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QUASI-GEOSTROPHIC VORTICES IN CIRCUMSTELLAR DISKS

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RESUMEN

Se discute la física de los vórtices en discos circunestelares asociados con objetos estelares jóvenes, elucidando las propiedades físicas básicas de esos sistemas tormentosos localizados. Esos vórtices pueden intensificar la formación de planetas gigantes a través de una inestabilidad gravitatoria, permitiendo a los granos de polvo (elementos pesados) asentarse en el centro en una escala de tiempo corta. La misma inestabilidad gravitatoria se ve también intensificada debido a que los vórtices producen una mayor densidad superficial local en el disco. Además, pueden incrementar la disipación de energía en los discos y de ese modo afectar su acrecimiento.

ABSTRACT

We discuss the physics of vortices in circumstellar disks associated with young stellar objects and elucidate the basic physical properties of these localized storm systems. Many different types of vortices can exist in circumstellar disks. Vortices may enhance giant planet formation via gravitational instability by allowing dust grains (heavy elements) to settle to the center on a short time scale; the gravitational instability itself is also enhanced because the vortices create a larger local surface density in the disk. In addition, vortices can enhance energy dissipation in disks and thereby affect disk accretion.

Key words: **STARS: FORMATION — HYDRODYNAMICS**

1. INTRODUCTION

Circumstellar disks play an important role in the star formation process (see, e.g., the reviews of Shu, Adams, & Lizano 1987; Bertout 1989; Beckwith & Sargent 1993). Through many recent studies, we now have a reasonably good understanding of the basic physical properties of these disks, such as total disk mass, temperature distributions, and radial size (e.g., Rydgren & Zak 1985; Rucinski 1987; Kenyon & Hartmann 1987; Beckwith et al. 1990; Adams, Emerson, & Fuller 1990). However, many different physical processes occur in these systems and it is not yet known which processes are most important for disk evolution. In this work (based on a recent paper: Adams & Watkins 1995), we study the basic physics of vortices, which are essentially long-lived storm systems. We explore possible effects that vortices have on disk evolution and dynamics. In particular, we show that vortices can enhance the formation of giant planets within the mechanism of gravitational instability. Vortices can also enhance energy dissipation and thereby affect disk accretion.

The most well known and well studied example of a fluid vortex in an astrophysical setting is the Great Red Spot of Jupiter (e.g., Ingersoll 1990). This stable swirling storm system has lived for many dynamical time scales. Although the exact nature of the Great Red Spot remains somewhat controversial (see, e.g., Marcus 1993 and Petviashvili & Pokhotelov 1992 for different perspectives), the basic physics is known and provides a starting point for this present work. In particular, we focus on quasi-geostrophic vortices and leave more complex vortex models for future work. Similar vortices arise in most rotating fluid systems, such as the atmosphere and oceans of the Earth (e.g., Ghil & Childress 1987). We note that vortices have also been studied in the context of galactic disks (Korchagin & Petviashvili 1985; Korchagin & Ryabtsev 1991). These previous studies have not included the self-gravity of the system and have not considered nearly Keplerian rotation curves such as those found in circumstellar disks (see, however, Von Weizsäcker 1944). In addition, two recent papers (Tanga et al. 1995; Barge & Sommeria 1995) also discuss planet formation and dust settling in vortices.

2. GENERAL FORMULATION

In this section, we outline the derivation of the basic equation of motion for vortices in circumstellar disks. We make a number of approximations along the way. We consider the vortices to be small compared to the radial position r in the disk; we thus invoke a local approximation. We can then separate the vortical motion from the mean flow of the disk. We introduce both the vorticity and the stream function and obtain equations in terms of these physical quantities.

We consider disturbances which are small compared to the radial extent of the disk. As a general trend, the vortex size is roughly comparable to the scale height H of the disk and for thin disks $H/r \ll 1$. We thus work in terms of the variables x and y centered on a point (r_0, ϕ_0) rotating with the disk (see Figure 1).

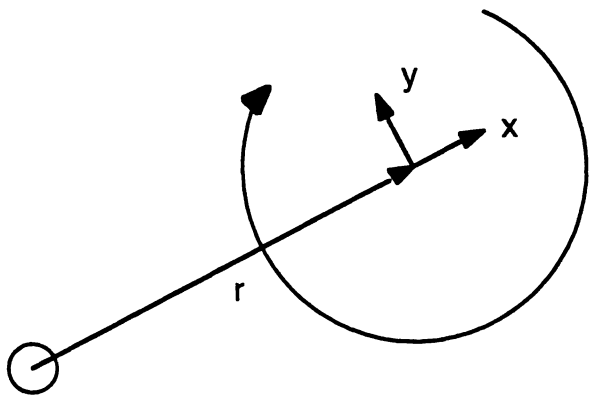

Fig. 1.— Schematic diagram of local coordinate system.

