

### COMPARATIVE STUDY OF SYMPLECTIC MAPPINGS IN RESONANT SYSTEMS

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In this work, three kinds of mappings are compared. This maps are applicable to the averaged differential equations system corresponding to the 3/1 asteroidal resonance, and they are: Wisdom's map (1982), Hadjidemetriou's implicit scheme (1991), and an explicit map (equivalent to the previous one) based on a Lie generating function. The hamiltonian model used is that of Henrard & Caranicolas (1990).

In order to compare the maps, they are developed in power series of the perturbing mass  $\mu$ . Wisdom's map is expanded in Taylor series, while Hadjidemetriou's map is explicitated through the use of Lagrange series. The last map is constructed as a Lie series in the small parameter  $\mu$ . The comparison between these developments, shows that they are equal up to the first order in  $\mu$ . The differences that appear beyond the first order are related to two facts: first, Lagrange series are not identical to Lie series, except at the first order; second, Hadjidemetriou's map considers just the unperturbed hamiltonian in the integrable part of the problem, while Wisdom's map includes also the secular terms of the hamiltonian in the integrable part.

These differences up to first order are not so serious because we are using an averaged hamiltonian model which is valid only to first order in  $\mu$ . Besides, this hamiltonian is expanded only to the second order in Jupiter's eccentricity, and  $\mathcal{O}(e_J^2) \approx \mathcal{O}(\mu)$ . Comparison between the numerical results obtained by Wisdom and Hadjidemetriou respectively, shows that they are very similar. So we conclude that the solutions are conditioned just by the model and not by the kind of map that is used.

- Hadjidemetriou, J.D. 1991, in *Predictability, Stability and Chaos*, ed. A.E. Roy, (Plenum Press), p. 157-175  
 Henrard, J., & Caranicolas, N.D. 1990, *Cel. Mech. Dyn. Astr.*, 47, 99-121  
 Wisdom, J. 1982, *AJ*87 (3), 577-593

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### TIME-FREQUENCY ANALYSIS FOR RESONANT MOTION AND RESULTS FOR PLUTO

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For a particle in a mean-motion resonant orbit perturbed by several planets, the librational motion is not defined by a unique frequency but by a set of frequencies which may be undistinguishable in the case of a chaotic evolution. In order to determine the behavior of the librational motion, we propose to analyze the time evolution of spectra obtained on short time spans. We propose a time frequency analysis model specially devised for the determination of low frequencies. The most popular methods, the Fourier and wavelets transforms, are inefficient in low frequencies where the leakage due to the constant term of the time series modifies the spectra. There is another class of methods without these inconveniences based on the fit of a general model to the data. We adapt one of these methods (Ferraz-Mello 1981, *AJ*, 86, 619 and Foster 1995, *AJ*, 109, 1889) to the case of numerical integrations and use it for the analysis of the dynamical behavior of resonant particles. In this method, we don't need to calculate numerically continuous integrals, eliminating another source of errors. The time evolution of the instantaneous librational motion allows us to see if the motion has secular or chaotic trends. In the case of Pluto, we have found a small chaotic component in the time evolution of the libration amplitude.

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### DYNAMICS OF THE 2:3 EXTERIOR RESONANCE WITH NEPTUNE

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We have studied the dynamical behavior of the exterior 2:3 mean-motion resonance with Neptune using two main tools. First, in the frame of the planar restricted three-body problem, with an elliptic orbit for Neptune, we numerically integrate the exact equations of the motion of particles located in the resonance. Short-period terms are eliminated by

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