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CRAB PULSAR-NEBULA INTERACTIONS

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RESUMEN

Discutimos el espacio de parámetros esperado en la vecindad del nudo de emisión más próximo al pulsar del Cangrejo. Luego analizamos la aceleración relativista de partículas en esta región y en otras partes. La aceleración resonante a través de líneas de campo magnético radiales energizan en forma significativa a las partículas cargadas del signo apropiado sobre aquellas del signo opuesto en lo que bien podría ser la ubicación del nudo, pero no hemos podido aún mostrar que deba de resultar un nudo de emisión.

ABSTRACT

We discuss the expected parameter space in the vicinity of the nearest knot of emission near the Crab pulsar. We then analyze the relativistic acceleration of particles in this region and elsewhere. Resonant acceleration by radial magnetic field lines significantly energize charged particles of the appropriate sign above those of the opposite sign in what could well be the location of the knot, but we have not yet been able to show that a knot of emission should result.

Key words: **ISM: SUPERNOVA REMNANTS (CRAB) — PULSARS: INDIVIDUAL (0531+21) — SHOCK WAVES**

1. INTRODUCTION

Theory has consistently predicted that pulsar action should generate relativistic winds. However, most pulsars are seen only in the radio frequency band. Worse, the radio pulsations —the spectacular signature of these objects— amounts to only a "dirt effect." Only about one part in 100 000 of the energy output, as estimated from the observed spin-down rates, is emitted in the radio bands. The Crab Nebula is even less efficient, with a thousand times even less beamed in the radio. However, the Crab Nebula is clearly filled with magnetized plasma that most plausible has originated from the pulsar, and rapid variations recently observed from *HST* show relativistic motions away from the pulsar. The central regions about the pulsar are characterized with quasi-stationary features ("wisps") that persist in place despite the putative wind (Michel 1995). We are, therefore, probably seeing generic flow structures that should help delineate the nature of the flow and test various theoretical expectations.

2. "TRANQUILITY PARK"

The pulsar action is generally thought to take place within the "light cylinder" (wave zone), a dated term left over from the days that centrifugal forces were thought to be dominant in the pulsar action (Goldreich & Julian 1969). However, this is nevertheless a characteristic scale dimension, being $c/\Omega = 1.5 \times 10^8$ cm for the Crab pulsar rotation rate of about 200 radians/s. Thus knot 1 is located about 1.5×10^8 wave zone distances away. The magnetic field at the wave zone is generally estimated to be about one mega-gauss, so the $1/r$ diminution of the wave field would then reduce the field to about 10^{-2} gauss or 10^3 γ (one γ being a nano-tessla, a common unit for terrestrial magnetospheric fields. The general field in the nebula is estimated to only be about a factor of 10 lower.

If indeed this region is dominated by emissions from the pulsar, we then expect that there is (1) a circularly polarized wave with $B_{wave} = 10^3$ γ, $\Omega_{wave} = 200$, and some plasma being driven away from the pulsar. Since the waves are generated by the component of a magnetic dipole orthogonal to the spin axis, one would additionally expect that an aligned component would be radially swept outwards by this plasma ("frozen-in-flux approximation"). Thus, a radial steady component of the magnetic field would also be expected. A perfectly radial component would die off as $1/r^2$, which would then be estimated to be $10^{-5}\gamma$, but the mere fact that these knots lie along the spin axis indicates that the flow is in the form of jets, and jet collimation could then give a significantly larger "radial" field. In any event, the rates, fields, and dimensions all suggest rather modest physical parameters in the knot 1 region, hence our nickname, "Tranquility Park."

These estimates only deepen the puzzle of why there is a knot of emission in this un-remarkable region. One possibility is a shock wave, but knot 2, about 10 times further out, already seems to represent a shock in the jet, and one generally does not need to shock a flow twice. If the radial field in this region were 1γ, however, the electron cyclotron frequency would be 180 radians/s, just about the same as the rotation rate of the circularly polarized wave. This value is much less than the wave field itself on the one hand, much larger than the $1/r^2$ estimate on the other, but not out of the question given the jet structure. The second possibility would then be plasma particles driven into resonance and radiating at or near the resonance region. Given that "180" is an expression for making a turn, this figure constitutes a convenient mnemonic for the cyclotron frequency in a standard field (1γ).

2.1. Theories

The above discussion leads us to distinguish between two types of astrophysical theory:

Type I: Use theory to TEST idea.

Type II: Use theory to MAKE idea work.