As the next simplification, we separate out the mean flow from the disturbance. In equilibrium, the disk is azimuthally symmetric and has no radial flow, i.e., $u = 0$ and $v = v_0 = r\Omega(r)$. Here, we want to separate the perturbed flow of the vortex from the unperturbed motion; we therefore write the azimuthal velocity v as the sum of parts:

$$v = v_0 + v_1 = r\Omega(r) + v_1 \,, \tag{2.1}$$

where $\Omega(r)$ is the unperturbed rotational speed as a function of radius. The equation of motion can then be written in the form

$$[\partial_t + r(\Omega - \Omega_0)\partial_y]\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} + 2\Omega(\hat{z}\times\mathbf{v}) + \nabla h_1 + \nabla\psi_1 + ur(\partial_x\Omega)\hat{y} = 0 \,, \tag{2.2}$$

where $\mathbf{v} = (u, v_1)$ is the perturbed velocity field. Notice the presence of the Coriolis term, $2\Omega(\hat{z}\times\mathbf{v})$, which arises because we are working in a rotating frame of reference.

We want to write the equations of motion in terms of vorticity, instead of velocity. This approach makes sense because we are interested in vortex solutions and vorticity is thus the relevant physical quantity. The vorticity ω is defined by

$$\omega \equiv \hat{z}\cdot(\nabla\times\mathbf{v}) \,, \tag{2.3}$$

i.e., we consider only the $\hat{z}$ component. We now take the curl of the force equation and combine it with the continuity equation. After some rearrangement, the resulting equation of motion becomes

$$\mathbf{D}\Big[\frac{\omega + 2\Omega + r\partial_x\Omega}{\sigma}\Big] = \mathbf{D}\,\Upsilon = 0 \,, \tag{2.4}$$

where we have defined the operator $\mathbf{D}$ to be the total advective derivative operator,

$$\mathbf{D} \equiv \partial_t + r(\Omega - \Omega_0)\partial_y + \mathbf{v}\cdot\nabla \,, \tag{2.5}$$

and where we have defined Υ to be the total vorticity per unit surface density. This quantity Υ, which we denote as the *vortensity*, has two contributions: the perturbative part and that of the mean flow. The equation of motion [2.4] implies that the *total* vortensity is advectively conserved.

For the vortices considered here, the fluid is nearly in a generalized type of geostrophic balance. Exact geostrophic balance occurs when pressure forces are balanced by Coriolis forces, so that the inertial forces are negligible. In this work, we generalize this concept to include gravity, so that both the Coriolis and gravitational forces are balanced by the pressure force (see Figure 2). In particular, we write the velocity $\mathbf{v}$ in the form

$$\mathbf{v} = \frac{1}{2\Omega_0}\left[\hat{z} \times (\nabla h_1 + \nabla \psi_1)\right] , \tag{2.6}$$

where Ω_0 is the value of the rotation rate in the disk at the origin of the local (x, y) coordinate system. In the geostrophic approximation, the vorticity can be written

$$\omega = \frac{1}{2\Omega_0}(\nabla^2 h_1 + \nabla^2 \psi_1) . \tag{2.7}$$

Thus, our equation of motion in the *quasi-geostrophic approximation* now becomes

$$\mathbf{D}\left[\frac{1}{2\Omega_0\sigma}(\nabla^2 h_1 + 4\pi G\rho_1) + \frac{2\Omega + r\partial_x\Omega}{\sigma}\right] = \mathbf{D}\Upsilon = 0 , \tag{2.8}$$

where we have used the Poisson equation to eliminate the gravitational potential in favor of the density. Equation [2.8] represents the fundamental equation of motion for vortices in the quasi-geostrophic model. Here, the total vortensity Υ is advectively conserved. Furthermore, equation [2.8] defines the relationship between the total vortensity and the density perturbation.

