

## THE TEMPERATURE DISTRIBUTION OF CIRCUMSTELLAR DISKS

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### RESUMEN

La temperatura es el parámetro más importante que determina el espectro de los discos circunestelares. Su distribución como función de la distancia a la estrella central se obtiene mediante el balance de los mecanismos de calentamiento y enfriamiento. Entre los mecanismos de calentamiento se encuentran la disipación viscosa intrínseca, la irradiación estelar directa, el calentamiento mediante choque debido a la envoltura cayendo hacia el centro y el calentamiento indirecto debido a luz dispersada por la envoltura. Hemos analizado en detalle cada uno de esos mecanismos, encontrando expresiones analíticas aproximadas que nos permiten evaluar su importancia en varias situaciones.

### ABSTRACT

The temperature is the single most important parameter that determines the spectrum of circumstellar disks. Its distribution as a function of distance to the central star is determined by the balance between the heating and cooling mechanisms. Among the former there are intrinsic viscous dissipation, direct stellar irradiation, shock heating by the infalling envelope and indirect heating via scattering from the envelope. We have analyzed in detail each of these mechanisms and found approximate analytic expressions which allow us to estimate their importance in several circumstances.

*Key words:* STARS: CIRCUMSTELLAR MATTER — STARS: FORMATION

### 1. INTRODUCTION

The spectrum of circumstellar disks is mainly determined by its temperature distribution. This property is strictly true in the case of optically thick disks. In thermal equilibrium the temperature is given by the balance between heating and cooling.

In most cases, the main contribution to the heating of accretion disks around pre-main-sequence stars is viscous dissipation. For a steady, geometrically thin accretion disk with uniform accretion rate of mass  $\dot{M}$ , the power dissipated per unit surface area is given by

$$F_{vis} = \frac{3GM_*\dot{M}}{8\pi R^3} \left[ 1 - \left( \frac{R_*}{R} \right)^{1/2} \right] \quad (1)$$

where  $M_*$  is the stellar mass and  $R_*$  its radius.  $R$  denotes the radial distance.

If the disk is optically thick in the vertical direction and the only, or most important, heating mechanism is viscous dissipation, the effective temperature distribution in the disk is found by equating the dissipation rate  $F_{vis}$  to the black body flux,

$$\sigma T_{eff}^4(R) = F_{vis}(R) \quad (2)$$

One finds,

$$\begin{aligned}
 T_{eff}(R) &= T_{0,vis} \left\{ \left( \frac{R_*}{R} \right)^3 \left[ 1 - \left( \frac{R_*}{R} \right)^{1/2} \right] \right\}^{1/4} \\
 &\approx T_{0,vis} \left( \frac{R_*}{R} \right)^{3/4} \quad \text{for } R \gg R_*
 \end{aligned}
 \tag{3}$$

where

$$T_{0,vis} \equiv \left( \frac{3GM_*\dot{M}}{8\pi R_*^3\sigma} \right)^{1/4}$$

For an observer at a distance  $D$  whose line of sight makes an angle  $i$  with respect to the normal of the disk plane, the observed flux at frequency  $\nu$  will be

$$S_\nu = \frac{2\pi \cos i}{D^2} \int_{R_*}^{R_d} B_\nu[T_{eff}(R)] R dR \tag{4}$$

where  $B_\nu$  is the Planck function and  $R_d$  is the outer radius of the disk.

For frequencies in the range  $kT_{eff}(R_d) \ll h\nu \ll kT_{0,vis}$  and  $T_{eff}(R)$  given by Eq. (2),

$$S_\nu \propto \nu^{1/3}. \tag{5}$$

At lower frequencies,  $h\nu \ll kT_{eff}(R_d)$

$$S_\nu \propto \nu^2, \tag{6}$$

and at higher frequencies,  $h\nu \gg kT_{0,vis}$

$$S_\nu \propto \nu^3 \exp(-h\nu/kT_{0,vis}). \tag{7}$$

In general (e.g., Adams, Lada, & Shu 1988) if

$$T_{eff}(R) \propto \left( \frac{R}{R_*} \right)^{-q} \tag{8}$$

then,

$$S_\nu \propto \nu^{(3-2/q)} \tag{9}$$

in the intermediate frequency range.

Several mechanisms will contribute to the heating of the disk in addition to viscous dissipation. Most of them are due to the presence of the central star and the infalling envelope. Among them, there are direct stellar irradiation, radiation scattering from the envelope, the boundary layer between the star and the disk, and shock heating by the infalling envelope.

In the following we will consider each of these heating mechanisms and give approximate analytic expressions for the corresponding heating power. The details of the results presented here are given in D'Alessio (1995) and Moreno & Cantó (1995).

