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THE STRUCTURE OF COLLAPSED OBJECTS AND THE EQUATION OF STATE OF MATTER AT HIGH DENSITIES

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RESUMEN

Se hace un breve resumen de los últimos desarrollos en el estudio de las propiedades de la materia a densidades y temperaturas extremas ($\rho \geq 10^{11}$ g cm^{-3}, $0 \leq T \lesssim 60$ MeV). Se discute la posibilidad de imponer cotas a las propiedades de la materia mediante el uso de *tests* basados en datos astrofísicos: masas límites, enfriamiento, emisión de neutrinos, glitches, etc. Los resultados de estos *tests* hasta el presente parecen indicar que aún se está lejos de tener un conocimiento firme sobre este problema.

ABSTRACT

A brief abstract of the latest developments in the field of the properties of matter at extreme densities and temperatures ($\rho \geq 10^{11}$ g cm^{-3}, $0 \leq T \lesssim 60$ MeV) is presented. The possibility of constraining these properties by means of astrophysical tests: limiting masses, cooling, neutrino emission, glitches, etc. is discussed. The results of such tests seems to indicate that we are still far from having a firm knowledge about this problem.

Key words: **DENSE MATTER — EQUATION OF STATE — STARS–NEUTRON**

1. INTRODUCTION

The equation of state (EOS) of matter at high densities and temperatures is a decisive ingredient in our understanding of many astrophysical problems related to the behavior of neutron stars. Since the onset of an accretion induced collapse or in a core collapse supernova (SN), matter reaches densities from 10^{-3} to few times the nuclear matter saturation (ρ_{sat}) one ($\rho_{sat} = 2.7 \times 10^{14}$ g cm^{-3}) and temperatures from 0 up to ≈ 60 MeV (1 MeV $= 1.1604 \times 10^{10}$ K). Among the astrophysical phenomena involving the high density EOS are the dynamics and explosion mechanism of SN and its neutrino emission, the structure, formation, and evolution of neutron stars, the accretion induced collapse of white dwarfs, some properties of pulsars, gamma ray bursts, etc.

The problem of computing the high density EOS is not a trivial one, because we need to compute the minimum free energy configuration for a mixture of protons, neutrons, electrons and neutrinos, taking into account the formation of nuclei, and at densities above ρ_{sat} the appearance of heavier baryons like hyperons. Also, at high enough densities, the phase transition from nuclear matter into quark plasma may occur, which would have important effects in all the above mentioned astrophysical phenomena. The problem is clearly formidable, and we shall not review it in full detail because of lack of space. We shall review the most relevant recent developments in this field paying special attention to the astrophysical test to constraint the properties of matter at such extreme conditions. For a detailed discussion of the underlying physics see e.g., Vautherin (1991).

2. THE.EQUATION OF STATE

2.1. At Subnuclear Densities

In conditions of $\rho \leq \rho_{sat}$, we have to deal with a mixture of protons, neutrons, electrons and neutrinos. At temperatures $T \lesssim 15$ MeV most of protons and some neutrons arrange in heavy nuclei. The problem has been studied by many authors (see Bethe 1990 and references therein). If we assume that the timescale of evolution

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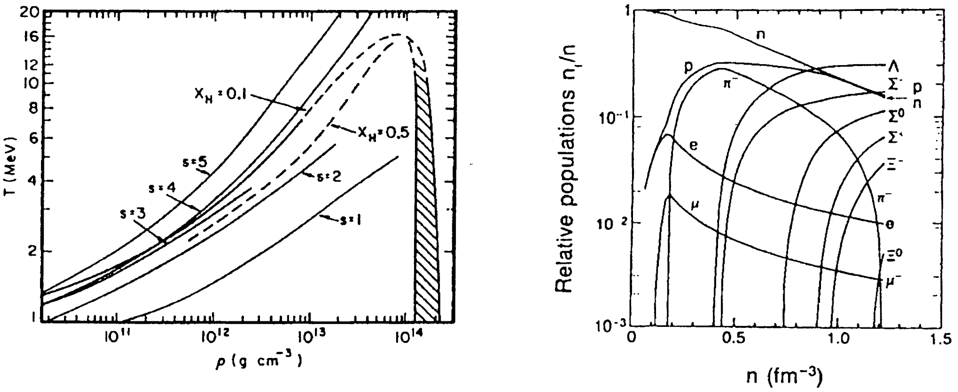