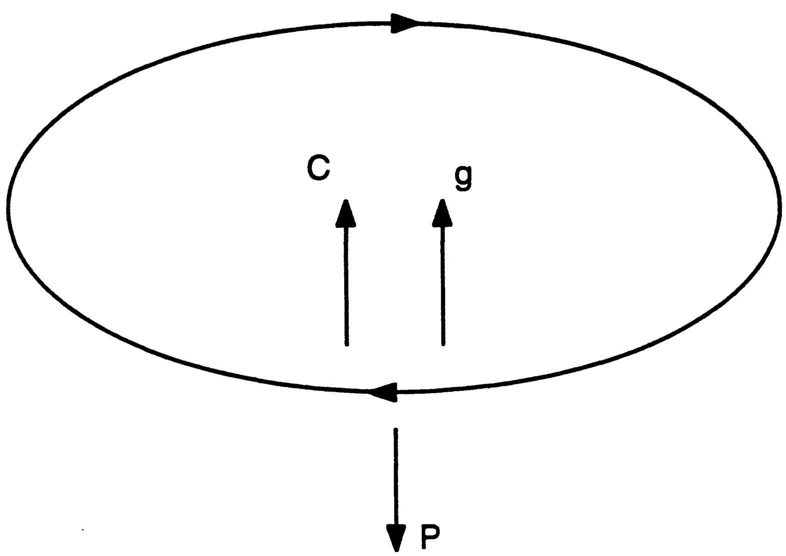

Fig. 2.— Schematic diagram illustrating geostrophic balance. For the case shown, both the Coriolis force (C) and the gravitational force (g) point inward, whereas the pressure force (P) is directed outward.

3. BASIC RESULTS

In this section, we consider the simplest type of vortices in circumstellar disks. We discuss the limiting case of point vortices and general linear vortices without self-gravity.

For the simplest case of a point vortex, the vorticity is localized at a point in space. Vortices of this type are the solutions to the simplest form of our equation of motion. For example, if we consider the rotation rate Ω and the total surface density σ of the disk to both be constant, then the equation of motion simplifies to the form

$$\mathbf{D}\omega = 0 . \tag{3.1}$$

In other words, in this limiting case, the vorticity itself is advectively conserved. This equation of motion has solutions which correspond to point vortices. Mathematically, we can write these solutions as

$$\omega = \Gamma \delta^2(\mathbf{x} - \mathbf{x}_0) , \tag{3.2}$$

where $\delta^2(\mathbf{x})$ is the two-dimensional Dirac Delta function and $\mathbf{x}_0$ is the position of the vortex. The quantity Γ, which determines the magnitude of the vorticity, is called the circulation.

Once the vorticity solution is specified in the form of equation [3.2], we can determine the physical structure of the vortex (Marcus 1993). We find the velocity field in a manner completely analogous to finding the magnetic field produced by a line-like current, i.e.,

$$\mathbf{v} = \frac{\Gamma}{2\pi\varpi} \hat{\varphi} , \tag{3.3}$$

where we have introduced local cylindrical coordinates centered on the position of the point vortex (ϖ is the local radial coordinate). The density enhancement is determined by the condition [2.6] of geostrophic balance. Notice that positive density enhancements correspond to negative values of the circulation Γ. In other words, the vortex motion must be clockwise (in the $-\hat{\varphi}$ direction) for a positive density vortex.

For the next higher order of approximation, we consider the case of vortices with no self-gravity and we consider only the leading order perturbations for Ω and σ. We also specialize to the case of a purely Keplerian rotation curve and we introduce a stream function Φ defined by

$$\Phi = h_1/2\Omega_0 \qquad \text{so that} \qquad \mathbf{v} = \hat{z} \times \nabla\Phi . \tag{3.4}$$

Departing slightly from the usual conventions (e.g., Marcus 1993), we define a Rossby wavenumber k_R, i.e.,

$$k_R \equiv \Omega_0/a . \tag{3.5}$$

For circumstellar disks, this Rossby wavenumber is the inverse of the disk scale height ($k_R = 1/H$). Putting all of these approximations together, we thus obtain the expression

$$\Upsilon = \frac{1}{\sigma_0(r)} \Big[\nabla^2\Phi - k_R^2\Phi + \alpha_0 x \Big] , \tag{3.6}$$

where the parameter $\alpha_0 = \Omega_0(2p-3)/4r$. Notice that in general α_0 can be either positive or negative, depending on whether the surface density $\sigma_0(r) \sim r^{-p}$ decreases faster or slower than the rotation curve $\Omega(r)$. For the particular case of $p = 3/2$, we obtain $\alpha_0 = 0$. Theories of the formation of protostellar disks suggest that disk density profiles will have indices in the range $1-2$ (see, e.g., Cassen & Moosman 1981; Terebey, Shu, & Cassen 1984). Thus, we expect α_0 to be small and we can take $p \approx 3/2$ and hence $\alpha_0 \approx 0$ as a starting approximation.