## 2. HEATING MECHANISMS

### 2.1. Direct Stellar Irradiation

First we consider the heating due to direct stellar irradiation. If the disk is geometrically thin and flat the amount of energy intercepted by the upper part of the disk at a distance  $R$  per unit area and time is (e.g., Adams et al. 1988),

$$F_{*,d} = \frac{\sigma T_*}{\pi} \left\{ \sin^{-1} \left( \frac{R_*}{R} \right) - \left( \frac{R_*}{R} \right) \left[ 1 - \left( \frac{R_*}{R} \right)^2 \right]^{1/2} \right\}, \quad (10)$$

where  $T_*$  is the stellar effective temperature. For  $R/R_* \gg 1$ ,

$$F_{*,d} \approx \frac{2\sigma}{3\pi} T_*^4 \left( \frac{R_*}{R} \right)^3 \quad (11)$$

An optically thick flat disk in which stellar irradiation is dominant will have an effective temperature distribution,

$$\begin{aligned} T_{eff}(R) &= \frac{T_*}{\pi^{1/4}} \left\{ \sin^{-1} \left( \frac{R_*}{R} \right) - \left( \frac{R_*}{R} \right) \left[ 1 - \left( \frac{R_*}{R} \right)^2 \right]^{1/2} \right\}^{1/4} \\ &\approx T_{0,d} \left( \frac{R_*}{R} \right)^{3/4} \end{aligned} \quad (12)$$

where,

$$T_{0,d} \equiv \left( \frac{2}{3\pi} \right)^{1/4} T_*$$

Therefore, it will show an spectrum,

$$S_\nu \propto \nu^{1/3} \quad (13)$$

in the intermediate frequency range.

Obviously, deviations from the flat geometry will increase the energy deposited by the star on the disk. This effect has been quantified by Kenyon & Hartmann (1987) who considered flared disks (that is, optically thick disks with concave upper surfaces) and the resultant temperature distribution.

The modified expression for the stellar flux intercepted by the disk is (see Kenyon & Hartmann 1987 for details)

$$F'_{*,d} = \frac{2\sigma T_*^4}{4} \int_0^{\phi_{max}} \int_0^{\theta_{max}} \sin \phi \left( \frac{c_1 \sin \phi \cos \theta + c_2 \cos \phi}{c_3} \right) d\theta d\phi \quad (14)$$

where

$$c_1 = \left[ \frac{dH}{dR} \frac{H}{d} + \frac{R}{d} \right], \quad c_2 = \left[ \frac{dH}{dR} \frac{R}{d} + \frac{H}{d} \right], \quad c_3 = \left[ 1 + \left( \frac{dH}{dR} \right)^2 \right]$$

$$d^2 = H^2 + R^2, \quad \phi_{max} = \sin^{-1} \left( \frac{R_*}{d} \right),$$

and  $\theta_{max}$  is a maximum angle that takes into account the occultation of the star by the disk. In the above formulae  $H = H(R)$  represents the photospheric height of the disk.

Once the disk photospheric height is specified, it is straightforward to determine the amount of stellar flux captured by the disk using Eq. (14).

In the case that the energy absorbed by the disk is radiated at the local black body temperature,

$$\sigma T_{eff}^4(R) = F'_{*,d}(R) \quad (15)$$

and the photospheric height is given by the hydrostatic scale height,

$$H = \left( \frac{c^2 R^3}{GM_*} \right)^{1/2} \quad (16)$$

where  $c$  is the sound speed calculated with  $T_{eff}$ . Eq. (14) can be solved iteratively to derive a self-consistent solution for  $H(R)$  and  $T_{eff}$  (Kenyon & Hartmann 1987).

The result of this procedure shows that the disk always remains thin ( $H/R$  and  $dH/dR \ll 1$ ) and thus Eq. (14) can be greatly simplified. Moreover, with such simplification an approximate analytic solution can be found for different regions in the disk (see Figure 1).

