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KINEMATICS AND VELOCITY ELLIPSOID OF THE SOLAR NEIGHBORHOOD WHITE DWARFS

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RESUMEN

Con el objeto de determinar la distribución de velocidades de las enanas blancas en el entorno solar utilizamos las componentes de la velocidad espacial. Utilizamos dos muestras, las más cercanas que 20 y 25 pc. Además, calculamos el movimiento solar y las dispersiones de velocidades para cuatro sub-muestras, a saber, DA, no-DA, enanas blancas calientes, y frías. La comparación de nuestros resultados para las muestras de 20 y 25 pc da como resultado una buena concordancia, mientras que los resultados de las comparaciones entre las otras sub-muestras no concuerdan. Se discute la dependencia de las dispersiones de velocidades y el movimiento solar de la composición química y la temperatura efectiva.

ABSTRACT

To determine the velocity ellipsoid of the solar neighborhood white dwarfs, we use the space velocity components of stars. Two samples of white dwarfs are used, the 20 pc and 25 pc samples. Beside the two main samples, the solar velocity and velocity dispersions are calculated for four subsamples, namely DA, non - DA, hot and cool white dwarfs. A comparison between the results of the 20 pc sample and those of the 25 pc sample gives good agreement, while the comparison between the other subsamples gives poor agreement. The dependence of the velocity dispersions and solar velocity on the chemical composition and effective temperatures is discussed.

Key Words: solar neighborhood — stars: kinematics and dynamics — stars: white dwarfs

1. INTRODUCTION

The majority of stars will eventually end their lives as white dwarfs. These faint stellar remnants can be used in many different investigations in astrophysics. White dwarf cooling processes have been used to date the globular star cluster M4 (Hansen et al. 2004; Hansen et al. 2002) and to independently determine the age of the galactic halo. Also, white dwarfs were used to determine the mass function of the cluster above the main-sequence turn-off (Richer et al. 2004 and Richer et al. 2002). Since all stars with a mass above 0.8 $M_\odot$ have evolved off the main-sequence in a 12 Gyr population, the white dwarfs represent our only link to the distribution of stars (i.e., the initial mass function) of intermediate and massive stars in such systems. White dwarfs are also astrophysically important when considering the chemical evolution of the Galaxy.

The velocity distribution of stars in the solar neighborhood has been characterized as an ellipsoid the centroid, size, and orientation of which vary systematically with the ages (and hence colors) of the stars under investigation (Hogg et al. 2005; Dehnen & Binney 1998).

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It has been known for a long time (Ogorpdnikov 1965) that, in the neighborhood of the Sun, the characteristic feature of stellar motion is the fact that the peculiar velocities have an axis of greatest mobility and this characteristic is represented most conveniently on the basis of an ellipsoidal law of velocity distribution.

In the present paper, we shall determine the velocity ellipsoid of solar neighborhood white dwarfs. We shall investigate the dependence of the velocity ellipsoid parameters on the number of stars, their spectral type and effective temperatures. The structure of the paper is as follows: § 2 deals with the method of computation and the data used. § 3 is devoted to the results and discussion. The conclusion is outlined in § 4.

2. DATA AND METHOD OF COMPUTATION

2.1. *Data*

The data used in the present computations are those of Sion et al. (2009) and Sion et al. (2014) for white dwarf within 20 and 25 pc of the Sun. The 20 pc sample contains a total of 126 candidate white dwarfs of different spectral types.

The 25 pc sample contains 141 candidates of spectral type DA and 68 of non-DA. The effective temperature ranges from 2600 K to 30510 K. The vector components of the space motions U, V and W are computed and tabulated.

The atmospheric parameters in the two samples were determined by different methods; i.e. photometric, spectroscopic and parallax observations.

In Table 1 we list the 25 pc white dwarfs list (209 candidate). The columns are labeled as follows: the WD number in Column 1, the spectral type in Column 2, the effective temperature in Column 3 and the space motions U, V and W in Columns 4, 5 and 6, respectively.

2.2. *Model*

To compute the velocity ellipsoid and its parameters for the solar neighborhood white dwarfs we follow the computational algorithm of Elsanhoury et al. (2013). A brief explanation of the algorithm will be given here.

The coordinates of the $i^{th.}$ star with respect to axes parallel to the original axes, but shifted to the center of the distribution, i.e. to the point $\overline{U}$, $\overline{V}$ and $\overline{W}$, will be $\left(U_i - \overline{U}\right)$; $\left(V_i - \overline{V}\right)$; $\left(W_i - \overline{W}\right)$, where U, V and W are the components of the space velocities and $\overline{U}$, $\overline{V}$ and $\overline{W}$ are the mean velocities defined as:

$$\overline{U} = \frac{1}{N}\sum_{i=1}^{N} U_i; \quad \overline{V} = \frac{1}{N}\sum_{i=1}^{N} V_i; \quad \overline{W} = \frac{1}{N}\sum_{i=1}^{N} W_i \tag{1}$$

N being the total number of the stars.

