

## A METHOD FOR LOCAL RECTIFICATION OF 2MASS POSITIONS WITH UCAC4

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### RESUMEN

Se propone rectificar localmente 2MASS con respecto a UCAC4 con el fin de disminuir las diferencias sistemáticas entre estos catálogos. Se desarrolla un método de rectificación que parte del cálculo de diferencias medias ponderadas 2MASS–UCAC4 en una cuadrícula regular en el cielo. Las correcciones que posteriormente se aplican a las posiciones 2MASS se obtienen mediante una interpolación *spline* de los valores medios calculados en la malla. El método se probó en cuatro campos de  $3^\circ \times 3^\circ$  en la zona ecliptical. Después de la rectificación en todos ellos las diferencias sistemáticas se reducen muy por debajo de las diferencias aleatorias. El catálogo 2MASS rectificado con el método propuesto puede considerarse como una extensión de UCAC4 para su uso en astrometría con una precisión de alrededor de 90 milisegundos de arco, y con errores sistemáticos despreciables, por ejemplo para la reducción astrométrica de imágenes CCD de pequeño campo.

### ABSTRACT

We propose to locally rectify 2MASS with respect to UCAC4 in order to diminish the systematic differences between these catalogs. We develop a rectification method that starts computing the weighted mean differences 2MASS–UCAC4 on a regular grid on the sky. The corrections that are later applied to 2MASS positions are obtained by a spline interpolation of the mean values calculated on the grid. The method is tested in four  $3^\circ \times 3^\circ$  fields in the ecliptical zone; after rectification in all of them the systematic differences are reduced well below the random differences. The 2MASS catalog rectified with the proposed method can be regarded as an extension of UCAC4 for astrometry, with an accuracy of around 90 mas in the positions, and with negligible systematic errors, for instance for the astrometric reduction of small field CCD images.

*Key Words:* astrometry — catalogs — methods: data analysis — methods: miscellaneous

### 1. INTRODUCTION

We are currently working to obtain astrometric positions on deep CCD images each covering a field  $6' \times 14'$ . This is a small field; therefore, an optimal astrometric reduction is difficult to obtain, either because of the small number of reference stars or because their spatial distribution on the image is non-uniform. This situation is common to most CCDs of similar or larger size. In deep CCD images—such as in our case—“bright” stars are saturated and they have to be discarded as reference stars.

All-sky reference catalogs dense and precise enough to enable the astrometric reduction of deep CCD images do not exist. The only available all-sky dense astrometric catalog is UCAC4; its average density is over 2000 stars per square degree (Zacharias et al. 2013) with large variations among different regions of the sky. Its limiting magnitude is  $\approx 16$ . Since in our images the stars with the highest positional accuracy are saturated

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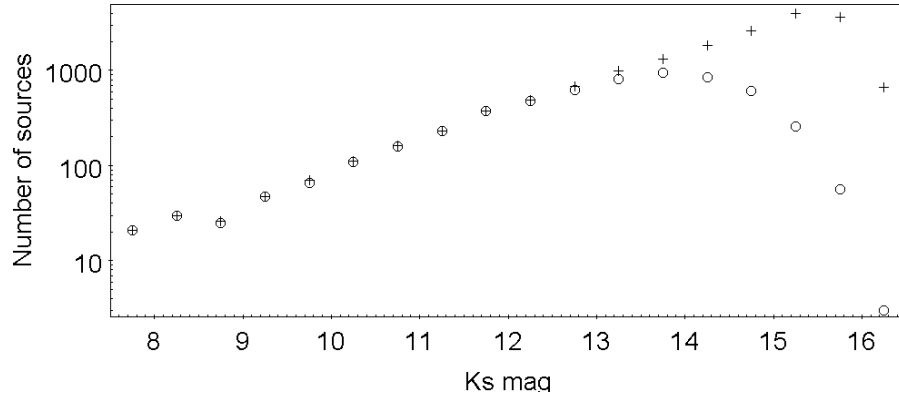


Fig. 1. Number of sources within a typical  $3^\circ \times 3^\circ$  field far from the galactic plane as a function of  $K_s$  magnitude. Circles: UCAC4. Crosses: 2MASS.

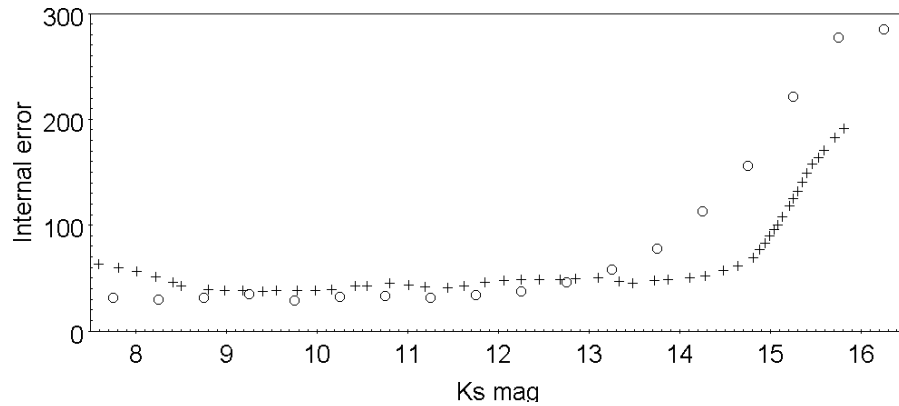


Fig. 2. Circles: median of UCAC4 internal errors within a typical  $3^\circ \times 3^\circ$  field far from the galactic plane as a function of  $K_s$  magnitude. Crosses: internal estimation of positional uncertainty in 2MASS from Skrutskie et al. (2006).

they cannot be used as reference stars. On the other hand, non-saturated stars have magnitudes at the faint end of the catalog and their positional accuracy is lower. Another available catalog is the 2MASS All-Sky Catalog of Point Sources. The more precise positions of this catalog have a  $K_s$  magnitude range between 13.5 and 15.5, with a positional accuracy of 70–90 mas (Skrutskie et al. 2006).

