2} \,, \tag{23}$$

where $n_0 = \rho_0/(\mu m_H)$ is the central number density of the self-gravitating HII region. It is a somewhat curious coincidence that we obtain $\lambda \approx \lambda_c = 1.60$

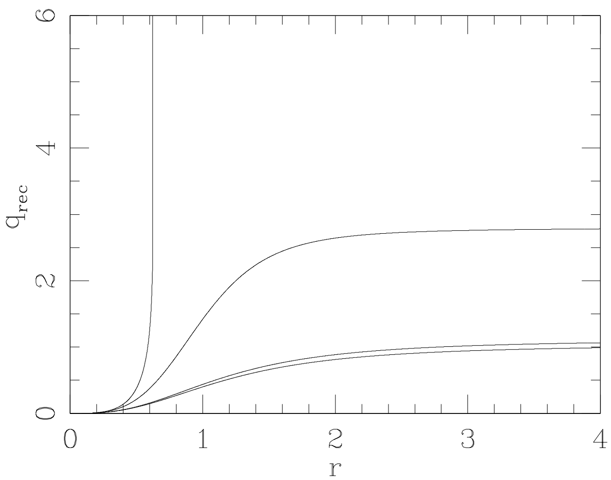

Fig. 4. Dimensionless recombination rate q_{rec} as a function of dimensionless radius r (see equation 22) for solutions with $\lambda = 0$ (bottom curve), 0.1, 1.0 and 2.0 (top curve). The solutions with $\lambda < \lambda_c = 1.60$ asymptotically tend to a finite recombination rate $q_\infty(\lambda)$ for $r \to \infty$.

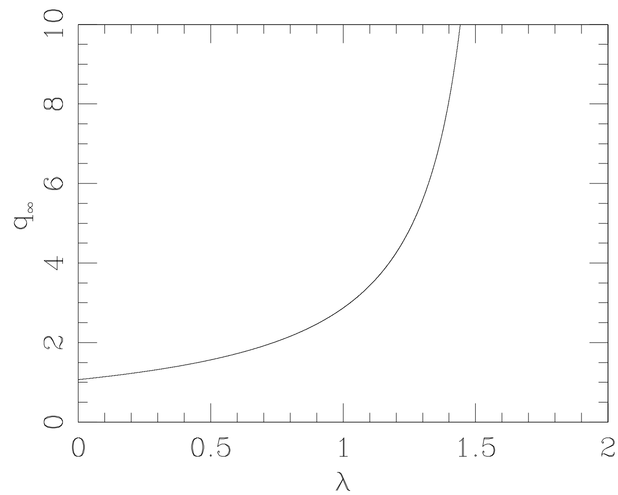

Fig. 5. Total dimensionless recombination rates q_∞ (of the density stratifications out to $r \to \infty$) as a function of the dimensionless parameter λ (see equation 22 and Figure 4).

(the critical value of λ for the transition between the two regimes of the solutions, see equation (23) for a $n_0 \approx 100$ cm^{-3} central density.

The dimensional recombination rate

$$Q_{rec} = \frac{4\pi R_0^3}{3} n_0^2 \alpha_H q_{rec}(R/R_0)\,, \qquad (24)$$

with $q_{rec}(r)$ given by equation (22) has to be balanced by the total rate S_* of ionizing photons produced by the central, stellar sources. As we see from Figure 5, $q_\infty \approx 1$ for $\lambda < 1/2$. For this range of λ, we can then set $q_{rec} \approx q_\infty \approx 1$ in equation (24) and use the resulting value of Q_{rec} to estimate the photon rate

$$S_{*,0} = 2.06 \times 10^{52}\mathrm{s}^{-1} \left(\frac{a}{10\,\mathrm{km\,s}^{-1}}\right)^3 \left(\frac{n_0}{10\,\mathrm{cm}^{-3}}\right)^{1/2} \qquad (25)$$

necessary for photoionizing the HII region out to $R \to \infty$. In other words, in order to maintain the photoionization of a self-gravitating HII region with a central density of 10 cm^{-3}, it is necessary to have the combined photoionization rate of ≈ 650 O5 stars.

The value of the core radius of the self-gravitating structure is obtained from equation (9):

$$R_0 = 186\,\mathrm{pc} \left(\frac{10\,\mathrm{km\,s}^{-1}}{a}\right) \left(\frac{10\,\mathrm{cm}^{-3}}{n_0}\right)^{1/2}\,, \qquad (26)$$

and the mass of the HII region within the core radius R_0 is

$$M_0 \approx \frac{4\pi R_0^3}{3} n_0 \mu m_H =$$

$$8.61 \times 10^6\, M_\odot \left(\frac{a}{10\,\mathrm{km\,s}^{-1}}\right)^3 \left(\frac{10\,\mathrm{cm}^{-3}}{n_0}\right)^{1/2}\,. \qquad (27)$$

The full mass of the HII region could be substantially larger, depending on the value of the maximum radius of the self-gravitating structure (see Figure 3).