The solutions for the stream function, the velocity field, and the surface density perturbation are similar to those of the point vortices discussed above. We can find these solutions as follows. We first construct generalized vortensity profiles Υ that satisfy the conservation condition of equation [2.8]. We can then invert the differential operator appearing in equation [3.6] to obtain the stream function Φ. Given the stream function, we can determine the velocity field of the vortex through the relation [3.4]. For example, for a given solution $\Upsilon(\mathbf{x})$ for the vortensity, the solution for the stream function is given by

$$\Phi(\mathbf{x}) = \sigma_0(r) \int d^2\mathbf{x}' G(\mathbf{x}, \mathbf{x}') \Upsilon(\mathbf{x}') , \tag{3.7}$$

where we have taken $\alpha_0 = 0$ and where $G(\mathbf{x}, \mathbf{x}')$ is the Green's function corresponding to the differential operator in equation [3.6]. This operator is simply the modified Helmholtz operator (in two dimensions) and the appropriate Green's function is made up of modified Bessel functions. Many different vortex solutions of this type can be constructed (see Adams & Watkins 1995 for further detail). Here, we only remark that the Rossby wavenumber k_R determines the length scale on which the stream function Φ changes; the velocity field $\mathbf{v}$ changes on this same length scale. This scale $\Lambda \sim 1/k_R$ is comparable to the scale height H in the disk.

4. EFFECTS OF SELF-GRAVITY ON VORTICES

In order to understand the effects of self-gravity on vortices, we proceed in the same fashion as in the previous section but we include the gravitational term. In this case we obtain

$$\Upsilon = \frac{1}{\sigma_0(r)}\left[\nabla^2\Phi - k_R^2\Phi + \frac{2\pi G\rho_1}{\Omega_0}\right], \tag{4.1}$$

where we have again taken $\alpha_0 = 0$. We now make a heuristic approximation to simplify the gravity term,

$$\rho_1 \approx \frac{\sigma_1}{Z} = \frac{\sigma_0}{a^2 Z} h_1 = \frac{2\sigma_0\Omega_0}{a^2 Z}\Phi, \tag{4.2}$$

where Z is an effective total height of the disk. We also define an effective Jeans wavenumber k_J through

$$k_J^2 \equiv 4\pi G\sigma_0/a^2 Z. \tag{4.3}$$

The expression for vortensity becomes

$$\Upsilon = \frac{1}{\sigma_0(r)}\left[\nabla^2\Phi - k_R^2\Phi + k_J^2\Phi\right] = \frac{1}{\sigma_0(r)}\left[\nabla^2\Phi - {k_{\rm eff}}^2\Phi\right], \tag{4.4}$$

where we have defined an effective Rossby wavenumber $k_{\rm eff}$ in the second equality.

We can now directly see the effects of gravity on the structure of the vortex. With no self-gravity, the Rossby wavenumber k_R determines the length scale on which the vortex structure can change. Gravitational effects appear in the equation with the opposite sign; as a result, gravity makes the effective Rossby wavenumber smaller and hence makes the effective Rossby radius larger.

An important crossover point occurs when the gravitational contribution exceeds the (old) Rossby wavenumber contribution and the total effective Rossby radius becomes imaginary. This crossover occurs when $k_R = k_J$, which implies that $4\pi G\sigma_0 = \Omega_0^2 Z$. If we also assume that the total effective height Z of the disk is twice the usual thermal scale height in the disk, i.e., $Z \approx 2a/\Omega_0$, we obtain the crossover condition in the form

$$\frac{\Omega_0 a}{\pi G\sigma_0} = Q_T \approx 2, \tag{4.5}$$

where Q_T is the stability parameter for gaseous disks (Toomre 1964). Here, when Q_T becomes less than 2, the effective Rossby wavenumber becomes imaginary. Notice that we have assumed that the rotation curve is purely Keplerian so that the epicyclic frequency κ is equal to the rotational frequency Ω. The parameter Q_T must be larger than unity in order for the disk to be stable to axisymmetric disturbances. Consideration of non-axisymmetric perturbations in circumstellar disks suggests that the stability parameter must be (at least) as large as $Q_T \sim 2$; otherwise the disks would be highly unstable and would have short lifetimes (e.g., Adams, Ruden, & Shu 1989; Shu et al. 1990; Adams & Benz 1992; Laughlin & Bodenheimer 1994). As a result, many young disk systems tend to live close to this crossover point. As disks evolve in time, they eventually lose mass to the central star, the value of Q_T increases, and the effective Rossby wavenumber $k_{\rm eff}$ approaches the true Rossby wavenumber k_R.