Figure 1 shows that using 2MASS as a reference catalog would overcome the lack of reference positions for the astrometric reduction of small fields in regions with poor coverage in UCAC4. For instance, in the interval  $14.5 < K_s < 15.0$  the number of sources in 2MASS is four times larger than in UCAC4, and in the interval  $15.0 < K_s < 15.5$  it is fifteen times larger. In addition Figure 2 suggests that the sources brighter than  $K_s = 13.5$  have better astrometric positions in UCAC4 than in 2MASS, while the opposite occurs for the fainter sources. Therefore 2MASS seems to be a good choice as a reference catalog in small deep fields, but it is not an astrometric catalog because of severe systematic errors in its positions.

The systematics can be seen in Figure 3 where we display position differences between 2MASS and other catalogs for different fields. The observed amplitudes of the systematic differences are hundreds of milli-arcseconds. These differences are well above the 2MASS dispersion given by the authors of this catalog.

When 2MASS is used as a reference catalog these systematic errors appear in the results of the astrometric reduction. In this work we propose to improve the quality of the 2MASS positions with respect to UCAC4 by means of a rectification method that removes the systematic differences between these catalogs. The proposed rectification of 2MASS would extend the range of accurate positions up to  $K_s = 15$  and with a lower accuracy

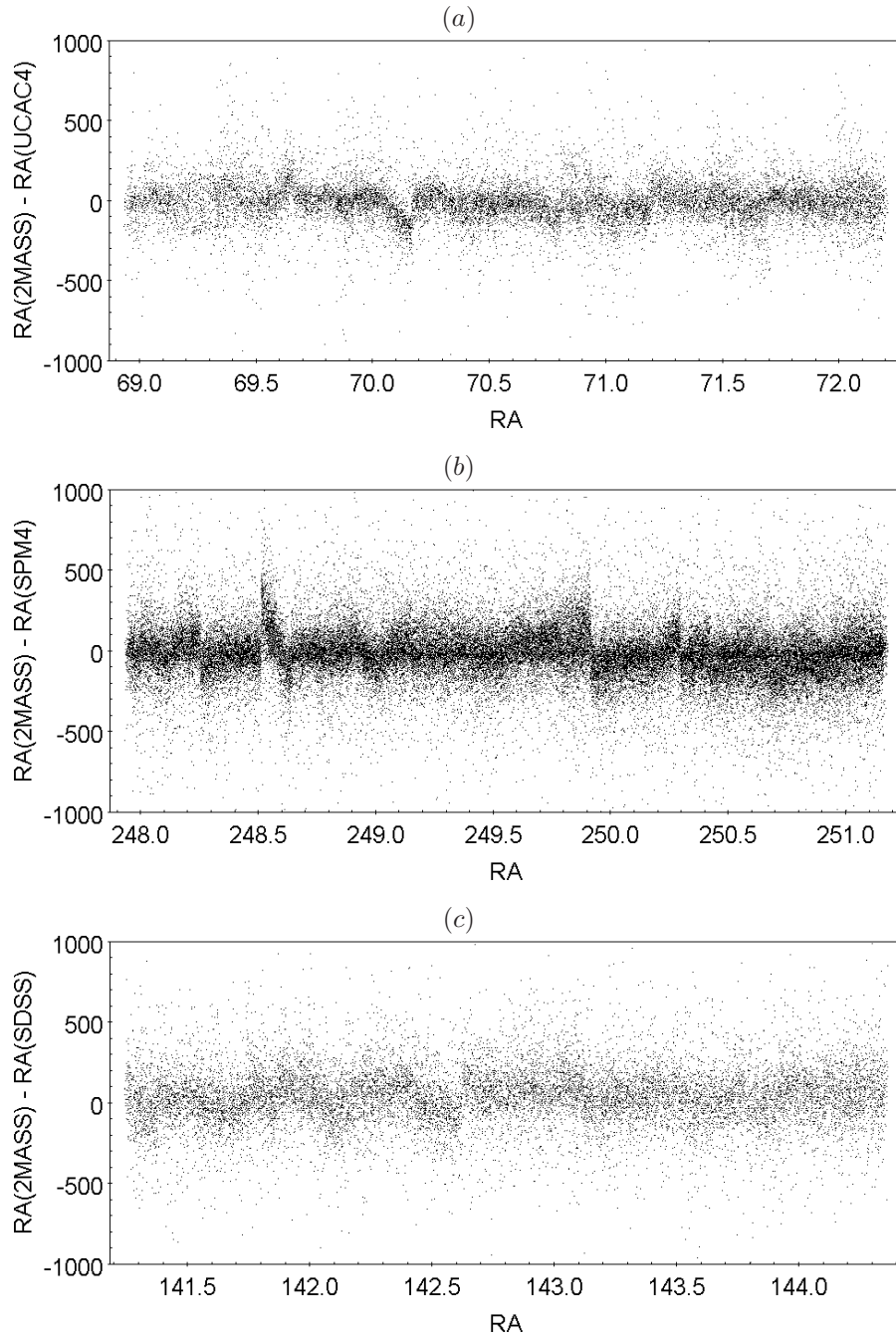


Fig. 3. Comparison of 2MASS positions with different catalogs over fields of  $3^\circ \times 3^\circ$ . (a) UCAC4, (b) SPM4, (c) SDSS-DR9.

up to  $K_s = 15.5$ . After this correction the astrometric reduction of small field CCD images could rely on 2MASS as a reference catalog.