Fig. 1. a) The phase diagram for nuclear matter as a function of ρ and T, the adiabats of 1 to 5 *k/baryon* are shown. Dashed lines are where 10% or 50% of matter is in heavy nuclei. Inside the shaded area there appear "bubbles" and at higher densities uniform nuclear matter (taken from Lamb et al. 1978, with permission). b) The relative population of particles as a function of the total baryon density as calculated by Glendenning (1985) in the mean field approximation (with permission.)

of the object under consideration is lower than the neutrino diffusion one (as in SN cores), it can be assumed that the mixture does not reach β- equilibrium. We show, in Fig. 1a the phase diagram for such conditions as calculated by Lamb et al. (1978)

2.2. ABOVE NUCLEAR DENSITY

At higher densities $\rho > \rho_{sat}$, the problem becomes more complicate because the chemical potential of the particles is so high that it is energetically allowed for heavier particles to appear. Such particles ($\mu, \Delta, \Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Theta^0, \Theta^-, \pi^-$, and $K^{\pm}$) are unstable at laboratory conditions but not at such high densities because they are Fermions and thus obey Pauli blocking which prevents them to decay back.

The first work considering the formation of hyperons in nuclear matter was published long ago (Ambartsumyan & Saakyan 1960). However, there are in the literature many EOSs which do not consider such particles and concentrated on the problem of a *neutron EOS* (see Shapiro & Teukolsky 1983 and references therein). Recently, it was shown that even in the case of knowing the neutron EOS with no uncertainty, the formation of hyperons would make the EOS much softer (due that the nucleons at the top of the Fermi are the ones that decay to form these particles) and uncertain (Kapusta & Olive 1989). The uncertainties are mainly due to the lack of knowledge of particle-particle interaction potentials. More recently, it was proposed the possibility that kaons form a condensate making the EOS even softer (Brown & Bethe 1994). A typical solution for this problem is presented in Fig. 1b where the abundance of the different particles at a given baryonic number at zero temperature as calculated by Glendenning (1985) is shown.

3. THE ASTROPHYSICAL TESTS OF THE EQUATION OF STATE

One of the most dramatic problems in the study of the EOS at high densities is the almost impossibility of testing it in laboratory. At high energy ion collisions, temperature is so high that it is very hard to extract useful information on the astrophysical conditions we are interested in. Thus, one is forced to employ astrophysical objects to test such EOS. From here on, we shall discuss the most important astrophysical tests to the EOS.

3.1. The Maximum Mass of Neutron Stars

One of the most obvious ways to impose constraints to the EOS is using the data compiled on the masses of neutron stars. Since sometime ago, neutron star masses have been measured with different (sometimes very high, see Fig. 2) degrees of accuracy depending upon the type of object in which a neutron star is present.

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From the theoretical point of view, for a given EOS, we may integrate the Oppenheimer - Volkoff equations of Relativistic stellar structure at zero temperature

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2}\frac{(1+\frac{P}{\rho\, c^2})(1+\frac{4\,\pi P r^3}{M_r c^2})}{(1-\frac{2GM_r}{r\, c^2})}\,; \qquad \frac{dM_r}{dr} = 4\pi r^2\rho\;, \qquad (1)$$

(where the symbols have their usual meaning). The central pressure is a free parameter and the surface is where P vanishes. In such way, a sequence of models can be constructed. For any EOS, the sequence will have a maximum mass model (like in the case of white dwarfs, but here the exact value is essentially due to the repulsive nucleon-nucleon interactions and not only to the exclusion principle). Such maximum mass model must have at least a mass of 1.44 $M_\odot$ (the mass of PSR1913+16 which is known with very little uncertainty) for the EOS not to be rejected. In Figure 2 we show the measured values of neutron objects together with their respective error bars. It is clear that neutron stars with masses of at least 1.44 $M_\odot$ exist, however, from the available observational data it is not clear which is the maximum allowed mass value.

One intriguing problem is the lack of a neutron object detected in SN 1987A. Such object *did* form as indicated by the epochal neutrino detections of Bionta et al. (1987) and Kirata et al. (1987). Most of researchers agree that the energy emitted in such way (99% of the total) was $\approx 2 \times 10^{53}$ erg and should be approximately the binding energy of the just formed object (i.e., a *proto* - neutron star). Many EOSs allow for objects with such a binding and so, this datum is not enough for selecting an EOS.