Let ξ be an arbitrary axis, its zero point coincident with the center of the distribution and let l,m and n be the direction cosines of the axis with respect to the shifted one; then the coordinates Q_i of the point i, with respect to the ξ axis are given by:

$$Q_i = l\left(U_i - \overline{U}\right) + m\left(V_i - \overline{V}\right) + n\left(W_i - \overline{W}\right). \tag{2}$$

Let us adopt, as the measure of the scatter components Q_i, a generalization of the mean square deviation, defined by

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} Q_i^2 \tag{3}$$

From equations (1), (2) and (3) we deduce after some calculations that

$$\sigma^2 = \underline{x}^T B \underline{x} \tag{4}$$

where $\underline{x}$ is the (3×1) direction cosine vector and B is the (3×3) symmetric matrix μ_{ij}, with elements μ_{ij}:

$$\begin{aligned}
\mu_{11} &= \tfrac{1}{N}\textstyle\sum_{i=1}^{N} U_i^2 - \left(\overline{U}\right)^2; & \mu_{12} &= \tfrac{1}{N}\textstyle\sum_{i=1}^{N} U_i V_i - \overline{UV};\\
\mu_{13} &= \tfrac{1}{N}\textstyle\sum_{i=1}^{N} U_i W_i - \overline{UW}; & \mu_{22} &= \tfrac{1}{N}\textstyle\sum_{i=1}^{N} V_i^2 - \left(\overline{V}\right)^2;\\
\mu_{23} &= \tfrac{1}{N}\textstyle\sum_{i=1}^{N} V_i W_i - \overline{VW}; & \mu_{33} &= \tfrac{1}{N}\textstyle\sum_{i=1}^{N} W_i^2 - \left(\overline{W}\right)^2.
\end{aligned} \tag{5}$$

TABLE 1

DATA OF 209 WD STARS (25 PC) STUDIED BY SION ET AL. (2014)

No.	WD No.	Sp. Type	$T_{\rm eff}$	U	V	W
1	0000 − 345	DCP8.1	6643	-11.9	-43.7	3.2
2	0008 + 424	DA6.8	7380	-10	-1.9	-17.4
3	0009 + 501	DAH7.6	6502	-28.1	8.6	-24.1
4	0011 − 134	DAH8.4	5992	-75.5	-31.1	-9.8
5	0011 − 721	DA7.8	6325	10.3	-23.2	12.2
6	0029 − 031	DA11.3	4470	68.3	-23	2.6
7	0038 + 555	DQ4.6	10900	39.4	-2	-11.9
8	0038 − 226	DQpec9.3	5529	-24.9	-4.6	-1.2
9	0046 + 051	DZ7.4	6215	-2.8	-53.6	-30.3
10	0108 + 048	DA6.4	8530	38.2	-0.1	11.8
11	0108 + 277	DA9.6	6428	-11.9	0.3	-9.9
12	0115 + 159	DQ5.6	9119	-21.4	-27.2	-32
13	0121 − 429	DAH7.9	6299	-0.7	-46.8	11.1
14	0123 − 460	DA8.5	5898	24.3	-91	24.9
15	0134 + 883	DA2.8	18311	-11.1	5.1	6.1
16	0135 − 052	DA6.9	7118	16.8	-37.2	-1.6
17	0141 − 675	DA7.8	6248	-21.8	-22.8	21.5
18	0148 + 467	DA3.8	14005	2.6	1.8	8.3
19	0148 + 641	DA5.6	9016	12.3	-11.2	-6.8
20	0208 + 396	DAZ6.9	7264	43	-62.9	-6.7
21	0213 + 427	DA9.0	5507	37.8	-67.3	-19.8
22	0227 + 050	DA2.7	18779	-0.1	-8.9	7.1
23	0230 − 144	DA9.2	5477	-20.7	-43.3	-13.3
24	0231 − 054	DA3.7	13550	68.5	-6.7	-56.4
25	0233 − 242	DC9.3	5312	-24.2	-34.1	-9.8
26	0236 + 259	DA9.2	5500	29.1	-21.1	-8.9
27	0243 − 026	DAZ7.4	6839	8.9	-51.6	-34.5
28	0245 + 541	DAZ9.5	5319	-16.4	10.9	-24.2
29	0255 − 705	DAZ4.7	10560	22.2	-82	-0.6
30	0310 − 688	DA3.3	16865	0.1	-4.2	2.8
31	0311 − 649	DA4.0	11945	7.9	-12.8	9.5
32	0322 − 019	DAZ9.9	5195	-21.5	-66.8	-20.7
33	0326 − 273	DA5.4	8483	46.7	-29.7	47.2
34	0341 + 182	DQ7.7	6568	-16	-94.9	-23.8
35	0344 + 014	DC9.9	5170	-7.9	-43.8	-4.9
36	0357 + 081	DA9.2	5478	-25.3	-4.3	-36.8
37	0413 − 077	DA3.1	17100	-44.2	-34.8	-73
38	0416 − 594	DA3.3	14000	-4.9	-5.7	-1.6
39	0419 − 487	DA8	6300	-1.2	-91.1	-57.4
40	0423 + 044	DA	5140	-6.9	-80.5	17.2
41	0423 + 120	DA8.2	6167	5.3	18.1	3.8