Since we are interested in obtaining astrometric positions on the ecliptical zone from deep CCD images, we plan to apply this rectification method to 2MASS in a strip a few degrees wide along the Ecliptic. The rectified 2MASS will also be useful for studies of Solar System objects, mainly for precise astrometry of small bodies.

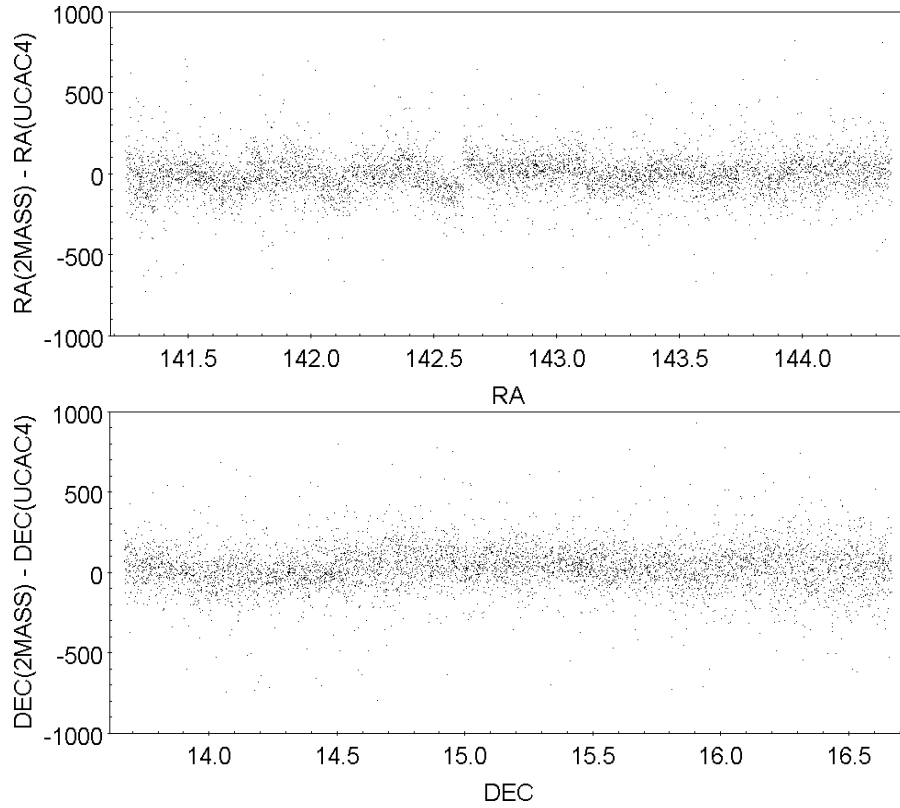


Fig. 4. Differences 2MASS–UCAC4 depending on right ascension and declination.

## 2. THE METHOD

Figures 3 and 4 show that the systematics in RA are very different from those in DEC: the RA systematics show strong discontinuities with abrupt changes in the value of the mean, a behaviour very different from that of the DEC systematics, which vary continuously in a smooth way, with a longer period of variation. Thus, the RA systematics have a higher spatial frequency than the DEC systematics.

We propose to locally rectify 2MASS with respect to UCAC4 in order to diminish the systematic differences 2MASS–UCAC4. Similar efforts were done with other catalogs by Stock & Abad (1988) for example. The different behavior in both coordinates is related to the 2MASS tiling strategy of observations in scans covering a sky region  $8.5'$  wide in RA and  $6^\circ$  long in DEC.

We define a working field of  $3^\circ \times 3^\circ$  on the sky. This size is selected in order to include several 2MASS tiles, because their systematic errors change from tile to tile. On that field we match the catalogs UCAC4 and 2MASS with a coincidence radius of  $1''$ ; a proper motion correction to the UCAC4 positions is not necessary because the epochs of these catalogs are quite close. We calculate the differences  $(\alpha_{2MASS} - \alpha_{UCAC4}, \delta_{2MASS} - \delta_{UCAC4})$  for the stars in common.

On the working field we define a square grid of step  $\rho$ , aligned with the coordinate axes RA and DEC. On each node of the grid  $(\alpha_g, \delta_g)$  we calculate the weighted mean difference  $(\alpha_{2MASS} - \alpha_{UCAC4}, \delta_{2MASS} - \delta_{UCAC4})$  inside a *smoothing area* centered on that node. As result of this procedure we obtain the mean systematic differences between 2MASS and UCAC4 evaluated on the nodes of a grid of size  $3^\circ \times 3^\circ$  that will be used for the rectification of 2MASS.

In order to decide the size of the grid step  $\rho$ , we performed the rectification of one field using values for  $\rho$  ranging from  $0.75'$  to  $2'$ . We finally adopted  $\rho = 1'$  because we found that with  $\rho < 1'$  there was no noticeable improvement in the rectification, and with  $\rho > 1'$  the differences with the highest spatial frequency were not properly corrected.

We also tested the effect on the rectification of the grid shift and we found that 90% of the differences between the corrections performed with the grid shifted in both coordinates are smaller than 7 mas, and less than 0.7% of them are larger than 30 mas. Therefore the effect of the grid shift is not significant, since it is well below the errors of UCAC4 positions.