5. PROGRADE/RETROGRADE ASYMMETRY

Within Keplerian circumstellar disks, vortices with positive circulation behave differently than vortices with negative circulation. As we have discussed previously, in the present context, a vortex with a *positive* density perturbation must have a *negative* circulation in order to achieve geostrophic balance. In other words, a vortex with a positive density perturbation must rotate clockwise in the *rotating* frame of reference in which we have been working. Now, suppose a planet (or other secondary body) forms within a vortex. This planet would naively appear to be rotating in a retrograde sense. However, this apparent retrograde rotation (say, at a rate Ω_P) must be corrected for the fact that the frame of reference is rotating as well in the opposite direction (at the local Keplerian rotation rate Ω). As long as the "local" rotation rate Ω_P is smaller than the Keplerian rotation rate Ω, the secondary body will rotate in a prograde sense in the inertial frame of reference. Notice also that if $\Omega_P \geq \Omega$ (which implies an actual retrograde rotation of the planet), then the approximations leading to our equations of motion break down.

Another important difference between vortices with positive and negative circulation is their lifetime and stability. In a Keplerian disk, the mean flow velocity decreases with radial distance to the star. In our local frame of reference centered on the vortex, the mean flow velocity is negative for $x > 0$ and positive for $x < 0$. As a result, vortices with negative (clockwise) circulation rotate "with the flow", whereas vortices with positive (anti-clockwise) circulation must directly oppose the flow; these latter vortices are defined as "adverse". Detailed numerical studies (cf. the review of Marcus 1993) show that adverse vortices are quickly ripped apart into long filaments and fragments, i.e., their lifetimes are very short. On the other hand, vortices of the other type tend to merge and grow into coherent structures. A complete discussion of the lifetime and stability of vortices is beyond the scope of this present work. We stress, however, that the vortices with positive density perturbations (negative circulation) are the ones most likely to live a long time in a Keplerian flow.

6. VORTEX GENERATION

Perhaps the most important unresolved issue is the manner in which these vortices are generated in the first place. Thus far, we have found many different solutions to the equations of fluid dynamics; these solutions represent *possible* behavior of the circumstellar disk. However, before the astrophysical importance of vortices can be established, one must understand the generation mechanism. In particular, the time scales for vortices to grow and decay are important. In this section, we (crudely) estimate a time scale for the vortensity (or vorticity) to change in a circumstellar disk.

One can argue that all differentially rotating fluid systems naturally produce vortical motions and hence vortices must be important at some level. For example, the Earth's atmosphere, the Earth's oceans, and the atmospheres of the giant planets all produce many different kinds of vortices such as those studied here. In addition, many types of vortices can be easily generated in laboratory experiments (Nezlin & Snezhkin 1993). Thus, one might naively expect vortices to arise naturally in circumstellar disks.

One important constraint on vortex generation is provided by Kelvin's circulation theorem (e.g., Ghil & Childress 1987; Shu 1992). This theorem states that for *inviscid barotropic flow*, the number of vortex lines that thread a given area (that moves with the fluid) remains unchanged with time. Thus, for the case of circumstellar disks which are nearly barotropic, vorticity is not automatically "generated" in these systems. In the present context (see equation [2.8]), the *total* vorticity (per unit surface density) is advectively conserved. This total includes both the vorticity of the perturbation and that of the mean flow. Thus, vorticity can be freely exchanged between the mean flow and the perturbation, but cannot be directly generated (or destroyed). We also note that, except under rather special circumstances, the mean flow is not unstable and does not spontaneously transfer its vorticity into perturbations (vortices).

If the flow is not exactly barotropic, then vorticity can be generated directly. Consider, for example, the force equation [2.2] with a standard pressure term of the form $\rho^{-1}\nabla p$. When we take the curl of the force equation to obtain the equation of motion for the vorticity, we obtain a forcing term of the form

$$\mathbf{F}_V = \frac{1}{\rho^2}\nabla\rho \times \nabla p = \frac{R}{\rho}\nabla\rho \times \nabla T\,, \tag{6.1}$$

where we have used the ideal gas law $p = \rho RT$ in obtaining the second equality. Clearly, for a barotropic equation of state $p = p(\rho)$, this new term vanishes. However, this forcing term will be nonzero whenever the surfaces of constant temperature do not line up exactly with the surfaces of constant density. Such a situation can occur in a circumstellar disk. In the absence of disk accretion energy, one important heating source for the disk will be reprocessing of stellar photons; this energy source naturally leads to nearly axisymmetric surfaces of constant temperature. On the other hand, gravitational instabilities naturally produce spiral patterns and hence non-axisymmetric surfaces of constant density.