No.	WD No.	Sp. Type	$T_{\rm eff}$	U	V	W
106	1121 + 216	DA6.7	7434	57.8	-25.3	-21.5
107	1124 + 595	DA4.8	10747	-12.2	1.3	6
108	1134 + 300	DA2.4	22469	9.2	-4.6	-2.9
109	1142 − 645	DQ6.4	7966	-52.1	25.3	6.6
110	1149 − 272	DQ8.1	6200	24.9	-8.5	-0.8
111	1202 − 232	DAZ5.8	8767	3	6.1	8.6
112	1208 + 576	DAZ8.6	6200	-45.5	-10.6	23
113	1214 + 032	DA8.0	6272	64	-11.2	2.3
114	1223 − 659	DA6.6	7594	0.7	0.1	-8.4
115	1236 − 495	DA4.3	11599	24	-17	-8.6
116	1241 − 798	DC/DQ	9556	38.1	-36.9	33.3
117	1257 + 037	DA9.0	5616	-6.2	-69.4	-25.6
118	1309 + 853	DAP9	5440	-16.3	-1.3	15.4
119	1310 + 583	DA4.8	10544	-16.3	6.7	5.7
120	1310 − 472	DC11.9	4158	-133.6	81.4	-51.5
121	1315 − 781	DC8.8	5619	-30.2	34	-41.7
122	1315 − 781	DC8.8	5619	-23.5	26.4	-32.3
123	1327 − 083	DA3.6	14571	49.2	-76.6	-7.4
124	1334 + 039	DA11	4971	87.5	-122.2	8.5
125	1344 + 106	DAH7.1	7059	54.6	-62.3	14.1
126	1337 + 705	DAZ2.5	20464	43.7	-15.5	29.9
127	1339 − 340	DA9.5	5361	55	108	177.4
128	1344 + 572	DA3.8	13389	27.3	26.8	37.5
129	1345 + 238	DA11	4581	67	-47.4	20.1
130	1350 − 090	DAP5	9518	-21.7	-23	-23.2
131	1401 + 457	DC19	2600	16.9	-27.6	5.5
132	1425 − 811	DAV4.2	12098	-7.9	-26.2	-44
133	1436 − 781	DA8.1	6270	29.7	-32.5	23.3
134	1444 − 174	DC10.2	4982	33	-61.4	17.6
135	1532 + 129	DZ6.7	7500	-1.8	-26.2	3.9
136	1538 + 333	DA5.6	8940	12	-5.2	11.8
137	1544 − 377	DA4.8	10610	5.5	-22.4	7.6
138	1609 + 135	DA5.4	9041	-25.2	-35.3	-17.4
139	1620 − 391	DA2.1	25985	-1.3	2.8	-3.2
140	1625 + 093	DA7.3	7038	-23.9	-46.7	-12.3
141	1626 + 368	DZA6.0	8507	35.1	15.5	35.6
142	1632 + 177	DAZ5.0	10225	-2.7	2.3	-5.3
143	1633 + 433	DAZ7.7	6608	-12.7	-4.2	-13.9
144	1633 + 572	DQ8.2	5958	45.4	-3	54.5
145	1639 + 537	DAH6.7	7510	-3.6	-19.1	8.9
146	1647 + 591	DAV4.1	12738	-6	-3	-5.4

TABLE 1 (CONTINUED)
DATA OF 209 WD STARS (25 PC) STUDIED BY SION ET AL. (2014)

No.	WD No.	Sp. Type	$T_{\rm eff}$	U	V	W
42	0426+588	DC7.1	7178	0.9	-35.4	-3.7
43	0431-279	DC9.5	5330	11.5	-28.8	33.3
44	0431-360	DA10.0	5153	10.8	-19.6	27
45	0433+270	DA9.0	5629	-0.1	-19.8	7
46	0435-088	DQ8.0	6367	-25.1	-63.2	-21.6
47	0457-004	DA4.7	10800	-6.1	-23.8	2.5
48	0503-174	DAH9.5	5300	32.2	42.5	41
49	0511+079	DA7.7	6590	-14.5	-9.2	-30.6
50	0532+414	DA6.8	7739	1.2	12.4	-13.4
51	0548-001	DQP8.3	6070	4.6	6.3	10.5
52	0552-041	DZ10.0	5182	-25.3	-65.7	-20.2
53	0553+053	DAP8.7	5785	-12.2	-19.8	-31.2
54	0615-591	DB3.2	16714	-11.3	-0.1	7.1
55	0618+067	DA8.1	5940	-9	-26.9	49.9
56	0620-402	DZ6	5919	-13.9	-25	-7.2
57	0628-020	DA	6912	61	-34.5	-27.2
58	0628-020	DA	6912	-5.2	-7.7	-17.7
59	0642-166	DA2	25967	-0.6	-10.2	-14.5
60	0644+025	DA6.8	22288	9.4	15.1	-30.9
61	0644+375	DA2.4	22288	-19.7	-39.6	-32.7
62	0651-398A	DA7.0	7222	5.2	28.7	8.9
63	0655-390	DA7.9	6311	2.7	2.3	-28.2
64	0657+320	DA10.1	4888	-31.7	-52.9	13.2
65	0659-063	DA7.7	6627	-19.7	-37.5	-29.5
66	0706+377	DQ7.6	6590	-7.2	-13	-31.6
67	0708-670	DC9.9	5097	4.7	4.9	-19.9
68	0727+482.1	DA10.0	4934	-14.6	-40.5	-14.7
69	0727+482.2	DA10.1	4926	-14.9	-41.1	-14.9
70	0728+642	DAP11.1	5135	-4.5	-9.2	3.9
71	0736+053	DQZ6.5	7871	0.6	-12	-18.4
72	0738-172	DZA6.6	7650	-30.5	-29.2	30.2
73	0743-336	DC10.6	4462	49.4	67.6	61.4
74	0747+073.1	DC10.4	4366	-76.7	-126.3	-49
75	0747+073.2	DC12.0	4782	-76.7	-126.3	-49
76	0749+426	DC11.7	4585	-17.8	-29.6	5.6
77	0751-252	DA9.8	5085	19.1	15.5	-13.8
78	0752-676	DA8.8	5735	-29.3	-15.4	13.6
79	0753+417	DA7.3	6880	-13.8	-28.6	-3.5
80	0805+356	DA7.3	6900	0.5	-4.9	-6.6
81	0806-661	DQ4.9	10205	-17.2	-6.5	7.3
82	0810+489	DC6.9	7300	-14.5	-22.8	8.6