We use as weight function a two-dimensional Gaussian

$$w(x, y) = w_x(x) \times w_y(y)$$

where

$$x = \alpha - \alpha_g \text{ and } y = \delta - \delta_g, \quad w_x(x) \text{ and } w_y(y),$$

are computed as

$$w_x(x) = (\exp(-x^2/2\sigma_x^2) - \exp(-K))/\sigma_x, \quad \text{if } |x| \leq 3\sigma_x;$$

$$w_x(x) = 0, \quad \text{if } |x| > 3\sigma_x;$$

$$w_y(y) = (\exp(-y^2/2\sigma_y^2) - \exp(-K))/\sigma_y, \quad \text{if } |y| \leq 3\sigma_y;$$

$$w_y(y) = 0, \quad \text{if } |y| > 3\sigma_y.$$

The constant  $K$  was selected in such a way that  $w_x(x) = 0$  for  $x = 3\sigma_x$  and  $w_y(y) = 0$  for  $y = 3\sigma_y$ , which gives  $K = 9/2$ . Here  $\sigma_x$  and  $\sigma_y$  are the widths in RA and DEC of the weight function. With this definition the size of the *smoothing area* was  $6\sigma_x \times 6\sigma_y$ . The numerical value of  $\sigma_x$  was always between  $0.5'$  and  $1'$ . Because of the grid step  $\rho = 1'$ , the lower limit ensured that all the stars were considered in the averages. On the other hand the upper limit allowed the proper correction of sharp discontinuities (the systematic differences of high spatial frequency). The numerical value of  $\sigma_y$  was computed by assuming that  $\sigma_x = 1'$  and requiring that the average number of stars within the smoothing area be always greater than, or equal to, 15. With this number of stars the random errors of the mean differences are expected to be less than 30 mas. Because of the size of the grid step, if the computed value of  $\sigma_y$  was smaller than  $0.5'$  then  $\sigma_y = 0.5'$  was adopted, with the same value for  $\sigma_x$ . If the resulting  $\sigma_y$  was smaller than  $\sigma_x$  then their values were swapped. Notice that with this choice of  $\sigma_x$  and  $\sigma_y$  the smoothing area was rectangular, with  $\sigma_x \leq \sigma_y$  always. We did not use a circular smoothing area because in low density regions the minimum number of stars would require a radius larger than  $1'$  and the sharp discontinuities would not be properly corrected.

In order to perform the rectification, for each position  $(\alpha_{2MASS}, \delta_{2MASS})$  we compute the correction  $(\Delta\alpha, \Delta\delta)$  by means of a two-dimensional cubic spline interpolation of the mean differences on the nodes of the grid. The corrected coordinates  $(\alpha_{2MASS-R}, \delta_{2MASS-R})$  are computed by

$$\alpha_{2MASS-R} = \alpha_{2MASS} - \Delta\alpha, \quad \text{and} \quad \delta_{2MASS-R} = \delta_{2MASS} - \Delta\delta.$$

Notice that although the size of the working field is  $3^\circ \times 3^\circ$ , the useful area is smaller because close to the borders the correction is erroneously estimated due to the asymmetry in the calculation of the mean differences and in the interpolation, since part of the smoothing area is outside the working field.

### 3. APPLYING THE METHOD

In order to evaluate the quality of the proposed procedure we applied the rectification method on four fields  $3^\circ \times 3^\circ$  in the ecliptical region (our region of interest). The coordinates of the center of Zone 1 were arbitrarily selected. The other three fields were selected approximately at ecliptic longitude  $+90^\circ$ ,  $-90^\circ$  and  $+180^\circ$  from Zone 1, and they were called Zone 1 + 90, Zone 1 - 90 and Zone 1 + 180 respectively. The centers of these fields and the number of sources on each one are listed in Table 1.

Figure 5 shows the differences between the 2MASS coordinates and UCAC4 before and after rectification in Zone 1 + 90. This is the field with the lowest density. Notice that the differences that depend on RA are more

TABLE 1  
ANALYZED FIELDS

Field	RA	DEC	2MASS sources	UCAC4 sources	UCAC4 mean density <sup>a</sup>
Zone 1	70.57200	22.58722	62934	15701	0.485
Zone 1 + 90	142.80417	15.17056	17456	5891	0.182
Zone 1 – 90	334.82500	–8.99275	20571	5932	0.183
Zone 1 + 180	249.55508	–21.22264	144092	25081	0.774

<sup>a</sup>Sources/arcmin<sup>2</sup>.

TABLE 2  
COMPARISONS WITH UCAC4<sup>a</sup>

Field	Before rectification		After rectification	
	$\Delta RA$	$\Delta DEC$	$\Delta RA$	$\Delta DEC$
Zone 1	$-10 \pm 145$	$2 \pm 138$	$0 \pm 100$	$0 \pm 96$
Zone 1 + 90	$-5 \pm 126$	$29 \pm 137$	$0 \pm 81$	$0 \pm 83$
Zone 1 – 90	$-6 \pm 148$	$39 \pm 145$	$0 \pm 94$	$0 \pm 89$
Zone 1 + 180	$-18 \pm 156$	$6 \pm 134$	$0 \pm 104$	$0 \pm 95$

<sup>a</sup>Mean differences and standard deviations in mas.