Examples of type II theories are not hard to find as witness the wild variety of ideas published for explain gamma-ray bursts, for example. Surely the bulk of these would ordinarily be dismissed if straightforward assessments were made. The arguments above would, in themselves, be sufficient for a type II publication. The work we will discuss is instead aimed at a type I assessment of whether the above modeling is sufficient to explain knot 1. Moreover, we need to know what the conditions are for resonance if the particles are relativistic. Does relativistic motion reduce or increase the fields need for resonance?

3. ACCELERATION IN LOW FREQUENCY WAVES

A number of approximate estimates have been made regarding acceleration of the plasma in a pulsar wind (reviewed in Michel 1991). Here we examine some new exact solutions. Although our parameter space is nominal, particles released into this space are accelerated to *huge* energies. What is particularly interesting is that the simple estimates one might make for such energies can be *wrong*. Consider first the circularly polarized wave, with fields given by time-varying phase

$$\phi = \Omega t - kz + \phi_0, \tag{1}$$

where we now use Cartesian coordinates with z being the distance along the spin axis. The wave fields are then

$$\begin{aligned} B_x &= -B_0 \sin\phi, && (2)\\ B_y &= B_0 \cos\phi, && (3)\\ E_x &= cB_0 \cos\phi, && (4)\\ E_y &= cB_0 \sin\phi, && (5) \end{aligned}$$

corresponding to counter-clockwise rotation in the x-y axis. Here we will neglect the possible B_z field. The textbook solution would be to note that the particle velocity must vary harmonically with ϕ and therefore, the solution should be circular particle motion. Indeed, the motion

$$\begin{aligned} v_x &= v_0 \sin\phi, && (6)\\ v_y &= v_0 \cos\phi, && (7)\\ v_z &= 0, && (8) \end{aligned}$$

is an exact "particle-on-a-string" solution to the Lorentz force, because the electron circles at the "tip" of the electric field vector, balancing electrical force with centrifugal force. Since $\mathbf{v} \parallel \mathbf{B}$, the magnetic terms vanish in the Lorentz force. We would then write the force balance as

$$mv_0\omega = ecB_0, \tag{9}$$

or

$$v_0 = c\frac{eB_0}{m\Omega} = c\frac{\omega_c}{\Omega}. \tag{10}$$

However, we immediately notice that in a field of $10^3\gamma$, the cyclotron frequency (ω_c) is about 10^3 times Ω, and v_0 cannot possibly have that velocity.

Notice here that we have a problem, not usually pointed up in plasma physics texts, with electromagnetic waves at low frequencies; even at modest wave amplitudes the electric field can be "on" so long that electrons become relativistic. The usual relativistic patch is simply to introduce the Lorentz factor and write

$$\gamma m v_\perp \Omega = ecB_\perp, \tag{11}$$

and if the Lorentz factor is large we can approximate the velocity to be c, giving

$$\gamma \approx \frac{\omega_\perp}{\Omega} \equiv g. \tag{12}$$

But an electron had such a Lorentz factor, it would have an energy of 500 MeV! Where did the electron "get" all this energy to run in a circle? Note that we have changed the subscript 0 to $\perp$ to emphasize the motion involved. Here $\omega_\perp \equiv eB_\perp/m$, the non-relativistic definition of the cyclotron frequency.

This question illustrates a hidden problem owing to the relativistic nature of the issue. The hidden problem can be traced to a ubiquitous problem with Physics texts, namely that they frequently present the student with only the *inhomogeneous* solutions to linear differential equations. A classic example is the driven, damped harmonic oscillator, where the driven amplitude is solved for and the homogeneous solution (which is of no interest to the author) is entirely neglected. For non-linear differential equations, what would be called "inhomogeneous" are called *singular solutions.* They both have the same definition: no free parameters. Our particle-on-a-string solution is an example; everything is solved for, but it requires the particle to move at fixed velocity in a circle. But what if the particle had started from rest? That's what one needs the homogeneous solutions for!

If one looks at plasma physics texts, one sees exactly this error made repeatedly. Wave propagation through a plasma always has the homogeneous solutions discarded. This is the assumption that the average velocity of the plasma particles are zero, a plausible assumption without the wave! But it is maintained even with the wave. One can even make up physically realizable problems that would be done incorrectly. Consider a single electromagnetic pulse where E_x is briefly non-zero, but only of one sign. The standard approach would be to Fourier transform the pulse, modify the transform for the plasma dispersion relation, and transform back, giving a smeared pulse with a tail. In fact, such a pulse would accelerate electrons in the $-x$ direction and ions in the $+x$ direction and attenuate itself in (quickly) giving all its energy to the plasma.