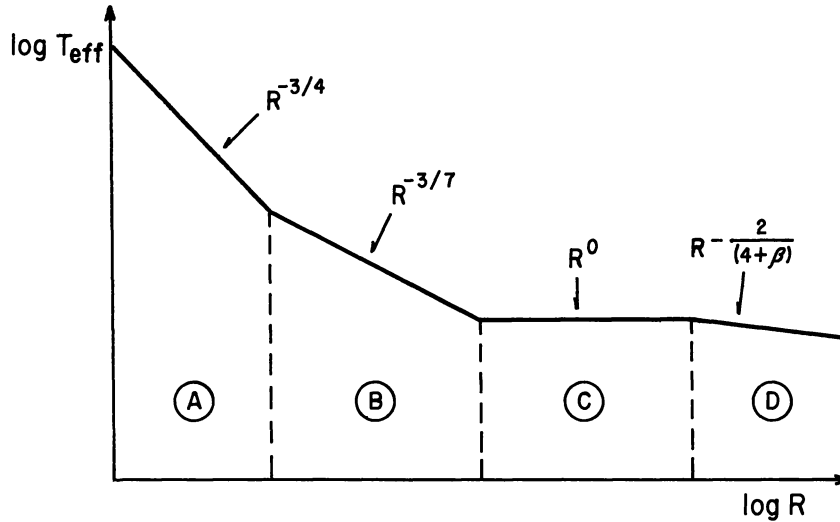


Fig. 1.— Schematic representation of the effective temperature distribution of a geometrically thin self-consistent flared disk. Each region is described in the text.

*Region A:* The disk is optically thick both to the stellar radiation and to its own radiation. Flaring is unimportant. Then

$$T_{eff}(R) = \frac{T_*}{7^{1/4}} \left( \frac{R}{R_*} \right)^{-3/4} \quad (17)$$

Above,  $\lambda \equiv (kT_* R_*/mGM_*)^{1/2}$  and  $m$  is average mass per particle.

*Region B:* The disk is also optically thick to both the stellar and to its own radiation, however, flaring important.

$$T_{eff}(R) \propto R^{-3/7} \quad (18)$$

*Region C:* The disk is optically thick to the stellar radiation but optically thin to its own radiation. Flaring is important.

$$T_{eff}(R) = cnt \quad (19)$$

*Region D:* The disk is optically thin to both the stellar and its own radiation.

$$T_{eff}(R) \propto R^{-2/(4+\beta)} \quad (20)$$

where  $\beta$  is the exponent of the dust opacity.

The boundary between regions A and B is marked by  $H \sim R_*$ . For  $M_* = 1 M_\odot$ ,  $R = 3 R_\odot$ ,  $T_* = 4,000$  K, this occurs at  $R \simeq 36 R_*$ . Thus, one expects that the spectrum in the near IR will not be affected by a self-consistent flaring; that is, one expects,

$$S_\nu \propto \nu^{1/3}.$$

## 2.2. Scattering from the Envelope

One of the consequences of dusty envelopes around the star and the circumstellar disk will be to scatter the stellar and the disk light back into the disk (Natta 1993). The contribution to the heating of the outer parts of the disk from this scattered light may be significant (Butner, Natta & Evans 1993, Keene & Masson 1990, Natta 1993, 1995 [in these Proceedings]).

We follow Natta (1993) in considering the case of optically thin, spherically symmetric, dusty envelopes in which the number density of scattering particles varies as a power-law function of the distance to the central star,

$$n_d(r) = n_d^* \left( \frac{r}{R_*} \right)^{-\alpha} \quad (21)$$

The star has a temperature  $T_*$  and radius  $R_*$ . The disk is assumed to be geometrically thin and flat.

If the only source of energy is the central star (the case studied by Natta 1993), the heating rate per unit area at a distance  $R$  (on the disk) due to scattered radiation is given by,

$$F_{sc}^*(R) = \frac{\sigma T_*^4}{4\pi} \kappa_p^{(s)}(T_*) R_*^2 \int_0^{2\pi} \int_0^{\pi/2} \int_{r_{min}}^{r_{max}} \frac{n_d(r) r \cos\theta \sin\theta}{(r^2 + R^2 - 2rR\sin\theta \sin\phi)^{3/2}} dr d\theta d\phi \quad (22)$$

where  $\kappa_p^{(s)}$  is the Planck mean cross section for scattering, and  $r_{min}$  and  $r_{max}$  are the inner and outer radii of the envelope.

After some simplifying assumptions (D'Alessio 1995), Eq. (22) can be written as,

$$F_{sc}^*(R) = \frac{\sigma T_*}{16} \tau_* \left( \frac{R_*}{R} \right)^{\alpha+1} B(\alpha, R/r_{min}) \quad (23)$$

where,

$$\tau_* \equiv \kappa_p^{(s)}(T_*) n_d^* R_*$$

and,

$$B(\alpha, R/r_{min}) \equiv \left\{ \begin{array}{ll} \Lambda_*(\alpha) & \text{if } \alpha < 2 \\ 4 \ln \left( \frac{R}{r_{min}} \right) & \text{if } \alpha = 2 \\ 4(\alpha - 2)^{-1} \left( \frac{R}{r_{min}} \right)^{\alpha-2} & \text{if } \alpha > 2 \end{array} \right\} R > r_{min}$$

$$B(\alpha, R/r_{min}) \equiv \frac{4}{(\alpha + 1)} \left( \frac{R}{r_{min}} \right)^{\alpha+1} \quad R < r_{min}$$

$$\Lambda_*(\alpha) \equiv \int_0^\infty \frac{(x^2 + 1)}{x^{1+\alpha}} \ln \left[ \frac{(1 + x^2)^2}{(1 - x^2)^2} \right] dx$$

and is tabulated in Table 1.