TABLE 3  
COMPARISONS WITH EXTERNAL CATALOGS<sup>a</sup>

Field	Before rectification		After rectification	
	$\Delta RA$	$\Delta DEC$	$\Delta RA$	$\Delta DEC$
Zone 1	$-18 \pm 204$	$42 \pm 190$	$-5 \pm 200$	$40 \pm 185$
Zone 1 + 90	$38 \pm 195$	$82 \pm 194$	$44 \pm 191$	$53 \pm 185$
Zone 1 – 90	$-24 \pm 193$	$58 \pm 184$	$-18 \pm 183$	$22 \pm 177$
Zone 1 + 180	$-17 \pm 181$	$-8 \pm 164$	$3 \pm 175$	$-15 \pm 166$

<sup>a</sup>SDSS-DR9 in Zone 1, Zone 1 + 90, Zone 1 – 90; SPM4 in Zone 1 + 180. Mean differences and standard deviations in mas.

easily recognizable than those that depend on DEC. Therefore in subsequent comparisons only the differences that depend on RA will be shown.

Since UCAC4 was the reference catalog for the rectification, we also evaluated it by comparison with an external catalog. We computed the differences between 2MASS and SDSS-DR9 (Ahn et al. 2012) in the same field, before and after rectification; the results are shown in Figure 6.

Figure 7 shows the differences between 2MASS coordinates and UCAC4 before and after the rectification in Zone 1+180. This is the field with the highest density. Figure 8 shows the comparison with another catalog, SPM4 (Girard et al. 2011) for the same field. Zone 1 and Zone 1 – 90 are not displayed because the results of the rectification method were very similar to those found for the other fields.

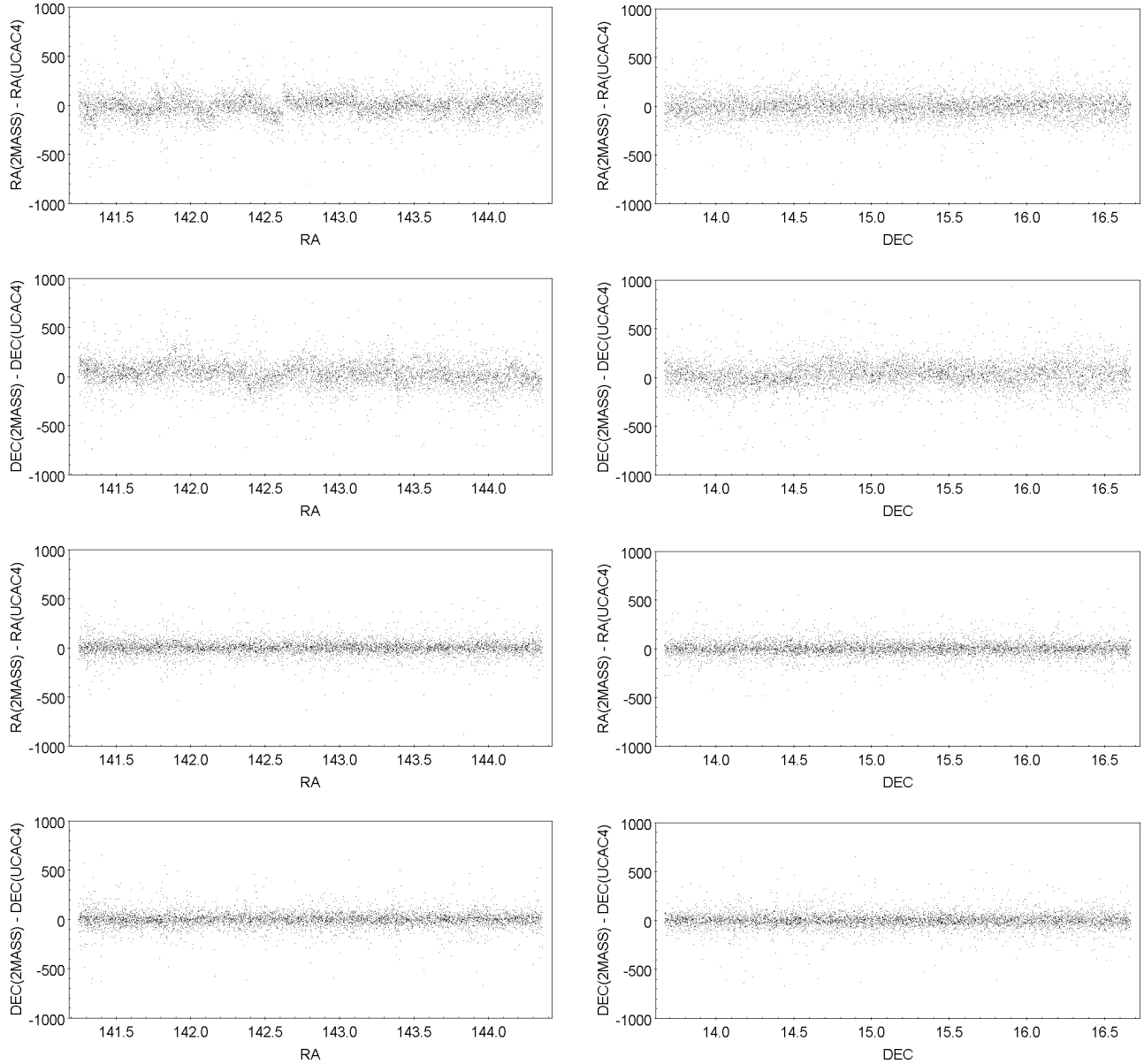


Fig. 5. Differences 2MASS–UCAC4 in RA and DEC, depending on both coordinates in Zone 1 + 90. Upper two panels: before rectification. Lower two panels: after rectification.