No.	WD No.	Sp. Type	$T_{\rm eff}$	U	V	W
147	1655+215	DAB5.4	9179	-38	-39	-17.7
148	1655+215	DAB5.4	9179	-25.9	-35.9	-18.4
149	1658+440	DAP1.7	30510	-6.6	33	28.2
150	1705+030	DZ7.7	6584	-16.6	-23.1	-13.4
151	1748+708	DQ9.0	5570	5.8	-10.8	34.2
152	1756+143	DA9.0	5466	-36.3	-86.3	49.1
153	1756+827	DA6.9	7214	8.8	-30.3	106.4
154	1814+134	DA9.5	5251	-47.7	-72	-8.6
155	1817-598	DA5.8	4960	-15.1	-22.9	2.3
156	1820+609	DA10.5	4919	-9.8	-9	-19.7
157	1829+547	DQP8.0	6345	2.8	-0.7	24.3
158	1840+042	DA5.8	9090	32	-23.6	19.9
159	1900+705	DAP4.2	11835	6.7	5.5	6
160	1917+386	DC7.9	6459	-5.2	-5.7	-8.6
161	1917-077	DBQZ4.9	10396	-5.4	-6.4	-0.5
162	1919+145	DA3.3	15280	-4.2	-5	-0.7
163	1935+276	DA4.2	12130	16	10.2	-31.7
164	1953-011	DC6.4	7920	-32.3	-31.9	3.7
165	2002-110	DA10.5	4800	40.4	9.8	-76.6
166	2007-303	DA3.5	14454	-22	-18.3	18
167	2008-600	DC9.9	5080	-17.7	-62.3	-3.3
168	2008-799	DA8.5	5800	18	-6.5	-20.7
169	2011+065	DQ7	6400	-64.2	-24.8	-17
170	2032+248	DA2.4	19983	-36.9	-24.9	-2.8
171	2039-202	DA2.5	19207	15.9	-3.9	-30.9
172	2039-682	DA3.1	15855	0.7	-17.8	-9.9
173	2040-392	DA4.5	10830	-13.6	-25.2	-0.3
174	2047+372	DA3.6	14070	13.4	5.5	-1.4
175	2048+263	DA9.7	5200	-79.5	104.1	-13.4
176	2048+263	DA9.7	5200	-37.6	-16.2	11.8
177	2048-250	DA6.6	7630	5.3	-13.6	-20
178	2054-050	DC10.9	4620	31.1	-11.4	-54.1
179	2105-820	DA4.7	10620	12.5	-17.4	3.6
180	2115-560	DA6	9736	17.5	-18.6	-31.4
181	2117+539	DA3.6	13990	-0.8	2.1	18.3
182	2118-388	DC9.6	5244	9.3	-6.3	-11.6
183	2126+734	DA3.8	16104	3.2	11.8	-23.5
184	2133-135	DA5.0	9736	11.7	-12.8	-20.3
185	2138-332	DZ7	7240	-15	-8.3	8.4
186	2140+207	DQ6.1	8200	-28	-18.9	-17.1
187	2149+021	DA2.8	17353	-28.4	-3.2	-37.7

TABLE 1 (CONTINUED)
DATA OF 209 WD STARS (25 PC) STUDIED BY SION ET AL. (2014)

No.	WD No.	Sp. Type	$T_{\rm eff}$	U	V	W
83	0810+489	DC6.9	7300	-9	-15.1	5
84	0816-310	DZ7.6	6463	-45	-52.7	-37
85	0821-669	DA9.8	5088	16.8	4.5	4.4
86	0827+328	DA6.9	7490	4.1	-45.4	-6.6
87	0839-327	DA5.5	9081	38.2	26.6	4.6
88	0840-136	DZ10.3	4874	14	0	-21
89	0843+358	DZ6	9041	7.6	-5.5	-13.6
90	0856+331	DQ5.1	9920	21.4	0.9	-24.3
91	0912+536	DCP7	7235	21	-42.5	-24.3
92	0946+534	DQ6.2	8100	20.2	-7.3	-16.1
93	0955+247	DA5.8	8621	19.1	-40.3	-11.8
94	0955+247	DA5.8	8621	6.1	-28.1	-17.7
95	0959+149	DC7	7200	-39.4	56.1	127.2
96	1009-184	DZ8.5	6036	35.1	-6	-26.8
97	1009-184	DZ8.5	6036	36.1	-8.8	-26.4
98	1012+083.1	DA7.5	6750	38.4	10.1	-12.2
99	1019+637	DA7.2	6742	-14.7	17.9	3.5
100	1033+714	DC10.3	4727	138.8	-73.2	-58
101	1036-204	DQP10.2	4694	36	17.9	20.7
102	1043-188	DQ8.1	5780	110.7	-71.1	107.8
103	1055-072	DA6.8	7491	42.3	-10.4	-16.3
104	1105-048	DA3.5	15141	-17.3	-37.8	-29.8
105	1116-470	DC8.6	5801	24.3	-9.2	-7