Table 1

$\alpha$	$\Lambda_*(\alpha)$	$\Lambda_v(\alpha)$
0	8.37	0.514
0.5	7.67	0.458
1.0	8.37	0.514
1.5	12.06	0.815

Then the temperature distribution of a passive disk with scattering heating can be approximated by,

$$T(R) \approx T_{0,*} \left[ \left( \frac{R}{R_*} \right)^{-3} + \frac{3\pi}{32} \tau_* B(\alpha, R/r_{min}) \left( \frac{R}{R_*} \right)^{-(\alpha+1)} \right]^{1/4} \quad (24)$$

In Figure 2 we compare the results of this analytic approximation with the exact results derived from a direct (numerical) integration of Eq. (22).

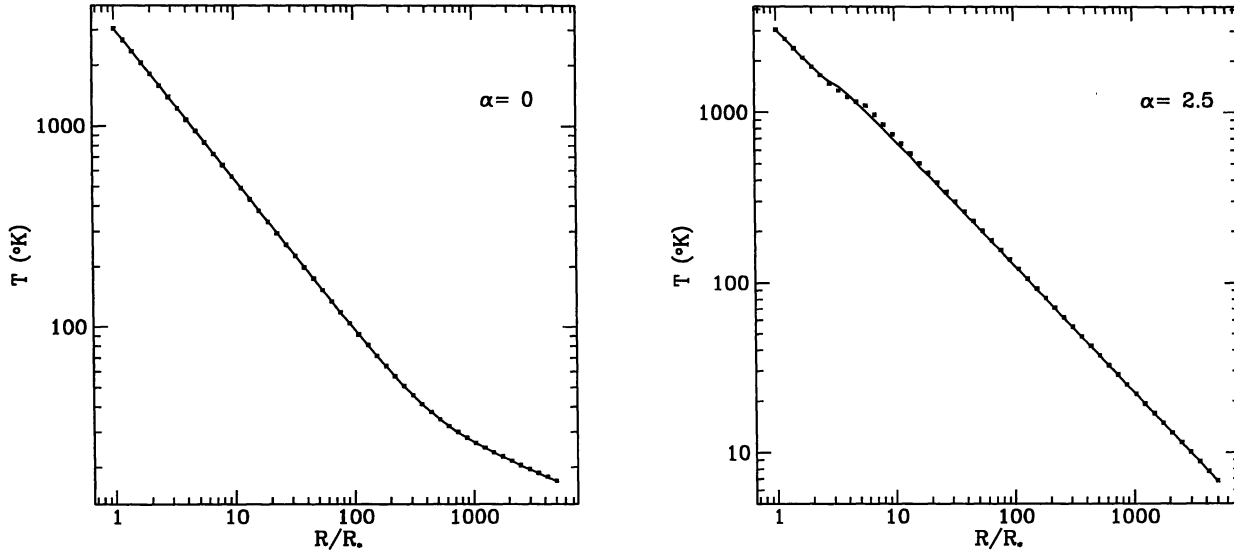


Fig. 2.— Radial temperature distribution of a passive disk heated by a central star with  $T_* = 4500$  K. The exact temperature (dots) and the approximation described in the text (continuous line) are shown for an envelope with  $\tau_* = 0.2$ ,  $r_{min} = 3 R_*$  and  $r_{max} = 10^5 R_*$ . The left panel corresponds to a power law exponent of the envelope density  $\alpha = 0$ , and the right panel corresponds to  $\alpha = 2.5$ .

We now consider the contribution of the radiation emitted by the disk which is scattered back into the disk. The rate of heating due to this scattered radiation is given by,

$$F_{sc}^D(R) = \sigma \int_{R_{min}}^{R_{max}} \int_{r_{min}}^{r_{max}} \int_0^{\pi/2} \frac{n_d(r) l r^4 \cos^2 \theta \sin \theta T^4(l) \kappa_p [T(l)]}{(r^2 + l^2)^{3/2} (r^2 + R^2)^{3/2}} {}_2F_1(a^2) {}_2F_1(b^2) d\theta dr dl \quad (25)$$

where  $R_{min}$  and  $R_{max}$  are the inner and outer disk radii, and  ${}_2F_1$  is the hypergeometric function  ${}_2F_1(3/4, 5/4, 1, z)$  and

$$a \equiv \frac{2lr \sin \theta}{r^2 + l^2} \quad \text{and} \quad b \equiv \frac{2Rr \sin \theta}{r^2 + R^2}$$

and  $T(l)$  is the disk temperature at a distance  $l$  from the star.