The lack a "PSR 1987A" has induced Bethe & Brown (1995) to speculate about the possible collapse of such object to a Black Hole (BH) (see also Glendenning 1995). Employing numerical models of the SN1987A explosion they concluded that considering the "fall back" after bounce, the neutron object should have 1.56 $M_\odot$ and this should be approximately the actual M_{max} for neutron objects. If so, stars more massive than $\approx 18\ M_\odot$ should collapse to BH without contributing to the galactic nucleosynthesis. This would be helpful in getting agreement between the observed and calculated oxygen abundances. Nevertheless for this to be tenable we must be able to account for the *very long* duration of the neutrino detections of SN 1987A ($\approx$ 13 s for the Kamiokande detector). This should be the time step between the bounce and BH subsidence. The numerical models of proto - neutron star evolution performed by Keil & Janka (1995) show us this not to be easy, and suggest that something unexpected may occur especially at later times.

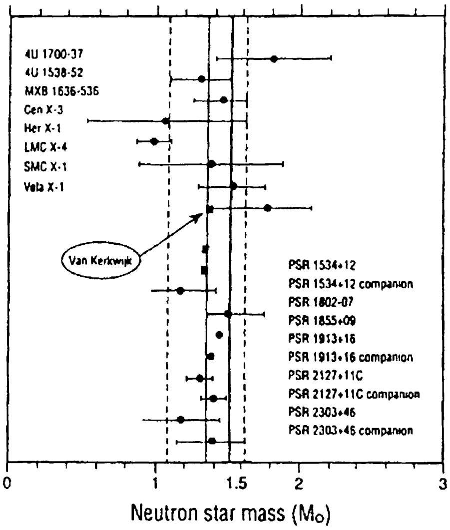

Fig. 2. The masses of measured neutron stars with their corresponding error bars. Note that the upper mass limit is rather uncertain. Taken from Bethe & Brown (1995) with permission.

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3.2. The Cooling of Neutron Stars

Just after the formation of a neutron star, it has a very large thermal content to be released. Such process is the cooling of neutron stars which has been considered by many researchers as one of the most powerful tools in extracting information about the EOS.

For this purpose we need to compute the evolution of such objects, which is possible only if we know many properties of the stellar matter at such conditions i.e., the EOS, neutrino emissivity, thermal conductivity, etc. We may calculate curves of e.g., surface temperature versus time, then if we know the temperature and age of some objects, we may select between a set of EOSs.

This is a very attractive proposal. However, unfortunately, at present we are faced to a very large uncertainty on the neutrino emissivity (which represents the main cooling agent at early stages of evolution) of the matter that may prevent us to get useful conclusions. Such uncertainty is that, at present, we do not know if the direct URCA process $n \rightarrow p + e^- + \bar{\nu_e}$; $p + e^+ \rightarrow n + \nu_e$ is possible (Lattimer et al. 1991) in neutron stars, or if, as believed up to some year ago, only the modified URCA $n + n \rightarrow n + p + e^- + \bar{\nu_e}$; $p + n + e^- \rightarrow n + n + \nu_e$ occurs. This is related to our uncertainty on the abundance of protons per baryon X_p in dense matter, a quantity dependent upon the EOS. Direct URCA is possible only if $X_p \geq 1/9$ (Pethick, 1991). It is important to note that many recent nuclear EOSs allow for the occurrence of direct URCA reactions (see Lattimer et al. 1991 for details)

Also there is the possibility that the condition upon X_p for direct URCA being fulfilled from a critical density ρ_{URCA} on. If so, it will allow for stars having a central density $\rho_c \geq \rho_{URCA}$, to cool down faster than the lower mass objects which will undergo the modified URCA process. Page & Applegate (1992) computed the cooling of neutron stars under the hypothesis that $\rho_{URCA} = \rho_c$ for an object of 1.35 $M_\odot$, their results are shown in Fig. 3a for the case of no superfluidity and in Fig. 3b for neutron superfluidity for different values of the energy gap. It is to be noted that these objects have a magnetosphere which in principle may "contaminate" the (X-ray) observations. This effect may be so important that we may not be observing temperatures due to the cooling process itself but to the magnetospheric emission (Ögelman 1993). At present it seems hard to extract some conclusion about the EOS coming from cooling studies at its present status. Much theoretical and observational work is required to overcome this situation.