No.	WD No.	Sp. Type	$T_{\rm eff}$	U	V	W
188	2151-015	DA6	8400	-59	52.4	-85.8
189	2154-512	DQ8.3	6100	-12.4	-22.4	9.9
190	2159-754	DA5.6	9040	-27.6	7.6	18.5
191	2211-392	DA8.1	6920	46.6	-43.1	-61.1
192	2211-392	DA8.1	6920	57.8	-43.7	-44.5
193	2211-392	DA8.1	6920	46.6	-43.1	-61.1
194	2211-392	DA8.1	6920	57.8	-43.7	-44.5
195	2215+386	DC10.6	4700	48.7	-7.1	-20.6
196	2226-754	DC11.9	4230	-0.5	-88.5	71.5
197	2226-755	DC12.1	4177	-0.5	-88.5	71.5
198	2246+223	DA4.7	10647	42.4	-10.4	-16.9
199	2248+293	DA9	5580	111.2	-29.8	-43
200	2251-070	DZ12.6	4000	68	-48.2	-50
201	2253+054	DA9	5600	20.3	-33.8	-34.2
202	2311-068	DQ6.8	7440	-45.3	-1.7	7.4
203	2322+137	DA10.7	4700	3.2	-0.5	-0.3
204	2326+049	DAV4.3	12206	-31.6	-2.1	-1.3
205	2336-079	DA4.6	10938	-5.8	-13.4	-5.9
206	2341+322	DA4.0	13128	-18.8	5.6	-0.4
207	2347+292	DA9	5810	-18.3	2.7	-57.2
208	2351-335	DA5.7	8850	-62	-22.5	-38.2
209	2359-434	DA5.9	8648	13.9	-16.4	-27.6

The necessary conditions for an extremum are now

$$(B - \lambda I)\,\underline{x} = 0. \tag{6}$$

These are three homogenous equations in three unknowns, which have a nontrivial solution if and only if

$$D(\lambda) = |B - \lambda I| = 0, \tag{7}$$

where λ is the eigenvalue, and $\underline{x}$ and B are given as:

$$\underline{x} = \begin{bmatrix} l \\ m \\ n \end{bmatrix} \qquad \text{and} \qquad B = \begin{vmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{12} & \mu_{22} & \mu_{23} \\ \mu_{13} & \mu_{23} & \mu_{33} \end{vmatrix}.$$

Equation (7) is characteristic equation for the matrix B. The required roots (i.e. eigenvalues) are

$$\begin{aligned} \lambda_1 &= 2\rho^{\frac{1}{3}} \cos\tfrac{\phi}{3} - \tfrac{k_1}{3}; \\ \lambda_2 &= -\rho^{\frac{1}{3}} \left\{\cos\tfrac{\phi}{3} + \sqrt{3}\sin\tfrac{\phi}{3}\right\} - \tfrac{k_1}{3}; \\ \lambda_3 &= -\rho^{\frac{1}{3}} \left\{\cos\tfrac{\phi}{3} - \sqrt{3}\sin\tfrac{\phi}{3}\right\} - \tfrac{k_1}{3}, \end{aligned} \tag{8}$$

where

$$\begin{aligned} k_1 &= -\left(\mu_{11}+\mu_{22}+\mu_{33}\right), \\ k_2 &= \mu_{11}\mu_{22}+\mu_{11}\mu_{33}+\mu_{22}\mu_{33}-\left(\mu_{12}^2+\mu_{13}^2+\mu_{23}^2\right), \\ k_3 &= \mu_{12}^2\mu_{33}+\mu_{13}^2\mu_{22}+\mu_{23}^2\mu_{11}-\mu_{11}\mu_{22}\mu_{33}-2\mu_{12}\mu_{13}\mu_{23}. \end{aligned} \tag{9}$$

$$\begin{aligned} q &= \tfrac{1}{3}k_2-\tfrac{1}{9}k_1^2; \\ r &= \tfrac{1}{6}\left(k_1k_2-3k_3\right)-\tfrac{1}{27}k_1^3, \end{aligned} \tag{10}$$

$$\rho = \sqrt{-q^3}, \tag{11}$$

$$x = \rho^2 - r^2, \tag{12}$$

and

$$\phi = \tan^{-1}\left(\frac{\sqrt{x}}{r}\right). \tag{13}$$

Depending on the matrix that controls the eigenvalue problem [equation (6)] for the velocity ellipsoid, we establish analytical expressions of some parameters for the correlations studies in terms of the matrix elements μ_{ij} of the eigenvalue problem for the velocity ellipsoid (the velocity ellipsoid parameters, VEPs).