Returning to our particle on a string problem, it is simple to fix because if the initial conditions are that the particle was at rest at some instant (phase of the wave) then taking that phase to be $\phi = 0$ (other phases just correspond to rotating the coordinate system), we have instead that while

$$v_x = -v_\perp \sin\phi, \tag{13}$$

still, what is changed is

$$v_y = v_\perp(\cos\phi - 1), \tag{14}$$

and we get a cycloidal motion. However, the cycloidal motion has the mean velocity $< v_y >= v_\perp$, and the particle now interacts with the B field of the wave, which excites harmonic motion in the z axis. The facile solution to this added component is that

$$v_z = -gv_\perp \cos\phi, \tag{15}$$

but this again ignores initial conditions. If the particle were entirely at rest initially, then

$$v_z = gv_\perp(1 - \cos\phi), \tag{16}$$

but notice the g!

If we took this equation at face value, we would conclude that the x-y energies would correspond to Lorentz factors of g, but it looks as if the z axis would get Lorentz factors of g^2! It also suggests that the particle would be accelerated to such energies and then decelerated to rest again. It turns out (Michel & Li 1997), that the fully relativistic (hence non-linear) for particle motion in a circularly polarized wave can be *exactly* solved. A third surprise is that, in the fully relativistic treatment, the resonance condition is not changed in the least. It remains at the frequency for the *non-relativistic* cyclotron frequency. (Of course, that could be regarded as a shift in the sense that resonance is not at the cyclotron frequency of the relativistic electron, which is changing all over the place as it interacts with the wave.) In other words, if we define

$$h \equiv \frac{\omega_{\|}}{\Omega}, \tag{17}$$

with

$$\omega_{\|} \equiv \frac{eB_{\|}}{m}, \tag{18}$$

then resonance is where $h = 1$.

4. APPLICATION TO THE CRAB WIND

Because the solutions are exactly harmonic functions of phase ϕ, we can use a simple Euler integration to solve the Lorentz force equations. The solutions are particularly simple if the particle is picked up from rest. In the wind case, the rectilinear approximation quickly fails as the particle is accelerated out into weaker field regions. We simply attenuate the fields appropriately with radial distance, while solving for the particle motion. The effect of attenuation can be seen in Figure 1, where the particle finds itself with more energy than it can return to the weakened wave, and it keeps approximately half of that energy.

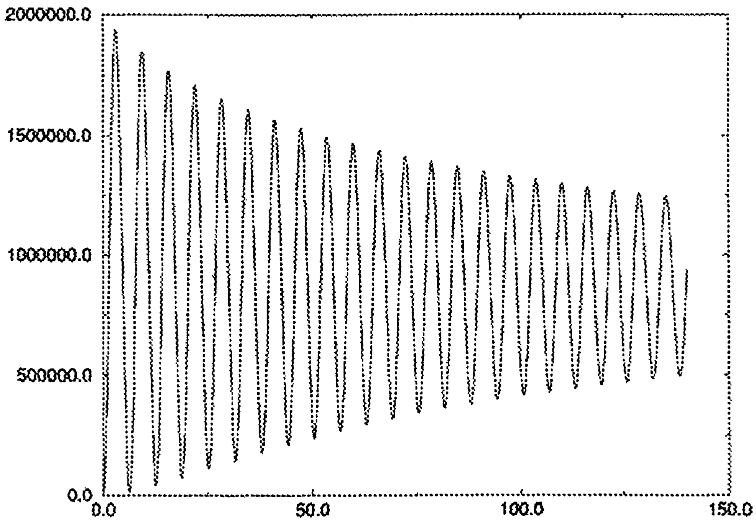

Fig. 1. Electron picked up at an altitude of 10^8 wave zone radii. The y-axis is the Lorentz factor and the x-axis is phase in radians.

The effect of adding a radial magnetic field is simply to cause the circular orbit to precess, as shown in Figure 2. If we combine the two results, we get rather unusual results as shown in Figure 3. As we move the pickup point inward, the electron gains most of its energy from the stronger fields at the pickup point and the attenuation becomes extremely rapid, producing behavior shown in Figure 4. The effect of resonance on pickup requires of course that the resonance condition be met within the interval, and since we conservatively assumed the radial magnetic field to decline as $1/r^2$, it necessitated moving even closer to 10^4 radii. The upper curve in Figure 5 has the radial field in the sense to resonate with the electron and the lower curve is the opposite sense.

