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STUDY OF A MULTIPOLE EQUATION FOR PULSAR EVOLUTION

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RESUMEN

Utilizamos la ecuación multipolar $\dot{\nu} = -g\nu^5 - r\nu^3 - s\nu$ consistente con índices de frenado entre 1 y 5, para modelar la evolución de pulsares. Aplicamos este modelo a pulsares con edades entre 10^3 y 10^5 años en el diagrama $P - \dot{P}$. Los pulsares evolucionan describiendo una curva cuyo valor mínimo en $\dot{P}$ ocurre para $n = 2$, y posteriormente evolucionan a regiones con $\dot{P}$ mayores. Como consecuencia, el modelo predice la existencia de pulsares en la parte superior derecha del diagrama $P - \dot{P}$, en una región centrada alrededor de $\log \dot{P} \approx -11$ y $\log P \approx 0.5$. Si existen, estos corresponderían a estrellas de neutrones "radio silenciosas".

ABSTRACT

The multipole equation $\dot{\nu} = -g\nu^5 - r\nu^3 - s\nu$, consistent with braking indexes between 1 and 5, is used to model the evolution of pulsars. This model is applied to the $P - \dot{P}$ diagram, for pulsars with ages between 10^3 and 10^5 years. Pulsars evolve describing a curve with the minimum value of $\dot{P}$ occurring when n = 2, then move to regions of higher $\dot{P}$. As a consequence, the model predicts the existence of pulsars in the right upper part of the $P - \dot{P}$ diagram in a region centered around $\log \dot{P} \approx -11$ and $\log P \approx 0.5$. If these exist, they should be "radio quiet" neutron stars.

Key words: **PULSARS: GENERAL**

1. INTRODUCTION

Pulsars are often described by the rotating magnetic dipole model, $\dot{\nu} = -k\nu^3$, which predicts a braking index $n \equiv \ddot{\nu}\nu/\dot{\nu}^2 = 3$. However, the four pulsars with properly measured braking indexes have $n < 3$ (Table 1). To explain these discrepancies, several physical processes have been invoked, like outflows of charged particles or magnetospheric currents (Blandford & Romani 1988). Furthermore, the evolutionary paths followed by the rotating magnetic dipole model are straight lines in the $P - \dot{P}$ diagram and these are not consistent with the observed $P - \dot{P}$ diagram (Camilo 1996). Knowing that n differs from 3, we introduce a wide range of possible values of n through a multipole rotation rate equation, $\dot{\nu} = -g\nu^5 - r\nu^3 - s\nu$ (hereafter *ME*), which we study in terms of its consistency with observed data. Integrating this equation backwards and forwards, we obtained evolutionary trajectories in $P - \dot{P}$ diagram for pulsars between 10^3 and 10^5 years of age.

2. THE MODEL

The dynamical evolution of pulsars is described here by the *ME* $\dot{\nu} = -g\nu^5 - r\nu^3 - s\nu$, where the coefficients g, r and s are directly related with the first and second braking indexes n and $m \equiv \dddot{\nu}\,\nu^2/\dot{\nu}^3$ through the matrix equation

$$\begin{pmatrix} 1 \\ n \\ m - n^2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 5 & 3 & 1 \\ 20 & 6 & 0 \end{pmatrix} \begin{pmatrix} g(\nu^5/-\dot{\nu}) \\ r(\nu^3/-\dot{\nu}) \\ s(\nu/-\dot{\nu}) \end{pmatrix}. \qquad (1)$$

The coefficients g, r and s are assumed constant and are calculated inverting (eq. 1). These coefficients are constrained to be positive, forcing the value of m to lie in the range $Max(n^2+3n-3, n^2+7n-15) < m < n^2+5n-5$, while n can go from 1 to 5. According to this, pulsars must be in a restricted region in the (n, m) diagram, shown in Figure 1. Time evolution is from high to low values of n.

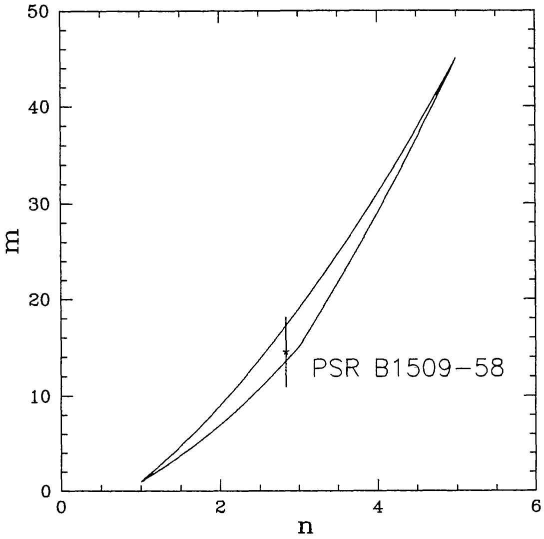

Fig. 1. Allowed values of braking indices n and m. A pulsar will evolve from the upper right to the lower left part. The only observational point is shown.