- The $\sigma_i;\ i=1,2,3$ parameters

The $\sigma_i;\ i=1,2,3$ parameters are defined as

$$\sigma_i = \sqrt{\lambda_i}\ . \tag{14}$$

- The l_i, $\mathrm{m_i}$ and n_i parameters

The l_i, m_i and n_i are the direction cosines for the eigenvalue problem. Then we have the following expressions for l_i, m_i and n_i:

$$l_i = \left[\mu_{22}\mu_{33}-\sigma_i^2\left(\mu_{22}+\mu_{33}-\sigma_i^2\right)-\mu_{23}^2\right]/D_i\ ; \qquad i=1,2,3, \tag{15}$$

$$m_i = \left[\mu_{23}\mu_{13}-\mu_{12}\mu_{33}+\sigma_i^2\mu_{12}\right]/D_i\ ; \qquad i=1,2,3, \tag{16}$$

$$n_i = \left[\mu_{12}\mu_{23}-\mu_{13}\mu_{22}+\sigma_i^2\mu_{13}\right]/D_i\ ; \qquad i=1,2,3, \tag{17}$$

where

$$\begin{aligned} D_i^2 = {} & \left(\mu_{22}\mu_{33}-\mu_{23}^2\right)^2+\left(\mu_{23}\mu_{13}-\mu_{12}\mu_{33}\right)^2+\left(\mu_{12}\mu_{23}-\mu_{13}\mu_{22}\right)^2 \\ & +2\left[\left(\mu_{22}+\mu_{33}\right)\left(\mu_{23}^2-\mu_{22}\mu_{33}\right)+\mu_{12}\left(\mu_{23}\mu_{13}-\mu_{12}\mu_{33}\right)+\mu_{13}\left(\mu_{12}\mu_{23}-\mu_{13}\mu_{22}\right)\right]\sigma_i^2 \\ & +\left(\mu_{33}^2+4\mu_{22}\mu_{33}+\mu_{22}^2-2\mu_{23}^2+\mu_{12}^2+\mu_{13}^2\right)\sigma_i^4-2\left(\mu_{22}+\mu_{33}\right)\sigma_i^6+\sigma_i^8. \end{aligned}$$

3. RESULTS

Based on the model described in the previous section, a Mathematica routine has been developed to compute the kinematics and velocity ellipsoid parameters. Figures 1-4 show the distribution of the space velocities of 209 white dwarfs (25 pc sample). The routine was run for all data and for the following subsamples:

- 126 WD (20 pc list).
- 209 WD (25 pc list).
- DA white dwarfs, with 141 candidates (from the 25 pc list).
- Non- DA white dwarfs, with 68 candidates (from the 25 pc list).
- Hot white dwarfs ($T_{\rm eff} \geq 12000\ K^{\circ}$) with 32 candidates (from the 25 pc list).
- Cool white dwarfs ($T_{\rm eff} < 12000\ K^{\circ}$) with 177 candidates (from the 25 pc list).

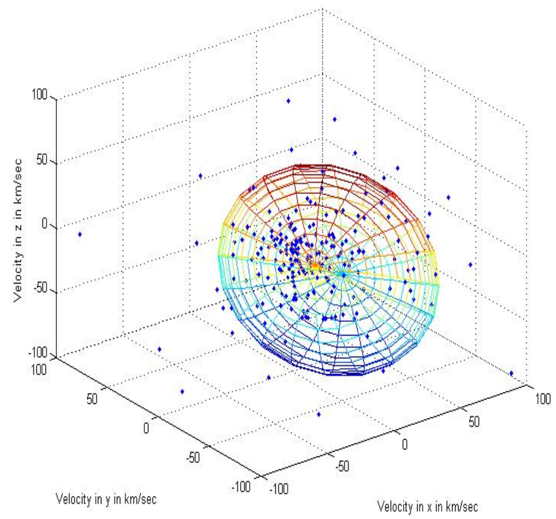

Fig. 1. Space velocity distribution of 209 WD (25 pc sample). The color figure can be viewed online.

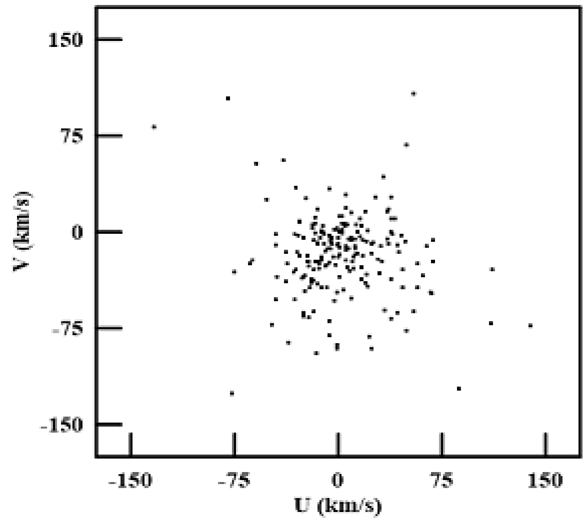

Fig. 2. $U-V$ velocity distribution of 209 WD (25 pc sample).

