

## COMPARATIVE STUDY OF SYMPLECTIC MAPPINGS IN RESONANT SYSTEMS

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In this work, three kinds of mappings are compared. This maps are applicable to the averaged differential equations system corresponding to the 3/1 asteroidal resonance, and they are: Wisdom's map (1982), Hadjidemetriou's implicit scheme (1991), and an explicit map (equivalent to the previous one) based on a Lie generating function. The hamiltonian model used is that of Henrard & Caranicolas (1990).

In order to compare the maps, they are developed in power series of the perturbing mass  $\mu$ . Wisdom's map is expanded in Taylor series, while Hadjidemetriou's map is explicitated through the use of Lagrange series. The last map is constructed as a Lie series in the small parameter  $\mu$ . The comparison between these developments, shows that they are equal up to the first order in  $\mu$ . The differences that appear beyond the first order are related to two facts: first, Lagrange series are not identical to Lie series, except at the first order; second, Hadjidemetriou's map considers just the unperturbed hamiltonian in the integrable part of the problem, while Wisdom's map includes also the secular terms of the hamiltonian in the integrable part.

These differences up to first order are not so serious because we are using an averaged hamiltonian model which is valid only to first order in  $\mu$ . Besides, this hamiltonian is expanded only to the second order in Jupiter's eccentricity, and  $\mathcal{O}(e_J^2) \approx \mathcal{O}(\mu)$ . Comparison between the numerical results obtained by Wisdom and Hadjidemetriou respectively, shows that they are very similar. So we conclude that the solutions are conditioned just by the model and not by the kind of map that is used.

- Hadjidemetriou, J.D. 1991, in *Predictability, Stability and Chaos*, ed. A.E. Roy, (Plenum Press), p. 157-175  
 Henrard, J., & Caranicolas, N.D. 1990, *Cel. Mech. Dyn. Astr.*, 47, 99-121  
 Wisdom, J. 1982, *AJ*87 (3), 577-593

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## TIME-FREQUENCY ANALYSIS FOR RESONANT MOTION AND RESULTS FOR PLUTO

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For a particle in a mean-motion resonant orbit perturbed by several planets, the librational motion is not defined by a unique frequency but by a set of frequencies which may be undistinguishable in the case of a chaotic evolution. In order to determine the behavior of the librational motion, we propose to analyze the time evolution of spectra obtained on short time spans. We propose a time frequency analysis model specially devised for the determination of low frequencies. The most popular methods, the Fourier and wavelets transforms, are inefficient in low frequencies where the leakage due to the constant term of the time series modifies the spectra. There is another class of methods without these inconveniences based on the fit of a general model to the data. We adapt one of these methods (Ferraz-Mello 1981, *AJ*, 86, 619 and Foster 1995, *AJ*, 109, 1889) to the case of numerical integrations and use it for the analysis of the dynamical behavior of resonant particles. In this method, we don't need to calculate numerically continuous integrals, eliminating another source of errors. The time evolution of the instantaneous librational motion allows us to see if the motion has secular or chaotic trends. In the case of Pluto, we have found a small chaotic component in the time evolution of the libration amplitude.

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## DYNAMICS OF THE 2:3 EXTERIOR RESONANCE WITH NEPTUNE

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We have studied the dynamical behavior of the exterior 2:3 mean-motion resonance with Neptune using two main tools. First, in the frame of the planar restricted three-body problem, with an elliptic orbit for Neptune, we numerically integrate the exact equations of the motion of particles located in the resonance. Short-period terms are eliminated by

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digital filtering. Assuming that the resulting solution corresponds to a system with two degrees of freedom, surface of sections are obtained for different values of the averaged energy. These surfaces show a very regular behavior without traces of diffusion in a time span of  $10^7$  years. For small amplitude librations, it is possible to obtain regular orbits up to  $e \sim 0.7$ . Second, we numerically integrate the equations of motion of particles including all outer planets, the spatial case. In this case, there is a drastic reduction in the space of initial conditions leading to stable librations. The maximum eccentricity allowed by the perturbations due to Uranus is  $e \sim 0.37$ . At small eccentricities, the evolution is strongly related to the law of structure. The positive branch is stable and the negative is unstable due to the effect of the motion of  $\varpi - \varpi_N$  and the overlap of the secular resonances  $\nu_8$  and  $\nu_{18}$ . This explain the different behavior found by several authors for particles evolving at approximately  $a \sim 40$  AU. Particles with initial libration centers located at the negative branch of the law of structure can be temporarily captured in  $\sigma$ -libration with Pluto-like values of  $e$  and  $i$ , but these orbits are not stable.

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#### THE NUMBER OF OBJECTS IN THE ORIGINAL ASTEROID BELT

Ricardo Gil-Hutton<sup>1,2</sup>

Modeling the collisional evolution of Ceres and Vesta we found that a moderate mass depletion by a factor of 6 to 7 occurred in the asteroid zone after the end of the accretion phase. In addition, the model provides a mean value of 0.25 to 0.35 for the fraction of leftover impact energy available for further mechanical work after a collision and, running the model for asteroids larger than 200 km, it provides a mean angular momentum at the end of the accretion period of  $0.108 \pm 0.044$  ( $\text{GM}^3\text{R}$ )<sup>1/2</sup> which agrees very well with low collisional evolution objects orbiting in the depleted region at about 4.3 AU from the Sun.

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#### PHOTOMETRY OF FAMILY ASTEROIDS

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The evolution of the asteroid belt is related to the process of minor planet family creation by collisions. The study of the rotational properties of their members (rotation periods, orientation of spin axis, precession periods, etc.) allows the modeling of the creation process and its evolution. In this paper, we present photometry of asteroids 558 Carmen, 613 Ginevra and 1124 Stroobantia obtained during the last years at Estación Astronómica Dr. Carlos Ulrrico Cesco and CASLEO, San Juan, Argentina. The rotational periods deduced were  $9.264 \pm 0.005$  hours for Carmen,  $16.447 \pm 0.009$  hours for Ginevra and  $16.393 \pm 0.006$  hours for Stroobantia.

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#### PHOTOMETRIC OBSERVATIONS OF (2060) CHIRON

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The comet (2060) Chiron, a Centaur Object, has unique characteristics: activity as far as 17 AU from the Sun, an estimated diameter of nearly 200 km, and a chaotic orbit. Due to these characteristics and to the fact that in February 1996 it will reach perihelion; a systematic photometric monitoring of (2060) Chiron has been underway since early 1994.

We present the results of the observations realized between 1994 and 1995 at the Observatório do Pico-dos-Dias (OPD, Brazil) and the Observatoire de Haute-Provence (OHP, France). The first observing run, in 1994, was performed using a 1.6-m telescope of the OPD on the 8th of February. The 95 campaign was performed on several small observing runs between March and July, on a 0.6-m and a 1.2-m telescope of the OPD and OHP, respectively.

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