It is clear that the evaluation of the rate of heating of the disk due to the scattered radiation of the disk by itself requires an iterative procedure (since the emission of the disk depends on its temperature which in turns depends on the rate of heating).

If one considers the case where the basic source of heating of the disk is viscous dissipation, then the temperature distribution in the disk will be given by,

$$\sigma T^4(R) = F_{vis}(R) + F_{sc}^D(R) \quad (26)$$

where  $F_{vis}$  is given by Eq. (1) and  $F_{sc}^D(R)$  by Eq. (25). In this case the iterative procedure mentioned above converges rather quickly. This implies that it is accurate enough to assume in the evaluation of  $F_{sc}^D(R)$  (Eq. [25]) the temperature distribution the disk would have if scattering heating were unimportant.

Again, making some simplifying assumptions (D'Alessio 1995), Eq. (25) reduces to,

$$F_{sc}^D(R) \approx \sigma T_{0,vis}^4 \tau_v \left( \frac{R_*}{R} \right)^{\alpha+1} C(T_{0,vis}) A(\alpha, R/R_{min}) \quad (27)$$

where,

$$\tau_v \equiv \kappa_p^{(s)}(T_{0,vis}) n_d^* R_*$$

$$A(\alpha, R)/r_{min} \equiv \left\{ \begin{array}{ll} \Lambda_v(\alpha) & \text{if } \alpha < 2 \\ \frac{1}{3} \ln \left( \frac{R}{r_{min}} \right) & \text{if } \alpha = 2 \\ \frac{1}{3}(\alpha - 2)^{-1} \left( \frac{R}{r_{min}} \right)^{\alpha-2} & \text{if } \alpha > 2 \end{array} \right\} R > r_{min}$$

$$A(\alpha, R/r_{min}) = \frac{1}{3(\alpha + 1)} \left( \frac{R}{r_{min}} \right)^{\alpha+1} \quad R < r_{min}$$

and  $C(T_{0,vis})$  is a slow varying function of  $T_{0,vis}$

$$C(T_{0,vis}) \equiv 0.5 \left( \frac{T_{0,vis}}{6500} \right)^{0.3}$$

The function  $\Lambda_v(\alpha)$  is tabulated in Table 1. Therefore, the temperature distribution in the disk can be approximated by,

$$T(R) \approx T_{0,vis} \left[ \left( \frac{R}{R_*} \right)^{-3} + \tau_v C(T_{0,vis}) A(\alpha, R) \left( \frac{R}{R_*} \right)^{-(\alpha+1)} \right]^{1/4} \quad (28)$$

In Figure 3 we compare the results using Eq. (25) with those using the analytic approximation (Eq. [28]). The parameters are  $M_* = 1 M_\odot$ ,  $R_* = 2.3 R_\odot$  and  $M = 10^{-6} \text{ yr}^{-1}$ .

### 2.3. The Boundary Layer

In a recent paper, Popham et al. (1993; hereafter PNHK, 1995 [in these Proceedings]) have reported physically self-consistent models for optically thick boundary layers around pre-main-sequence stars. They find that the boundary layer can be divided in two distinct regions. There is the dynamical boundary layer, a narrow region close to the star where the angular velocity drops from Keplerian to the much smaller stellar rotation. There, most of the boundary layer luminosity is generated by a vigorous viscous dissipation due to the strong angular velocity gradient. Only a small fraction of this luminosity is radiated locally from the surface of the disk. Most of the energy diffuses in the radial direction and is radiated from the disk surface in a much larger region, called the thermal boundary layer.