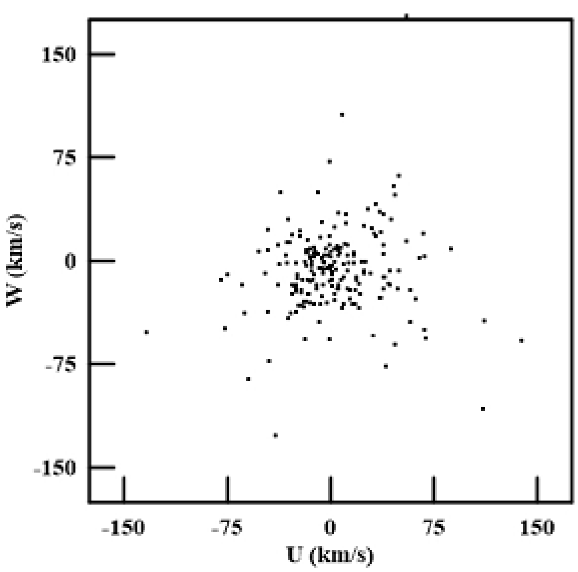

Fig. 3. $U-W$ velocity distribution of 209 WD (25 pc sample).

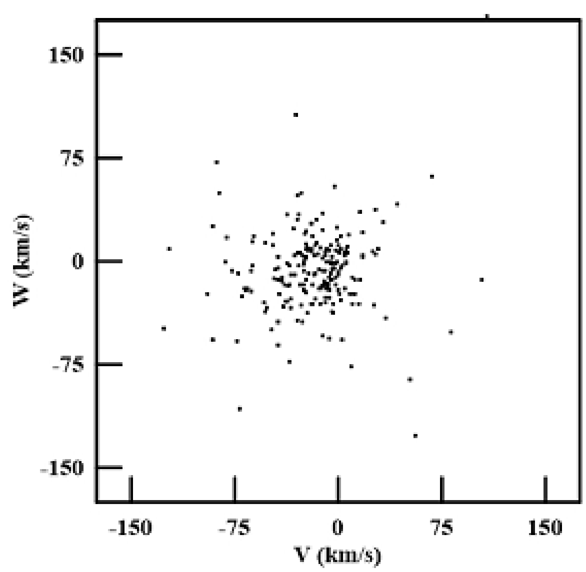

Fig. 4. $V-W$ velocity distribution of 209 WD (25 pc sample).

The results are listed in Tables 2 to 5. Row 1 shows the mean space velocities, Row 2 the dispersion in velocities, Row 3 the eigenvalues, Rows 4, 5 and 6 the l, m and n parameters, respectively.

In Table 6 we compare our results with results from different authors. We also show our results for different sub-samples. We tabulate σ_1, σ_2, σ_3, (σ_2/σ_1) and the solar velocity ($S_\odot$) obtained from our calculations. We also list results by different authors.

First we focused on the self-comparison between the two sets, the 20 pc (Table 2) and 25 pc (Table 3) samples. The velocity dispersions (σ_1, σ_2, σ_3) are comparable for the two samples, while the solar velocity is quite different. The 25 pc list has 167 % more stars than the 20 pc list, and the solar velocity is reduced by 22%.

Another comparison possible with the present results is that between two white dwarf subsamples of different spectral types, namely DA (141 candidates) and non-DA (68 candidates), listed in Table 4. Here the differences

TABLE 2

VEPS FOR ALL 126 WD (20 PC)*

VEPs			
$(\overline{U}, \overline{V}, \overline{W})$	1.1373	-21.9254	-5.62143
$(\sigma_1, \sigma_2, \sigma_3)$	40.9788	27.1258	34.8816
$(\lambda_1, \lambda_2, \lambda_3)$	1679.26	735.808	1216.73
$(l_1, l_2, l_3)_{\text{deg}}$	0.319174	0.12001	-0.940067
(m_1, m_2, m_3)	-0.916102	0.293066	-0.273624
(n_1, n_2, n_3)	-0.242664	-0.94853	-0.203481

*Sion et al. (2009).

TABLE 3

VEPS FOR ALL 209 WD (25 PC)*

VEPs			
$(\overline{U}, \overline{V}, \overline{W})$	2.48182	-18.6593	-6.94737
$(\sigma_1, \sigma_2, \sigma_3)$	40.3025	29.6298	34.5985
$(\lambda_1, \lambda_2, \lambda_3)$	1624.29	877.924	1197.06
$(l_1, l_2, l_3)_{\text{deg}}$	0.281297	0.381985	-0.880318
(m_1, m_2, m_3)	-0.892172	0.441952	-0.093314
(n_1, n_2, n_3)	-0.353414	-0.811644	-0.465116

*Sion et al. (2014).

TABLE 4

VEPS FOR DA WD AND NON-DA WD*

VEPs	DA (141)			non-DA (68)		
$(\overline{U}, \overline{V}, \overline{W})_{\text{km/sec}}$	4.15816	-17.2078	-6.28582	-0.994118	-21.6691	-8.31912
$(\sigma_1, \sigma_2, \sigma_3)_{\text{km/sec}}$	38.5354	26.6359	31.9558	44.0556	33.8479	40.1042
$(\lambda_1, \lambda_2, \lambda_3)_{\text{km/sec}}$	1484.98	709.473	1021.17	1940.89	1145.68	1608.35
$(l_1, l_2, l_3)_{\text{deg}}$	0.329185	0.524158	-0.785427	0.0805911	0.178776	-0.980584
$(m_1, m_2, m_3)_{\text{deg}}$	-0.85574	0.51723	-0.0134785	-0.959336	0.280912	-0.0276303
$(n_1, n_2, n_3)_{\text{deg}}$	-0.399182	-0.676559	-0.618807	-0.270518	-0.942936	-0.194145

*Sion et al. (2014).

between the two results are significant for both the velocity ellipsoid and the solar velocity. The differences for the ratio of the velocity dispersion (σ_2/σ_1) reflect the differences in the initial formation conditions for DA (with rich hydrogen atmospheres and metal cores) and non-DA white dwarfs (atmospheres with different chemical compositions).