We now consider the idealized case in which the surfaces of constant temperature are exactly axisymmetric. We will also assume that the disk has strongly growing spiral density perturbations so that the surfaces of constant density are tilted at an angle i with respect to axisymmetric surfaces. Spiral density wave theory in the WKBJ limit (e.g., Shu 1992) implies that the tilt angle is given by

$$i \sim \tan i \sim m/kr\,, \tag{6.2}$$

where m is the azimuthal wavenumber of the perturbation and k is the radial wavenumber. For global modes in circumstellar disks, we expect those with $m = 1$ to be the most important (Adams, Ruden, & Shu 1989;

Laughlin & Bodenheimer 1994). We also expect the radial wavelength to be comparable to the radius and hence $i \sim 1/2\pi$. Given this set of approximations, the generalized equation of motion for the vortensity can be written

$$\mathbf{D}\Upsilon \approx \frac{1}{2\pi\sigma}\frac{R}{\rho}|\nabla\rho|\,|\nabla T| - F_{DIS}\,, \tag{6.3}$$

where we have taken $\sin i \sim i \sim 1/2\pi$. For completeness, we have also included a dissipation term F_{DIS}.

Next, we want to estimate the time scale τ_V for vortensity in the disk to be generated by the effect described above. To make this estimate, we write $\mathbf{D}\Upsilon \sim \omega/\sigma\tau_V$. We write the density gradient term as $|\nabla\rho|/\rho \sim \beta/L$, where β is the amplitude and L is the size scale of the spiral perturbation. Finally, we write the temperature gradient term as $|\nabla T| \sim qT/r$, where q is the power-law index of the temperature profile. Solving for the time scale, we obtain the estimate

$$\tau_V \sim \frac{2\pi}{\beta q}\frac{\omega r L}{a^2}\,, \tag{6.4}$$

where a is the sound speed. Very roughly, we expect $\beta q \sim 1$, $\omega\Lambda \sim a$, and $L \sim \Lambda$, so that the time scale for the vortensity to change is $\tau_V \sim 2\pi r/a \sim 500$ yr. This time scale corresponds to several orbit times and is thus interesting for the evolution of circumstellar disks. We stress, however, that this argument is extremely crude and must be improved with an honest calculation.

7. ASTROPHYSICAL APPLICATIONS

In this section, we discuss applications of vortices to circumstellar disks. These vortices may enhance the formation of giant planets and may provide enhanced energy dissipation for disk accretion.

One of the main difficulties in forming giant planets through the mechanism of gravitational instability is that the planets are highly enriched in heavy elements (see, e.g., DeCampli & Cameron 1979; Stevenson 1982; Cameron 1988). In the solar nebula from which the planets formed, most of the heavy elements are expected to be in the form of dust grains. For most planet formation scenarios, the time scale for the dust grains to migrate to the center of the forming protoplanet is too long to explain the observed enrichment. In addition, a solid rocky core is found at the center of the giant planets; it is difficult for such a core to form at all in most models. However, vortices can, in principle, provide a mechanism for separating dust and gas on a shorter time scale and may provide a means of forming giant planets within the gravitational instability scenario.

In a rotating vortex system, the dust grains are not completely coupled to the gas. The grains do not directly feel the pressure forces, but they are coupled indirectly through drag forces. Following convention, we define a response time τ_d which determines the time scale on which dust grains respond to the force exerted by gas drag. For the regime of parameter space appropriate for circumstellar disks, the response time is given by

$$\tau_d = \frac{\rho_d \langle b\rangle}{2\rho a}\,, \tag{7.1}$$

where $\langle b\rangle$ is the average radius of the dust grains and ρ_d is the grain mass density (see, e.g., Weidenschilling & Cuzzi 1993). For typical conditions ($\rho_d \sim 2$ g/cm^3, $\langle b\rangle \sim 10^{-4}$ cm, $\rho \sim 10^{-12}$ g/cm^3, $a \sim 5 \times 10^4$ cm/s), this time scale is relatively short, $\tau_d \sim 2000$ s. Under these conditions, the dust grains quickly reach their terminal velocity, i.e., the force due to gas drag is balanced by the inward radial force (here due to Coriolis effects). This terminal velocity $v_{\rm term}$ is given by

$$v_{\rm term} = \tau_d \frac{dh_1}{d\varpi}\,. \tag{7.2}$$

In order to estimate the time scale for dust grains to settle to the center of the vortex, we use the point vortex solution found analytically in §3. If we use this solution in equation [7.2] and solve the resulting differential equation, we obtain the following simple expression for the time scale $\tau_{\rm settle}$,

$$\tau_{\rm settle} = \frac{\pi}{2}\frac{\Lambda^2}{\tau_d \Omega_0 |\Gamma|}\,, \tag{7.3}$$

where Λ is the assumed starting radius of the dust grain and is taken to be the vortex size. To obtain a numerical estimate for the time scale, we let $|\Gamma| \sim a\Lambda$ (corresponding to a weakly nonlinear vortex) and we let the vortex size be the inverse of the Rossby wavenumber, i.e., $\Lambda \sim a/\Omega_0$. We thus obtain the dust settling time scale

$$\tau_{\rm settle} = 2 \times 10^8 {\rm yr}\,(\Omega_0\, 100{\rm yr})^{-2}\,. \tag{7.4}$$