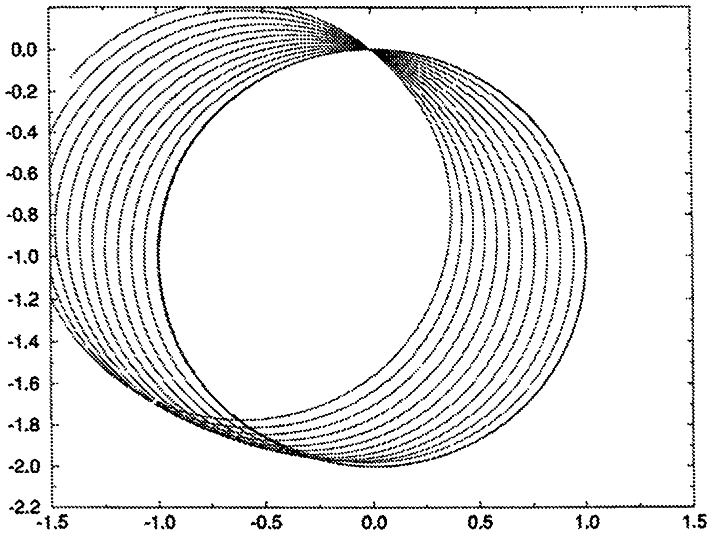

Fig. 2. Precession in a weak radial field. Particle always returns to its pickup point (the origin).

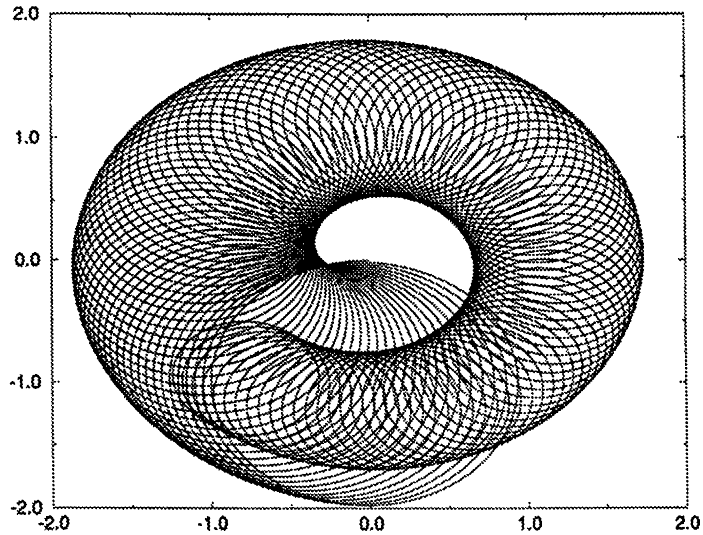

Fig. 3. Precession with attenuation.

Actually, almost identically the same curve is gotten in the latter case if the radial field is set to zero. Because the parameter range is now varying over huge factors, it has become essential to introduce variable step size. However, one trend is clear and that is that the asymptotic electron energy is becoming insensitive to how close to the pulsar it is picked up. The reason is that in close, the fields are dropping so rapidly with distance that the particle gets relatively little energy before being accelerated away. At large distances, the particle gets little

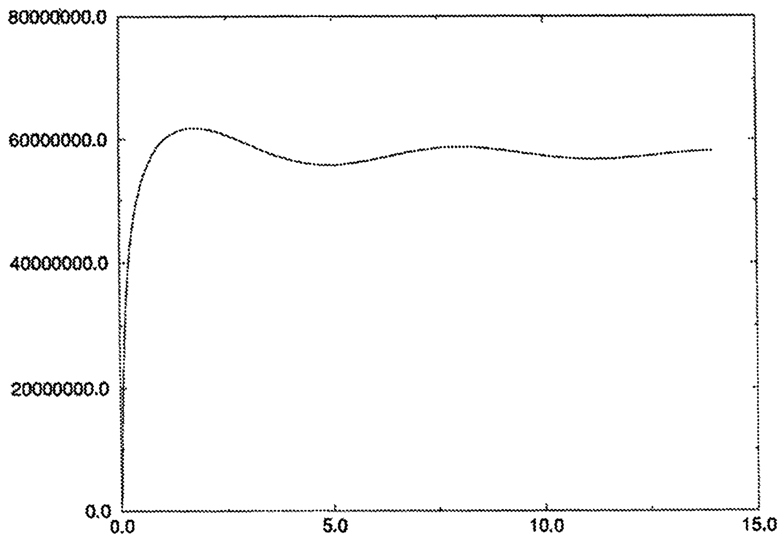

Fig. 4. Lorentz factor vs. phase for pickup from 10^6 radii. Notice that although the field weakens markedly, the electron remains coupled to some degree.