Integrating ME, the range of allowable values of m, for each value of n, defines a family of curves. The minimum value of $\dot{P}$ occurs when $n = 2$ and pulsars evolve to higher $\dot{P}$ values afterwards. Pulsars continue to move to a region in the $P - \dot{P}$ diagram where no radio pulsars have been observed (Figure 2*a*). Also, when $n \rightarrow 1$, the dynamical time, $t_{dyn} = P/2\dot{P}$, becomes independent of time, and therefore a bad estimator of age. This occurs for pulsars with frequencies below ~ 200 milliseconds (Figure 2*b*).

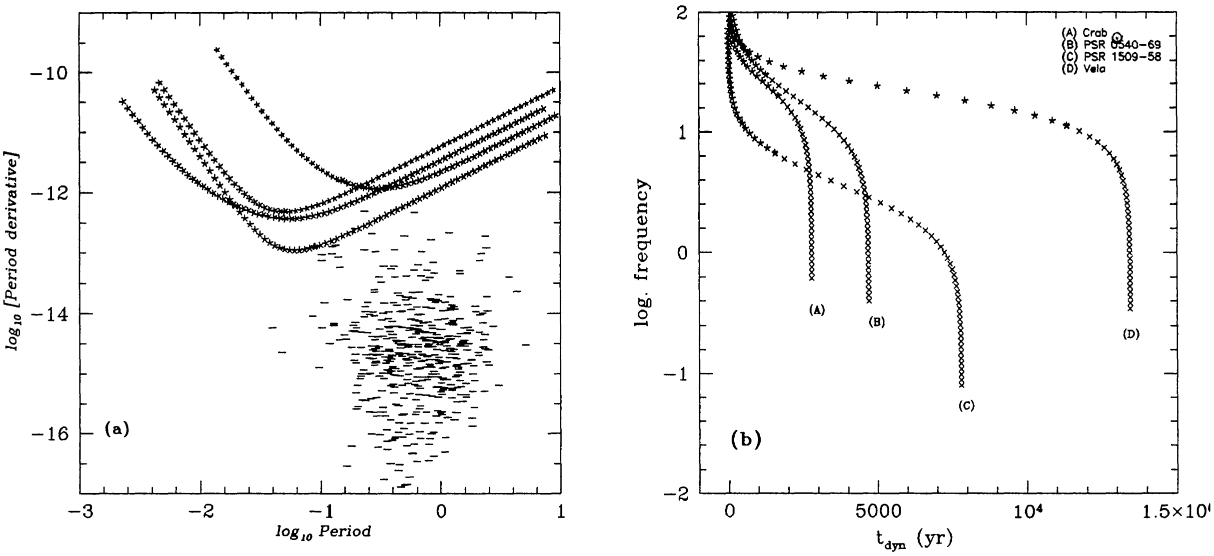

Fig. 2. (*a*). Typical evolutionary trajectories for the four youngest pulsars according to the multipole equation. Pulsars evolve from left to right and the minimum in $\dot{P}$ corresponds to $n = 2$. The small bars are the radio observed pulsars. (*b*) Frequency versus dynamical age according to the multipole model. Note that the dynamical age tends to a constant as $n \rightarrow 1$.

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3. EVOLUTION OF PULSARS WITH T_{DYN} BETWEEN 10^3 AND 10^5 YEARS

For the sample of the 21 youngest pulsars, those with $t_{dyn} \leq 10^5$ yrs, we integrated ME backwards from the present position to a reference dynamical age of $t_{dyn} = 1$ yr. We assumed n to be at present between 1.4 and 2. These trajectories define a birth region of pulsars consistent with the observed distribution in the $P - \dot{P}$ diagram (Figure 3*a*). In the $t_{dyn} = 1$ yr line, the extrapolated "initial" frequencies are in the range between 20 ms and 2 ms, all considered physically plausible.