This time scale is comparable to that expected for dust settling in a hydrostatically supported protoplanet. Furthermore, this time scale is too long to explain the observed element segregation in the giant planets in our solar system. To overcome this difficulty, either the grain size must be larger or the vortex strength (as measured by the circulation $|\Gamma|$) must be larger than assumed here. For example, suppose we require the dust settling time scale to be $\sim 10^6$ yr, roughly two orders of magnitude shorter than given above. This time requirement can be met provided that mean dust radius $\langle b \rangle$ and the circulation Γ satisfy the following constraint:

$$\frac{\langle b \rangle}{1\mu\mathrm{m}} \frac{|\Gamma|}{a/k_R} \geq 100 \,. \tag{7.5}$$

Since both of the above ratios are expected to be of order unity, we find that sufficient dust segregation to form giant planets requires somewhat extreme conditions. However, in principle, both the circulation and the grain size *can be* large and hence giant planet formation is at least *possible* in this scenario.

The above simple calculation can be generalized to obtain a more accurate description of dust settling in vortices. Recent papers (Tanga et al. 1995; Barge & Sommeria 1995) have performed numerical integrations of the equations of motion for dust particles in vortices and confirm that dust particles concentrate inside vortices on a relatively short time scale.

Finally, we note that vortices enhance the formation of giant planets via gravitational instability by increasing the local surface density. The perturbation requirements to form a secondary body in a circumstellar disk can be written in fairly general form (Adams & Watkins 1995). The basic result is that a moderately large perturbation, with density contrast $\beta = \Delta\sigma/\sigma \sim$3–5, is required in order to form a secondary body.

We now briefly discuss how vortices can affect energy dissipation and hence accretion in circumstellar disks. The overall effect of vortices is to provide additional avenues for energy dissipation in disks. We can identify three conceptually different ways for vortices to participate in energy dissipation: (1) annihilation of vortices, (2) energy dissipation within a single vortex, and (3) a cascade of vortices. However, we focus here on the second mechanism (see Adams & Watkins 1995 for further discussion).

In order to consider the dissipation of energy within a single vortex, we think of the vortex as a small scale analog of the circumstellar disk itself. As shown from our solutions to the equations of motion in the previous sections, the vortices are strongly differentially rotating. Now suppose the fluid has a viscosity ν, which we parameterize in the usual way according to

$$\nu = \frac{2}{3}\alpha a H \sim \alpha a \Lambda \,, \tag{7.6}$$

where Λ is the characteristic vortex size and α is the usual dimensionless "α parameter" (see, e.g., Pringle 1981). The viscous diffusion time scale τ_D is then given by

$$\tau_D \sim \frac{\Lambda^2}{\nu} \sim \frac{\Lambda/a}{\alpha} \sim \frac{1}{\Omega\alpha} \sim \frac{6\,\mathrm{yr}}{\alpha} \,, \tag{7.7}$$

where we have taken typical values to obtain the numerical estimate. Many studies of viscous disks suggest that $\alpha \sim 10^{-2} - 10^{-4}$ (e.g., Lin & Papaloizou 1980; Lin 1981), which implies a vortex dissipation time scale of $\tau_D \sim 6 \times 10^2 - 6 \times 10^4$ yr.

Because of the smaller size scale of the vortex, the energy dissipation rate is enhanced over the accretion time scale of the disk as a whole by a factor f given by

$$f \sim \left(\Lambda/r\right)^2 \sim 100 \,. \tag{7.8}$$

Thus, for a given fluid viscosity, vortices are more efficient at dissipating energy than the disk as a whole. As a result, vortices can play an important role in energy dissipation and hence disk accretion if a mechanism exists to efficiently transfer energy from the mean flow into the vortex (see §6).

8. SUMMARY OF RESULTS

In this work, we have begun a study of vortices in circumstellar disks. These vortices are basically storm systems in a generalized geostrophic balance, i.e., a balance between pressure forces, gravitational forces, and the Coriolis force. We have obtained a number of basic results concerning the physical properties of vortices in circumstellar disks. Notice that some of the results given below are taken from Adams & Watkins (1995) and are only summarized here:

[1] Many different types of vortex solutions are possible in circumstellar disks. In other words, circumstellar disks can exhibit many different types of vortical motions.

[2] Point vortices are the simplest type of vortex. Their properties can be found analytically and hence these vortices provide a prototype for understanding the basic physics of these systems.