The above considerations assume that we have $\lambda < \lambda_c = 1.60$ (implying $n_0 < 100$ cm^{-3}, see equation 23). The solutions with $\lambda > \lambda_c$ are less well behaved, since they have a density (and therefore a pressure) which diverges at a finite radius R_{max} (see Figure 2). If the photoionization rate of the central source(s) is large enough to balance the recombinations in the steeply increasing density vs. radius structure out to a Strömgren radius $R_S \approx R_{max}$ (see the $\lambda = 2$ solution of Figure 1), it will be necessary to have a very high pressure external, confining medium.

In the general situation in which such a high pressure confining medium does not exist, the $\lambda > \lambda_c = 1.60$ regime will not be hydrostatic, and the HII region might adopt an outward flowing, radiation pressure driven wind solution.

4. CONCLUSIONS

We have studied the hydrostatic configuration of an HII region, resulting from the balance between the pressure force, the self-gravity of the gas and the radiation pressure due to the photoionization processes. In this model, the gravitational force due to the central stars has been neglected, as this is a necessary requirement for obtaining solutions of the hydrostatic equation extending to the origin.

Clearly, the model is only relevant for HII regions ionized by a cluster with many O stars. We find that in order to photoionize a self-gravitating HII region out to infinity, it is necessary to have $\approx 650\,(n_0/10\,\mathrm{cm}^{-3})^{1/2}$ O5 stars (where n_0 is the central density of the HII region, see equation 25). A comparable number of O stars is necessary to ionize the HII region only out to its core radius (where the effects of the self-gravity become important).

If one assumes a Salpeter initial mass function, a cluster with ≈ 600 O stars has a total mass of $\approx 6 \times 10^4 M_\odot$ (see Leitherer & Heckman 1995). This mass is indeed much smaller than the total mass of the HII region (of $\approx 10^7 M_\odot$ within the core radius, see equation 27), so that it is indeed a reasonable assumption to neglect the gravity of the central stars.

We find a dimensionless parameter λ basically depending only on the central density n_0 of the cloud (see equation 23), which measures the relative importance of the radiation pressure associated with the photoionization processes. A sharp transition to a "radiation pressure dominated" regime is obtained for densities in excess of $n_{crit} = 100$ cm^{-3} (equation 23 with $\lambda = \lambda_c = 1.60$), in which the gas pressure of the HII region monotonically grows as a function of radius.

We argue that these $\lambda > \lambda_c$ solutions are unlikely to occur in astrophysical objects because of the high outside pressure that would be needed to confine them. Therefore, this regime of high radiation pressure will probably result in an outward flowing, wind-type solution. As this wind lacks an interior resupply (because the stellar winds from the sources contribute a negligible amount of mass), the central density will drop. Once a low enough central density is attained, the configuration will reach the $\lambda < \lambda_c$ regime, and a hydrostatic configuration with low outer pressure could be attained. Therefore, we might expect to observe a large number of giant HII regions with densities in the $n_0 \approx 10 \rightarrow 100$ cm^{-3} range, which have previously shedded mass in order to settle into a comfortable hydrostatic, self-gravitating regime with λ somewhat below $\lambda_c = 1.60$.

We should note that the formation of the extended, self-gravitating HII regions described in this paper will take a rather long time. Using a value $R_0 \approx (10\,\mathrm{cm}^{-3}/n_0)^{1/2} \times 200$ pc for the core radius (equation 26) and an expansion velocity $a = 10$ km s^{-1}, one obtains an expansion time $t_{exp} \approx (10\,\mathrm{cm}^{-3}/n_0)^{1/2} \times 2 \times 10^7$ yr. Clearly, in order to reach the hydrostatic situation it will be necessary to have a resupply of central O stars as a result of ongoing star formation.

The earlier, expanding regime of the HII regions can in principle be modeled with analytic, "expanding shell" models such as the one of (Matzner 2002) and (Krumholz & Matzner 2009), who included a radiation pressure term). However, it is not clear whether or not it will be possible to derive an analytic (or semi-analytic) model that will converge at long times to the hydrostatic solution (as has been done for the non-gravitating case by Raga et al. 2012a, b).

We should note that in the present models the presence of stellar winds from the ionizing photon stellar sources has not been considered. These winds could have important effects in many cases. Also, the effect of the gravity of the stellar sources (or of an associated dark matter distribution) has not been included. Finally, the effect of the radiation pressure on dust grains that could be present within the nebula has not been considered (Draine 2011 studies this effect in nebulae not subject to gravitational forces). Clearly, a substantial amount of work remains to be done.

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J. Cantó: Instituto de Astronomía, Universidad Nacional Autónoma de México, Apdo. Postal 70-264, C.P. 04510, México, D.F., México.

A. Esquivel and A. C. Raga: Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Apdo. Postal 70-543, C.P. 04510, México, D.F., México and Nordita, KTH Royal Institute of Technology and Stockholm University, Roslagstullsbacken 23, 10691 Stockholm, Sweden (esquivel, raga@nucleares.unam.mx).

G. Mellema: Department of Astronomy and Oskar Klein Centre, Stockholm University, AlbaNova, SE-10691 Stockholm, Sweden(garrelt.mellema@astro.su.se).

A. Rodríguez-González: Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Apdo. Postal 70-543, C.P. 04510, México, D.F., México (ary@nucleares.unam.mx).