PNHK model the entire accretion flow in the disk, the boundary layer and the star by a single set of equations: the conservation of mass, angular and radial momenta and an energy balance equation which takes into account the advection of entropy and the radial fluxes plus viscous dissipation of energy. This last equation is written as,

$$\dot{M} T \frac{dS}{dR} - \dot{M} \frac{\nu}{v} R \left( \frac{d\Omega}{dR} \right)^2 - \frac{d}{dR} (4\pi R H F_R) - 4\pi R F_V = 0 \quad (29)$$

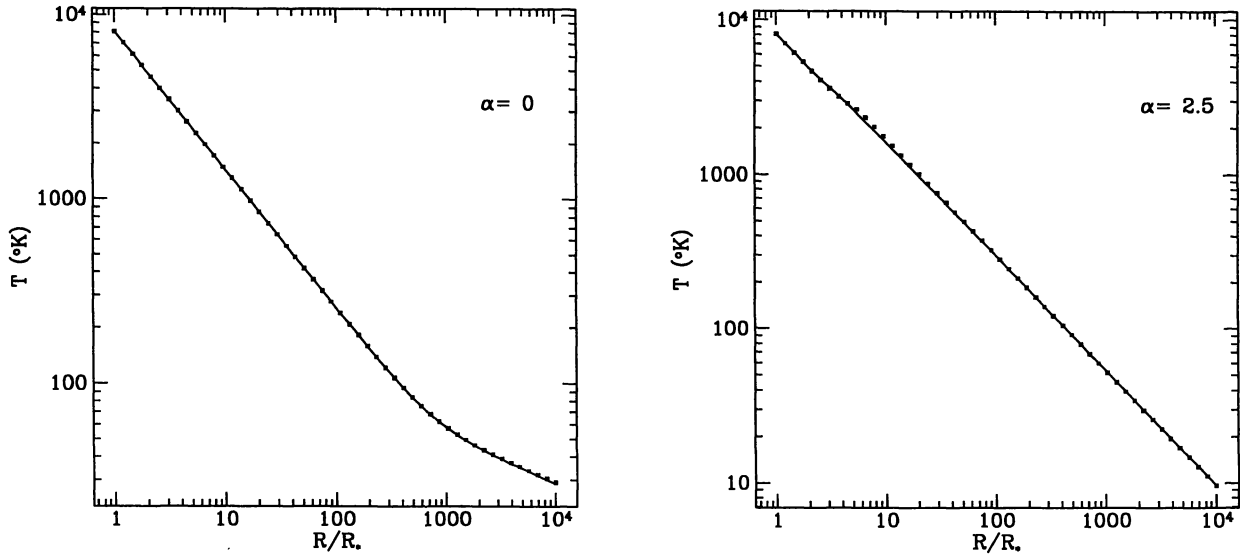


Fig. 3.— Radial temperature distribution of an accretion disk irradiated by itself. The exact temperature (dots) and the approximation described in the text (continuous line) are shown. Considering only viscous dissipation, the accretion disk temperature at the stellar surface is  $T_0 = 8100$  K, and the envelope has  $\tau_v = 0.2$ ,  $r_{min} = 3 R_*$  and  $r_{max} = 10^5 R_*$ . The left panel corresponds to a power law exponent of the envelope density  $\alpha = 0$ , and the right panel corresponds to  $\alpha = 2.5$ .

where  $T$  is the temperature,  $S$  the entropy,  $\dot{M}$  the mass accretion rate,  $\nu$  the viscosity coefficient,  $v$  the radial velocity,  $\Omega$  the angular velocity,  $H$  the vertical scale height and  $F_R$  and  $F_V$  the radial and vertical radiative fluxes respectively.

In our simple model (D'Alessio 1995) we neglect the advection of entropy and divide the boundary layer in a narrow dynamical boundary layer where the total BL luminosity is generated

$$L_{BL} = \frac{1}{2} \frac{GM_* \dot{M}}{R_*} \quad (30)$$

and a more extended thermal boundary layer where we assume that viscous dissipation is unimportant. There, Eq. (29) reduces to,

$$4\pi R F_v \approx -\frac{d}{dR}(4\pi R H F_R). \quad (31)$$

We have integrated Eq. (31) with the boundary condition,

$$(4\pi R H F_R)_{R_*} = L_{BL},$$

and the assumption that the thermal BL is optically thick (and therefore that the diffusion approximation holds for the radiative fluxes). For the Rosseland Mean Opacity we have used the analytic approximation of Bell & Lin (1993),

$$\kappa_R = 1.5 \times 10^{20} \rho T^{-\gamma} \quad (32)$$

in cgs units, where  $\gamma = 5/2$ .