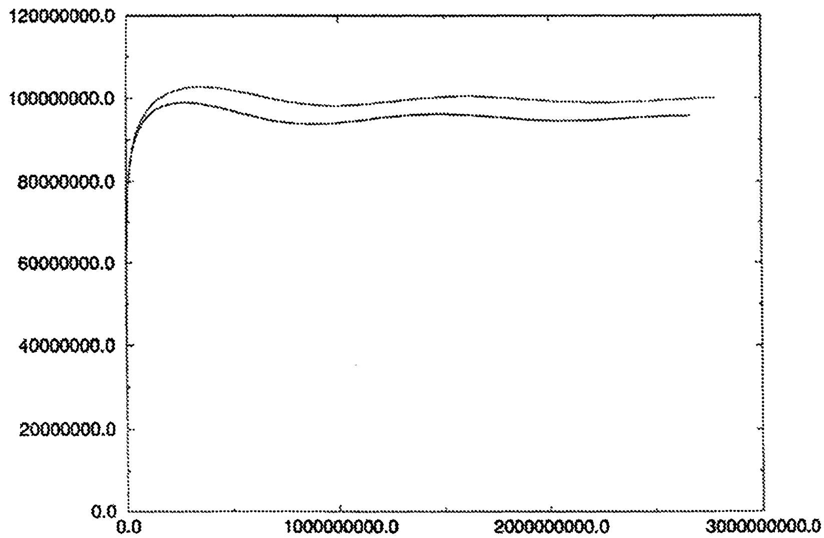

Fig. 5. Lorentz factor vs. distance in wave zone radii for resonant (upper) and non-resonant particles (lower).

energy because the waves become weak, even though there is plenty of distance over which the fields are roughly constant. Accordingly, a particle picked up near the pulsar gains most of its energy at the same intermediate distance. The effect of variable step size is shown in Figure 6, where the particle acceleration is also inhibited by the much stronger radial magnetic field (for simplicity, we took the wave and radial fields to be equal at the wind zone). Although the particle went essentially nowhere in this figure (about 4000 steps), it was out at nebular distances after 8000 steps.

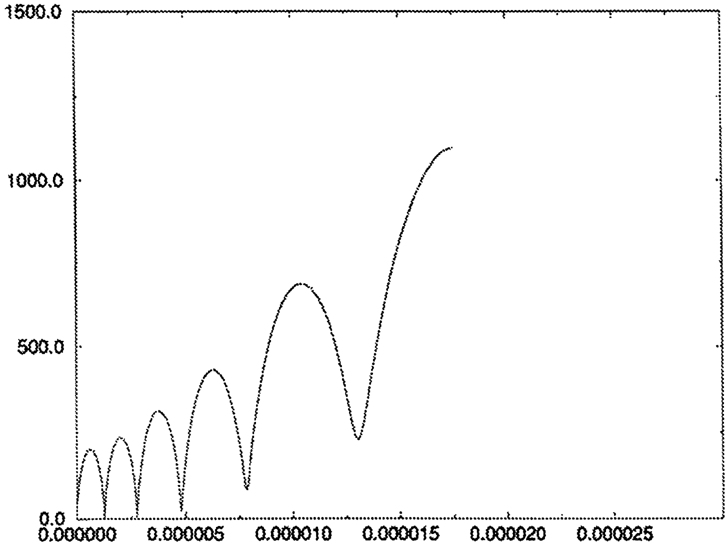

Fig. 6. Lorentz factor and z-displacement for pickup at 10 radii. The particles get relatively little energy owing to the strong B_z.

5. CONCLUSIONS

The nature of knot 1 remains uncertain, although there are a number of things to be checked. The distance at which the maximum energization is found is similar to the knot 1 distance. We plan to include synchrotron radiation in our codes to ensure that we are not missing interesting behavior that is not evident in the trajectory plots. The resonant region will also be examined, although it would not be at the knot 1 distance if we assume $1/r^2$ fall off of the radial magnetic fields. In any event, the wave-particle interaction itself is interesting in its many counter-intuitive features.

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