#### 4. EVALUATION OF THE METHOD

Tables 2 and 3 contain a brief statistical summary of the four fields analyzed in this work. The comparisons with UCAC4—regarded as an internal check—shown in Figures 5 and 7 demonstrate that the remarkable systematic differences between 2MASS and UCAC4 become negligible after the proposed rectification method is applied. As a consequence, it can be seen in Table 2 that the mean differences become null and the dispersions are significantly reduced, down to  $\lesssim 100$  mas, the typical positional uncertainty of 2MASS (Skrutskie et al. 2006).

The comparisons with external catalogs shown in Figures 6 and 8 demonstrate that prior to rectification the systematic differences are very similar to those found with UCAC4, although the dispersions are noticeably larger. After rectification the systematic differences are greatly reduced. However, slight systematic differences may still persist. In addition, the reduction of the dispersions and the variation in the mean differences shown in Table 3 are very slight. Since the dispersions are large, they are not significant.

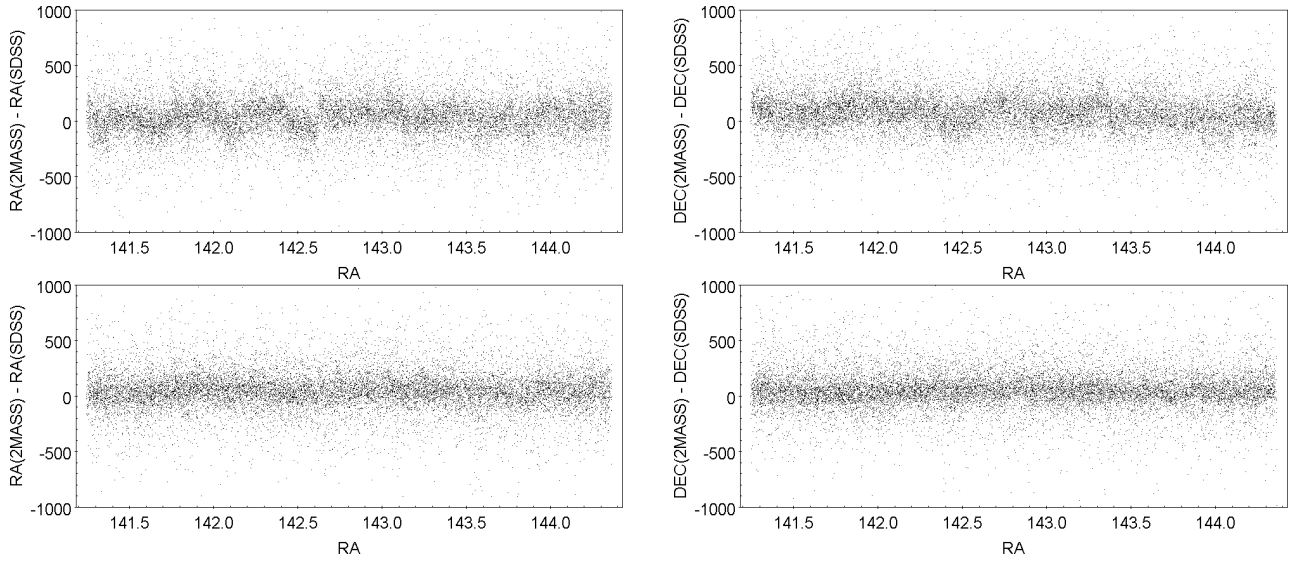


Fig. 6. Differences 2MASS–SDSS-DR9 in RA and DEC, depending on RA in Zone 1+90. Upper panel: before rectification. Lower panel: after rectification.

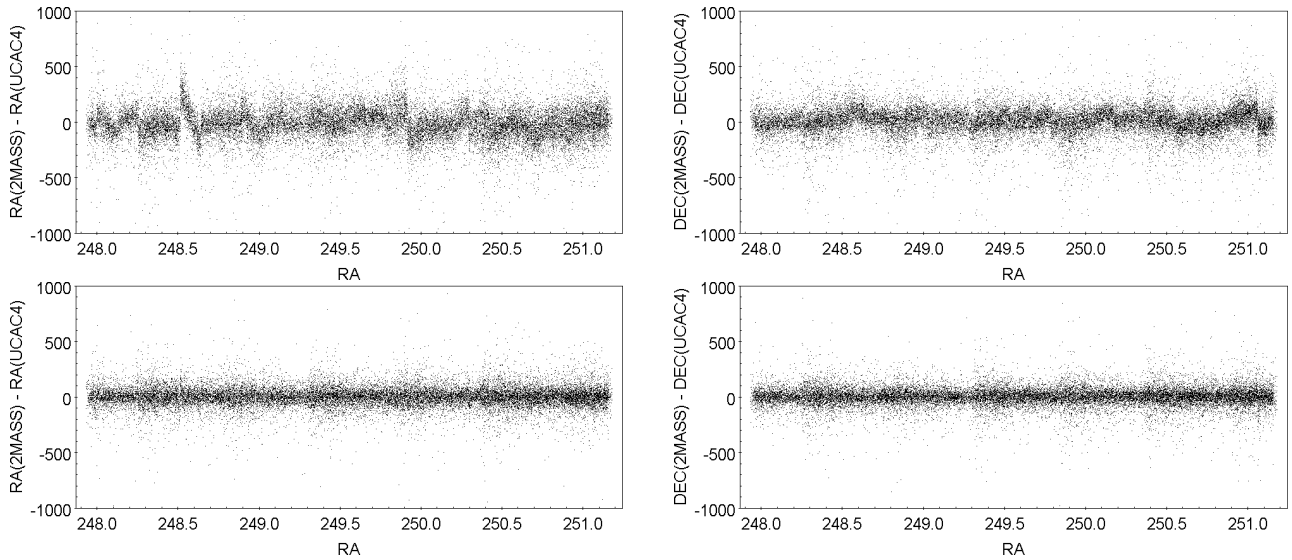


Fig. 7. Differences 2MASS–UCAC4 in RA and DEC, depending on RA in Zone 1+180. Upper panel: before rectification. Lower panel: after rectification.