3.3. Glitches in Pulsars

It is known from long ago that some pulsars suddenly spin up ("glitch"). This phenomenon was detected in many pulsars all of which are *not* millisecond ones (i.e., with periods $\geq$ 10 ms). It is currently accepted that

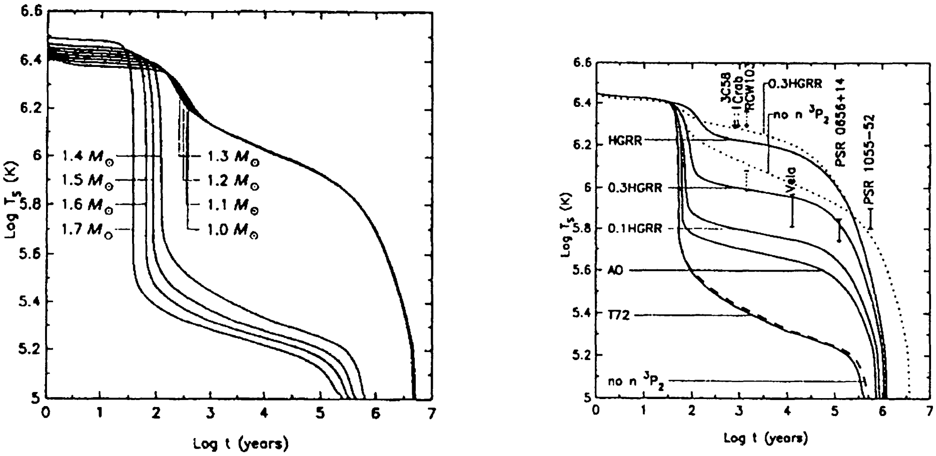

Fig. 3. a) The cooling of neutron stars of different masses with and without the direct URCA process. The critical mass value for direct URCA reactions was assumed to be 1.35 $M_\odot$. b) The cooling of a 1.4 $M_\odot$ neutron star for different values of the superfluid gap. The smaller the gap the smaller the differences compared to the case without superfluid, the temperature of some observed objects is also included. Taken from Page & Applegate (1992) with permission.

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glitches are related to the superfluid layer that should exist in neutron stars at densities $\rho \leq \rho_{sat}$. Superfluids cannot rotate as whole but mimic it by means of a vortex array. Such array has an equilibrium configuration corresponding to a given rotation rate Ω. This vortex array pins to crust nuclei with a finite energy, then as Ω decreases (due to the electromagnetic torque consequence of the magnetospheric emission), the array is not in equilibrium but departs from it more and more. When such departure is large enough, the vortex array decouples and pins again at a configuration corresponding to the actual Ω. The recoupling process is associated with an exponential decay in Ω with a timescale of tens of days.

From observations of the Vela pulsar, it is known that the fraction of moment of inertia in the superfluid dripped layer $\alpha = I_{sf}/I$ must be at least of $\alpha = 0.034$ (Datta & Alpar 1993). This fact may be employed to constraint the EOS because a neutron star should have at least such value of α in order to glitch the observed way.

The theoretical values of α may be calculated with a technique similar to the discussed in §3.1. α is dependent to the total stellar mass, then if we assume $M = 1.4\ M_{\odot}$, we can compare to Vela pulsar data. The method just described was applied to select EOSs from a large sample by Link, Epstein, & Van Riper (1992) and Datta & Alpar (1993) who conclude that some hyperonic soft EOSs may be ruled out with this procedure.

3.4. The Rotation of Neutron Stars

If we identify pulsars with neutron stars, pulsar rotation may be employed to constraint the EOS. Pulsars rotate with periods from 4 s to 1.56 ms (corresponding to PSR 1937+214). The second fastest pulsar (PSR 1957+20) has a period only 3% larger. This fact induced Friedman et al. (1988) to suggest this should not be accidental but an indication that this should be the fastest possible rotation rate. They solved the Genaral Relativistic equations for rotating objects and found a set of sequences. If we assume 1.56 ms to be the shortest possible period for a 1.4 $M_{\odot}$ object, it clearly favours stiff[2] EOSs. This is simple to understand: the stiffer the EOS, for a fixed mass, the larger the object, and thus, for a given rotation rate, the larger the centrifugal force. This method for studying the EOS will be more powerful if some faster pulsars (if they exist) are discovered.