The result is,

$$F_v(R) = \sigma T_{BL}^4 (1 - r/\Delta)^{2\lambda} \equiv F_{BL}(R) \quad (33)$$



where

$$\begin{aligned}\lambda &\equiv \frac{13}{2} + \gamma = 9 \\ r &\equiv \frac{R - R_{BL}}{R_*} \\ T_{BL} &\equiv C(\lambda)^{1/4} \left( \frac{L_{BL}}{4\pi R_*^2 \sigma} \right)^{1/4} g \\ C(\lambda) &\equiv \left( \frac{2\lambda + 1}{2} \right)^{\lambda/(2\lambda+1)} = 2.9 \\ \Delta &\equiv \frac{(2\lambda + 1)}{C(\lambda) g^4} \\ g &\equiv g_0 \alpha^{0.03} M_*^{0.09} R_*^{-0.02} \dot{M}^{-0.04}\end{aligned}$$

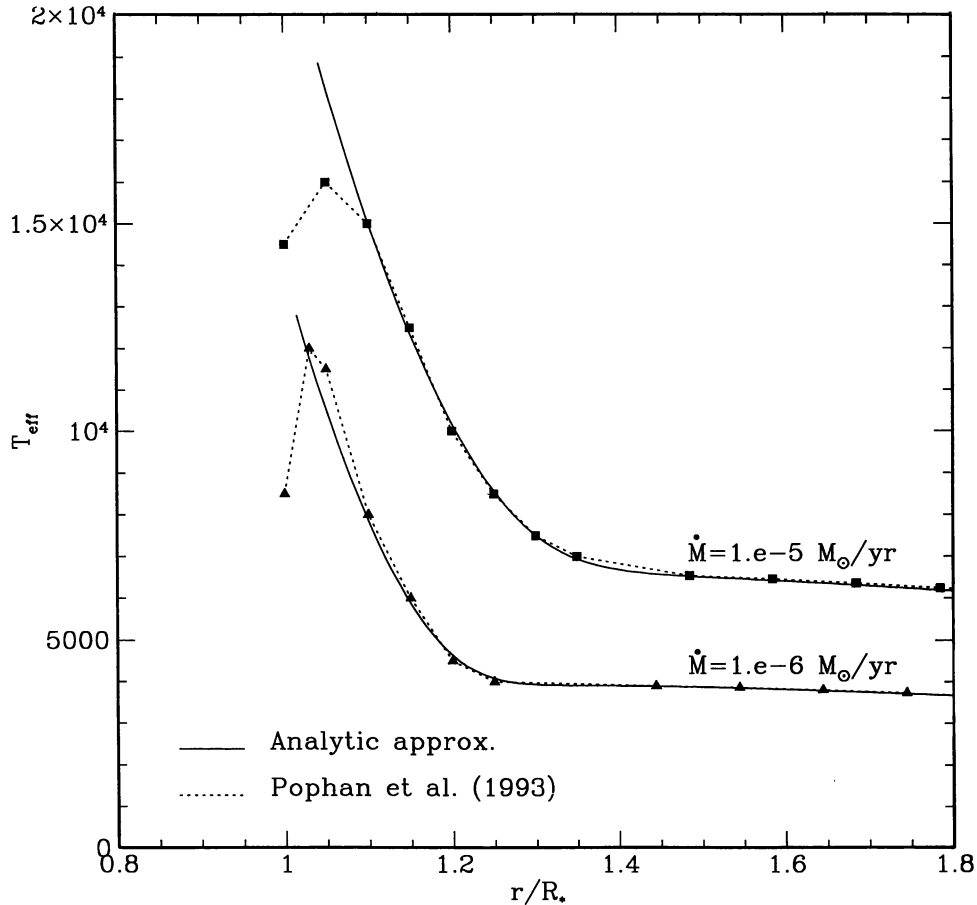


Fig. 4.— Boundary layer effective temperature as a function of radius. The exact result (dotted line) given by Popham et al. (1993) is compared with the analytic approximation (continuous line) described in the text for  $\dot{M} = 10^{-6} M_{\odot} \text{ yr}^{-1}$  (triangles) and  $\dot{M} = 10^{-5} M_{\odot} \text{ yr}^{-1}$  (squares). The parameters of the model are  $\alpha = 0.1$  and the stellar radius are:  $R_* = 2.33 R_{\odot}$  for  $\dot{M} = 10^{-6} M_{\odot} \text{ yr}^{-1}$  and  $R_* = 2.53 R_{\odot}$  for  $\dot{M} = 10^{-5} M_{\odot} \text{ yr}^{-1}$ .  $R_{BL} = 0.015 R_*$  for  $\dot{M} = 10^{-6} M_{\odot} \text{ yr}^{-1}$  and  $R_{BL} = 0.042$  for  $\dot{M} = 10^{-5} M_{\odot} \text{ yr}^{-1}$ .

where  $R_{BL}$  is the outer radius of the dynamical BL,  $\alpha$  is the viscosity parameter and  $g_0$  is a constant. If  $M_*$  is in solar masses,  $R_*$  is in solar radius and  $\dot{M}$  in solar masses per year,  $g_0 \simeq 1$ .