[3] In the absence of self-gravity, the size scale Λ for this type of vortex is determined by the inverse of the Rossby wavenumber, i.e., $\Lambda = a/\Omega$. For circumstellar disks, this size scale is comparable to the thermal scale height H in the disk.

[4] The leading order effect of self-gravity is to make the effective Rossby wavenumber smaller. This effect, in turn, makes the size of the vortex larger than in the case without gravity.

[5] Nonlinear effects can lead to qualitatively new behavior. In particular, under special circumstances, solitary wave solutions can arise.

[6] Magnetic fields can have two different effects on the evolution of vortices. The fields exert a force on the fluid and hence add an additional force term to the equation of motion. For the simplest case of a magnetic field in the vertical ($\hat{z}$) direction, this force corresponds to a magnetic pressure only. The second effect is to allow for ohmic dissipation; for expected parameters in circumstellar disks, however, this effect is small.

[7] Circumstellar disks can, in principle, generate vorticity (or vortensity) through baroclinic effects. These effects arise whenever the surfaces of constant density do not line up with the surfaces of constant temperature. Disks can naturally obtain nearly axisymmetric temperature profiles (from reprocessing of stellar photons) and non-axisymmetric density profiles (from self-gravitating spiral modes). The estimated time scale for the vortensity to change is $\sim$500 yr.

[8] Vortices can enhance the formation of giant planets through the mechanism of gravitational instability. In particular, dust grains settle to the center of these vortices. In order for the time scale for dust settling to be interesting for planet formation, either the dust grains must be quite large ($\langle b \rangle \gg 1\ \mu$m) and/or the vortex circulation must be very strong ($|\Gamma| \gg aH$). Vortices also enhance the formation of giant planets by gravitational instability by increasing the local surface density of the disk.

[9] We have derived a general argument which illustrates the conditions under which secondary bodies can form in circumstellar disks. For moderately massive disks associated with young stellar objects, a moderately large perturbation amplitude $\beta = \Delta\sigma/\sigma \sim 3$–5 is required to form a secondary.

[10] Planets formed in vortices should generally rotate in a prograde sense with respect to their orbits. In order to produce a planet with retrograde rotation, the circulation of the vortex must be sufficiently strong that the approximations used in this paper break down.

[11] Vortices can enhance the dissipation of energy in circumstellar disks and can thereby help disk accretion. For a given fluid viscosity, vortices dissipate energy faster than the disk as a whole. The physical reason for this enhancement is that the differential rotation in the vortex is strong and the size scale is short compared to the entire disk. Thus, vortices can enhance disk accretion, provided that some mechanism exists to transfer energy from the mean circumstellar flow into the vortices.

9. DISCUSSION

In this work, we have suggested that vortices may be important for both planet formation and for disk accretion. An analogous dual role has been claimed previously for gravitational instabilities in disks (e.g., Adams, Ruden, & Shu 1989), although the secondary bodies are usually considered to be "binary companions" for this latter mechanism. In either case, it might seem paradoxical that one physical process can produce two very different results – namely secondary bodies and disk accretion. However, this apparent contradiction can be resolved for both mechanisms as follows: Energy dissipation and hence disk accretion arises for "failed" structures, whereas secondary bodies can form only within "successful" structures.

We first consider energy dissipation and disk accretion. Self-gravitating spiral modes lead to disk accretion when they saturate at a fairly *low* amplitudes (Laughlin & Bodenheimer 1994), i.e., when the instabilities "fail" to achieve strong nonlinear growth. Similarly, vortices can lead to energy dissipation and hence help disk accretion when they dissipate their energy faster than they can grow, i.e., when they also "fail" to achieve strong nonlinear growth.

On the other hand, secondary bodies can be produced by either mechanism for the case of "successful" structures. Self-gravitating spiral modes can collapse to form secondary bodies when they successfully grow well into the nonlinear regime; a density contrast $\beta \sim 3 - 5$ is required (Adams & Watkins 1995; see also Bodenheimer & Laughlin 1995 in these Proceedings). Vortices can lead to giant planet formation if they are sufficiently long-lived (so that dust grains collect at the vortex center) and/or highly nonlinear (so that they must collapse gravitationally), i.e., if the vortices are "successful".

This work represents a preliminary step toward understanding the physics of vortices in circumstellar disks and related astrophysical systems. Many directions for future work remain. The most important unresolved issue is the mechanism which generates vortices. Another important related issue is the stability and lifetimes of vortices. Finally, we note that the overall goal of this work is to understand the dynamics and evolution of circumstellar disks as a whole. Vortices play only a partial role in this process; thus, one challenge for the future is to integrate vortices into the overall picture of disk evolution.

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