Although in the above figures there are no visible systematic differences after rectification, some small ones could survive under the scattering of the data. In order to quantify these residual systematic differences, we calculated the grid of mean differences between the rectified 2MASS and UCAC4 (referred to as “second grid” hereafter), and we compared it with the grid used for rectification (“first grid”). Figure 9 shows the values of the first and the second grid along one grid line in the center of Zone 1+180. The statistics of the values for the first grid are:  $\Delta RA = (-18 \pm 104)$  mas,  $\Delta DEC = (3 \pm 83)$  mas, and for the second grid they are:  $\Delta RA = (0 \pm 28)$  mas,  $\Delta DEC = (0 \pm 25)$  mas. The dispersions are similar to the smallest errors in the UCAC4 positions, and they can be regarded as a measure of the error of the rectification method.



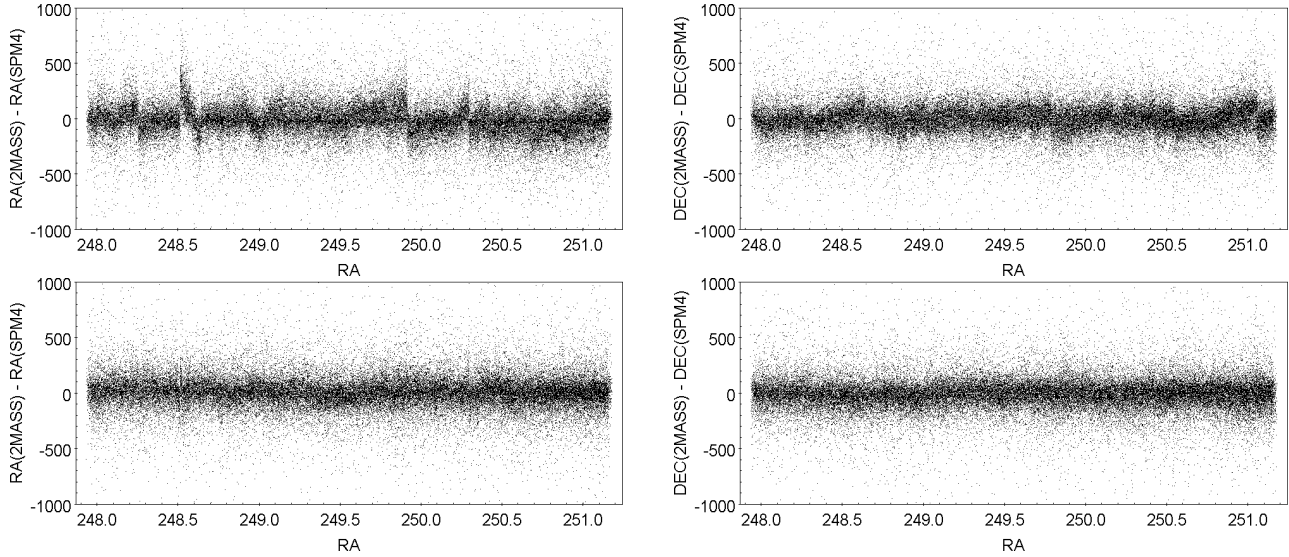


Fig. 8. Differences 2MASS–SPM4 in RA and DEC, depending on RA in Zone 1+180. Upper panel: before rectification. Lower panel: after rectification.

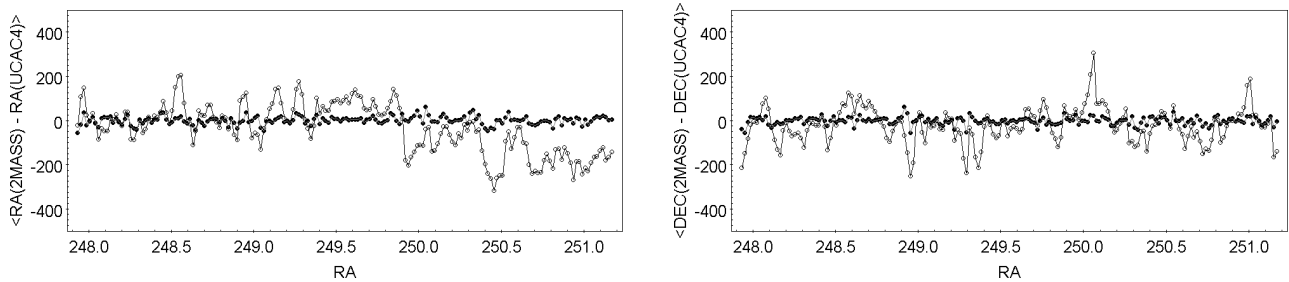


Fig. 9. Mean differences 2MASS–UCAC4 in RA and DEC along one grid line in Zone 1+180. Open circles: before rectification. Filled circles: after rectification.

Figure 10 displays the VPD of second grid values versus first grid values in both coordinates. The correlation between them provides information on the residual systematic differences after the rectification. As a first approximation they can be quantified by means of the slopes of the displayed distributions computed by linear least squares fits. The resulting slope in RA is  $0.101 \pm 0.001$  and in DEC it is  $0.137 \pm 0.001$ . These values suggest that the residual systematic differences are approximately 10% of the corrected ones.