Finally, we compared results for hot white dwarfs (32 candidates) and cool white dwarfs (177 candidates), listed in Table 5. Here again, the results for both the velocity ellipsoid parameters and the solar velocity are very discrepant. Perhaps this is due to the influence of the number of stars on the results; a conclusion about the variation of the velocity dispersions with the effective temperatures cannot be drawn.

Now we turn to the comparisons between our results and those of Wehlau (1957) for dwarfs within 25 pc of the Sun. As we see from Table 6 both velocity dispersions and solar velocity are spread over a large range.

TABLE 5

VEPS FOR HOT WD AND COLD WD*

VEPs	Hot WD (32)			Cool WD (177)		
$(\overline{U}, \overline{V}, \overline{W})_{\mathrm{km/sec}}$	-0.45312	-7.16562	-8.7125	3.01243	-20.7373	-6.62825
$(\sigma_1, \sigma_2, \sigma_3)_{\mathrm{km/sec}}$	29.1026	17.5027	23.3191	42.4863	30.9974	35.9743
$(\lambda_1, \lambda_2, \lambda_3)_{\mathrm{km/sec}}$	846.96	306.343	543.781	1805.09	960.836	1294.15
$(l_1, l_2, l_3)_{\mathrm{deg}}$	0.199976	0.0888788	-0.975761	0.310712	0.368454	-0.876185
$(m_1, m_2, m_3)_{\mathrm{deg}}$	0.532547	0.82607	0.184386	-0.902551	0.403464	-0.150397
$(n_1, n_2, n_3)_{\mathrm{deg}}$	0.822437	-0.556512	0.117862	-0.298094	-0.837531	-0.457909

*Sion et al. (2014).

TABLE 6

VELOCITY DISPERSIONS FOR DIFFERENT SPECTRAL TYPES

Spectral Types	σ_1	σ_2	σ_3	$S_\odot$	(σ_2/σ_1)	Reference
126 WD (2009)	40.97	27.12	34.88	22.66	0.66	This work
209 WD (2014)	40.30	29.63	34.60	20.06	0.74	This work
32 hot WD (2014)	29.10	17.50	23.31	11.28	0.60	This work
177 cold WD (2014)	42.48	30.99	35.97	21.97	0.72	This work
141 DA WD (2014)	38.53	26.63	31.95	18.78	0.69	This work
68 non-DA WD (2014)	44.05	33.84	40.10	23.23	0.76	This work
A0-F3	20.3	9.4	9.2	13.7	0.46	Wehlau (1957)
F4-F8	26.5	17.3	17	17.1	0.65	Wehlau (1957)
F9-G1	25.8	18.4	20	26.4	0.71	Wehlau (1957)
G2-G7	32.4	16.6	14.7	23.9	0.51	Wehlau (1957)
G8-K2	28.2	15.6	11	19.8	0.55	Wehlau (1957)
K3-K6	34.6	19.7	15.9	25	0.56	Wehlau (1957)
K8-M2	32.1	21	18.8	17.3	0.65	Wehlau (1957)
M3-M6	31.2	23.1	16.2	23.3	0.74	Wehlau (1957)

TABLE 7

OORT'S CONSTANTS

$A\left(\mathrm{km\,s^{-1}\ kpc^{-1}}\right)$	$B\left(\mathrm{km\,s^{-1}\ kpc^{-1}}\right)$	$(\sigma_2/\sigma_1)^2$
14.5	-12	0.65
12.6	-13.2	0.71
14.8	-12.4	0.67
11.3	-13.9	0.74

This could be interpreted as due to the different method of calculations and the number of stars in samples studied.

Important quantities in stellar kinematics are the Oort constants. The relation between these constants and the ratio (σ_2/σ_1) is given by $(\sigma_2/\sigma_1)^2 = -B/\left(A - B\right)$. In Table 7 we list the values of the constants A and B according to Olling and Merrifield (1998). Column 1 shows Oort constant A, Column 2 Oort constant B and Column 3 the ratio (σ_2/σ_1) calculated with A and B. As we see from the table, the ratio (σ_2/σ_1) has values in the range 0.65-0.74, in good agreement with our calculations.

4. SUMMARY AND CONCLUSION

In the present paper, the velocity dispersions and the solar velocity are calculated, using the white dwarfs within 20 pc and 25 pc. We have also performed calculations for four subsamples; DA, non-DA, hot and cool white dwarfs. The conclusions reached are the following:

- Increasing the number of white dwarfs by a factor $\simeq 2$, results in a decrease of the derived parameters by about 22%.
- The dependence of the derived values on the spectral type of the white dwarfs (DA and non-DA) is clear and reflects the dependence on the chemical composition and, consequently, on the age of the star.
- We could not determine the effect of the effective temperature on the velocity dispersions and on the solar velocity, because of the large difference in the number of the two subsamples (hot and cool white dwarfs).
- The comparison with published parameters for dwarfs within 25 pc of the Sun shows great discrepancies, which could be attributed to the type of stars used as well as to the method of calculations.

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