3.5. The Vibration of Neutron Stars

Another method to extract information about the EOS may be to study the pulsation of neutron stars. In the case of variable white dwarf stars, data on their light curves has been successfully employed to unveil their outer layers structure. In the case of pulsars this should be much more difficult, simply because we should be able to observe variability during the duty cycle, which is often very short. It imposes a cut in the spectrum of vibrations we may observe. Some microstructure in the profile of the emission of some pulsars has been proposed to be due to the vibration of the underlying neutron object. If so, we may expect this method to be useful. However, it has also been proposed that this may be due to oscillations in the magnetospheric plasma. In any case it is clear that observational data should be extremely hard to get.

Evidence of submillisecond structure has been reported by Bath et al. (1992) in a γ ray burst with a timescale of $\tau \leq 0.20$ ms and Finley, Ögelman, & Kiziloglu (1992) reported a pulsation period of $\tau \sim 0.385$ s in PSR 0656+14.

Theoretically, we can solve the relativistic equations for different pulsation modes. The radial and dipolar modes do not produce any gravitational wave radiation and because of this reason are the most probable to appear. Väth & Chanmugam 1992 have calculated the radial pulsation modes for a set of EOSs. As in the case of employing the SN1987A neutrino detections, most of the EOSs account for the above quoted observations (if are assumed to be due to neutron star oscillations). Consequently, at present this method is not powerful enough for selecting EOSs.

4. EXOTIC POSSIBILITIES

It is quite important for our discussion if there is a phase transition to quark - gluon plasma at densities lower than the corresponding to the centre of a maximum mass neutron star.

Once (if) formed the quark plasma it decays producing strange quarks by means of the weak decay $ud \rightarrow us$ to form a uds plasma of almost equal abundances in each flavour. Witten (1984) conjectured that such plasma

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[2] An EOS is considered stiff (soft) if for a given pressure exerts a big (small) pressure.

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(Strange Matter, SM) may have lower free energy than normal nuclear matter. This was reinforced by the calculation of Farhi & Jaffe (1984) who shown the existence of a wide window in the space of parameters space of the EOS inside which Witten's conjecture is indeed correct. Thus, matter inside neutron stars should be metastable against decaying to SM forming the so-called strange stars (e.g., Benvenuto, Horvath, & Vucetich 1991)[3]. The decay is mediated by a combustion process that should proceed as a detonation (Lugones, Benvenuto, & Vucetich 1994). This may be decisive in the mechanism of SN (Benvenuto & Horvath 1989). The argument is simple: let us assume that the conversion to SM releases $\sim 20\ MeV/baryon$ (Farhi & Jaffe 1984). The deconfined mass should be about one Chandrasekhar Mass $\sim 2 \times 10^{57}$ baryon. Thus, it releases $\sim 6 \times 10^{52}$ erg, more than enough to prompt a SN event.

Another interesting consequence of the SM hypothesis is that, because at zero pressure SM has a density of $\sim 2\ \rho_{sat}$, it allows for the existence of Strange Dwarf Stars. These objects have a SM core and an extended normal matter envelope with densities below the one of neutron drip. These objects have a mass-radius relation which has no discontinuity between white dwarf-like and strange stars-like objects, in clear contrast with the standard picture (Glendenning, Kettner, & Weber 1995). The objects that resemble white dwarfs have an evolution quite similar to the corresponding to white dwarfs, but if they suffer enough accretion they should explode as dim SN Ia with arbitrary mass values (Benvenuto & Althaus 1996).

5. CONCLUSIONS

We have briefly discussed the main recent results on the equation of state of dense matter and mainly concentrated on the different astrophysical tests performed in order to constraint their properties. Among these main tests, we have reviewed the employment of data on the maximum mass of neutron stars, cooling, glitches in pulsars, rotation and vibrational properties together with the difficulties inherent of each method.

A careful examination of the above discussion shows us that we are still far from having an accurate knowledge of the EOS. From one side the masses of compact stars and the lack of a "PSR 1987A" has been proposed to favour soft a EOS. From other methods like glitches and rotation some authors favoured stiff a EOS. Moreover, the uncertainty on the value of X_p does not allow us to be sure if direct URCA occurs in neutron stars or not and what is the role of the magnetosphere emission in the measured temperatures. It seems that much work remains to be done before a coherent description of EOS become available.

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[3] Some caveats (e.g., the impossibility of a strange star to glitch) have been posed on this possibility but all of them depend on the lack of low density bound states of SM (something like strange nuclei). However, it has not been proven this to be the actual case and so, these arguments should be taken with care (see Benvenuto et al. 1991).

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