In Figure 4 we compare our analytic approximation with the detailed numerical results of PNHK for the effective temperature. In the analytic approximation the effective temperature was calculated from,

$$\sigma T_{eff}^4(R) = F_{BL}(R) + F_*(R) + F_{vis}(R) \quad (34)$$

where we have used Eqs. (1) and (34) for  $F_{vis}$  and  $F_{BL}$  respectively and,

$$F_*(R) = \sigma T_*^4 \left[ \frac{C(\lambda)g^4}{2} \right] \quad (35)$$

is the stellar flux diffused through the thermal boundary layer.

#### 2.4. The Accretion Shocks

The accretion flow arrives at the disk at supersonic velocities. There, a shock is formed which thermalizes the velocity component perpendicular to the disk. The flow, however, retains its radial and rotational velocities through the shock, which are, in general, very different from the velocities of the disk (see, e.g., Cassen & Mossman 1981). The rotational velocity in the disk is close to Keplerian and the radial velocity is very small.

In the standard picture it is postulated that behind the shock there is a viscous layer through which the incoming flow is incorporated into the disk and the disk velocities are established (Cassen & Mossman 1981). However, since, in general, the viscous time scale in the disk is much larger than the rotational periods, an alternative, more likely, scenario emerges. In this scenario the flow behind the accretion shock moves inwards in an orbit nearly parallel to the disk plane until it reaches a second shock. In this shock the radial (inward) component of the velocity is thermalized and the flow settles down in a circular Keplerian orbit (see Figure 5). The back pressure of this shock is provided by the gas in the disk already in a Keplerian circular motion (see Moreno & Cantó 1995).

The power dissipated in each of these shocks per unit area are (Moreno & Cantó 1995),

$$F_{sh,1}(R) = \frac{GM_*\dot{M}}{16\pi R_D^3} \frac{(1-u)^{1/2}}{u^2} \quad ; u \leq 1 \quad (36)$$

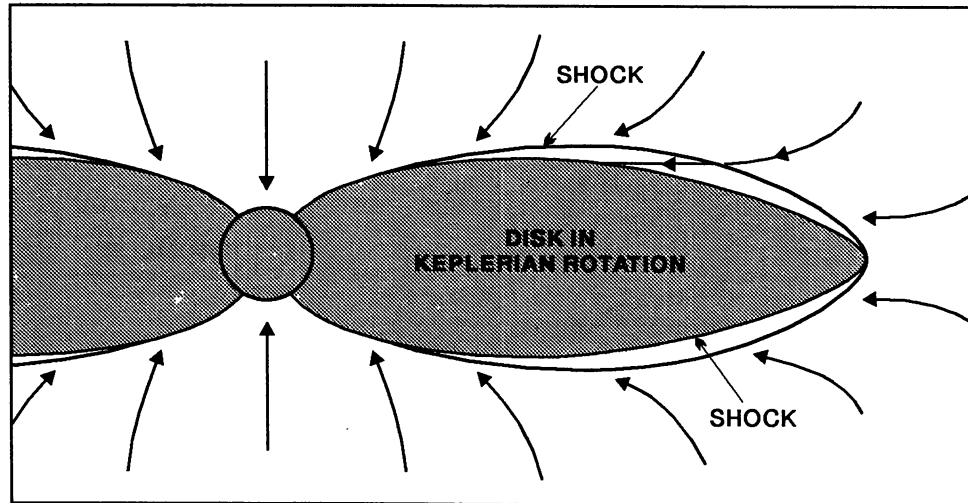


Fig. 5.— Schematic representation of the interaction of the accretion flow with the disk. In the outer shock the velocity component perpendicular to the disk is thermalized. Behind this, the flow moves inwards until a second shock is reached. In this second shock the radial velocity is thermalized and the flow settles down in a circular Keplerian orbit.

for the outer shock, and,

$$F_{sh,2}(R) = \frac{GM_*\dot{M}}{32\pi R_D^3} \frac{(1+u-u^{1/2})}{u^{5/2}(1-u^{1/2})^{1/2}} \quad ; u \leq 1 \quad (37)$$

for the inner shock. Above,  $R_D$  is the disk radius and  $u = R/R_D$ .

### 3. SUMMARY

In this paper we have analysed several mechanisms that contribute to the heating of circumstellar disks around pre-main sequence stars in addition to the well known viscous heating and heating by direct stellar irradiation. They are, scattering from the envelope of the stellar and disk radiation, the presence of a boundary layer between the star and the disk and shock heating by the infalling envelope.

For each of these mechanisms we give approximate analytic expressions for the corresponding heating power.

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