We also analyzed the resulting rms differences obtained with UCAC4 and SPM4 after successive applications of the rectification method in Zone 1+180; they are displayed in Figure 11. It can be seen that the rms differences obtained with UCAC4 always decrease while those with SPM4 decrease after the first rectification, but they increase after the next ones.

## 5. CONCLUSIONS

The proposed method was able to reduce the systematic differences between 2MASS and UCAC4 well below the random differences in all the fields analyzed in this work, despite their very different densities (from 0.182 to 0.774 sources per square arcminute).

As expected, after rectification the mean differences 2MASS–UCAC4 are null, and the rms differences are reduced from  $\approx 140$  mas to  $\approx 90$  mas. If we assume that the average error of positions in UCAC4 is  $\approx 30$  mas and that in 2MASS  $\approx 85$  mas, then the resulting rms differences are  $\approx 90$  mas, as obtained.

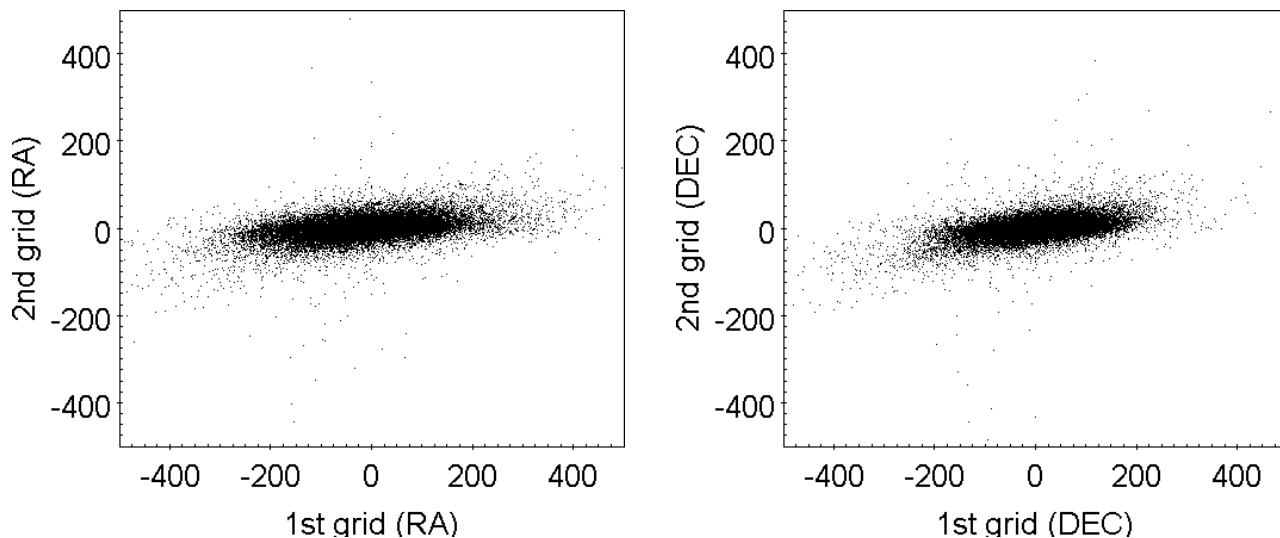


Fig. 10. VPD of second grid values versus first grid values in right ascension (left) and declination (right), in mas.

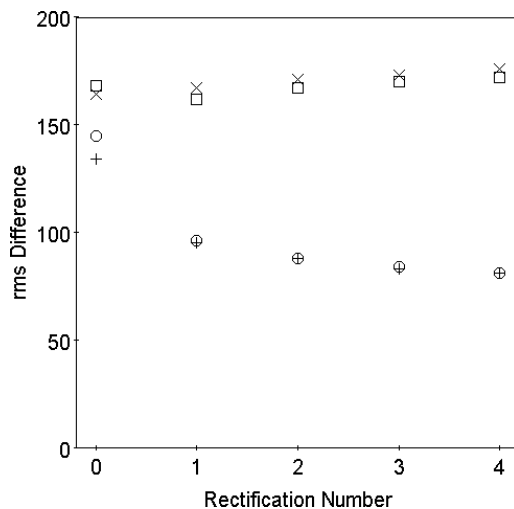


Fig. 11. Rms differences in mas between successive rectifications of 2MASS and other catalogs. Lower set: UCAC4. Upper set: SPM4. Open symbols: RA. Crosses: DEC.

The application of successive rectifications was tested only on Zone 1 + 180. It was found that the differences with UCAC4 always decreased, while the rms differences with an external catalog decreased after the first application, but they increased after further rectifications. This behavior suggests that successive rectifications could cause the rectified 2MASS to approach UCAC4 and its random errors. Therefore, they are not recommended.

The 2MASS catalog rectified with the proposed method can be regarded as an extension to magnitude  $K_s = 15$  of UCAC4. It is suited for astrometry with an accuracy around  $\approx 90$  mas in the positions and with a lower accuracy to magnitude  $K_s = 15.5$ , with negligible systematic errors. While our primary goal was to rectify 2MASS in the ecliptical zone to enable its use as a reference catalog for the astrometric reduction of

deep CCD images, we are considering performing the rectification of the complete 2MASS catalog. In the meantime we can perform on demand rectifications of particular fields requested by other authors.

Despite the fact that the rectification method was developed in the ecliptical zone, the method itself is not dependent on the region of the sky, so it could be applied in any other region if necessary, after adequate adaptation for regions close to the celestial poles.

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