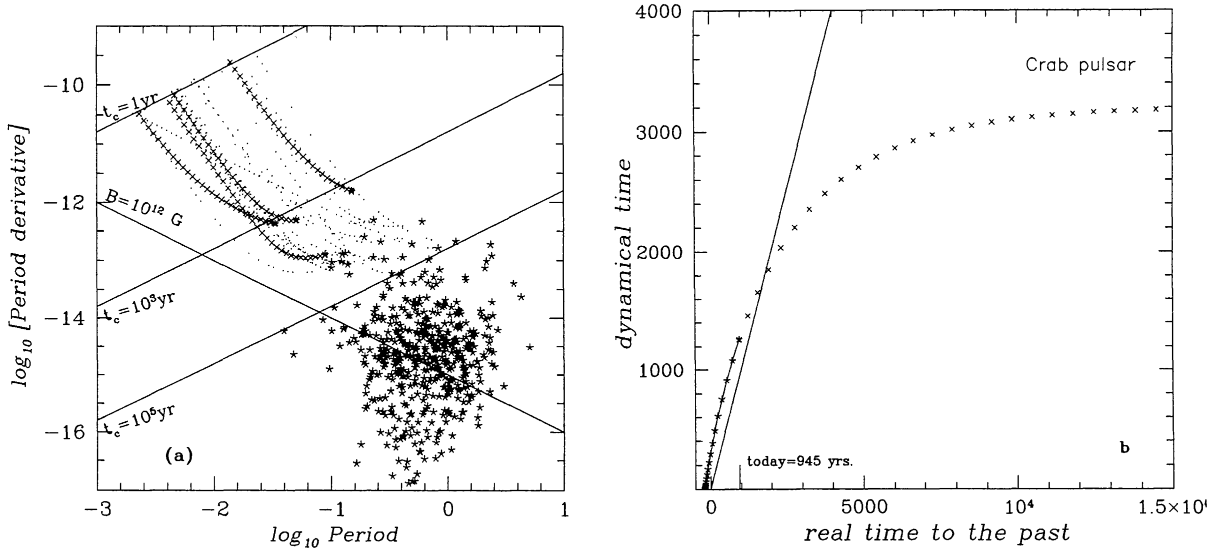

Fig. 3. (*a*). Trajectories for the 21 youngest pulsars evolved backwards to t_{dyn}=1yr. The four youngest pulsars are clearly indicated. (*b*) Comparison of t_{dyn} and real age for Crab.

4. THE PULSARS WITH BETTER MEASUREMENTS

For the four youngest objects n has been properly measured. Running the model backwards provides information about their initial conditions (see Table 1). Running the model forwards we find that they evolve to region in the $P - \dot{P}$ diagram centered around $\log \dot{P} \approx -11$ and $\log P \approx 0.5$, (see Fig. 2*a*). Note there are no observed points in this part of the diagram.

PSR B1509-58

The measured value of the secondary braking index, $m = 14.5 \pm 3.6$, is consistent with the model prediction: $13.56 \leq m \leq 17.23$. Choosing $n = 2.837$ (Manchester, Durdin, & Newton 1985), $m = 14.5$, and assuming an age of 1200 yr, the period at birth is found to be 68 ms. At present PSR B1509-58 is the only pulsar for which m is measured. Such measurements in the future for more pulsars, will enable us to test the narrow range of m values predicted by the ME.

Crab Pulsar

This pulsar is known to be born in 1054 A.D. and its braking index is $n = 2.509 \pm 0.001$ (Lyne, Pritchard, & Smith 1988). When integrating ME backwards to birth, the model predicts a birth period of 18 ms, almost independent of the value of m. For the Crab we can compare the chronological and dynamically derived ages (Fig. 3*b*). A linear fit between them, $t_{dyn} = 261 + 1.17 t_{chron}$ years, is a good aproximation for $t_{dyn} \leq 2000$ years. After that t_{dyn} approaches a constant value of $\sim$ 3000 years.

PSR B0540-69

This pulsar with braking index $n = 2.02 \pm 0.01$ (Manchester & Peterson 1989), is very close to reaching its $\dot{P}$ minimum. If we assume this pulsar has an age of 1600 yr we obtain a birth period of 20 ms, similar to that found for the Crab.

Vela Pulsar

In comparison with PSR B1509-58, the timing parameters are not so well known, mostly because of the glitch activity of Vela. The measured braking index, $n = 1.4 \pm 0.2$ (Lyne et al. 1996), indicates that $\dot{P}$ is growing, independently of our model. When integrating ME backwards for 11 000 yrs we obtain a relatively high birth period of 50 ms.

TABLE 1

THE YOUNGEST FOUR PULSARS

Pulsar	n	range of m	assumed age(yr)	t_{dyn} (yr)	initial P (ms)
Crab	2.509±0.001	10.82,13.64	950	1258	18
1509-58	2.837±0.03	13.56,17.23	1200	1553	68
0540-69	2.02±0.01	7.14,9.18	1600	1664	20
Vela	1.4±0.2	3.16,3.96	11000	11335	50

5. DISCUSSION

The birth region of pulsars in $P - \dot{P}$ diagram, artificially constrained here to t_{dyn}=1 yr, is dependent on the chosen values of n and m. A distribution of (t_{dyn}=1yr) periods predicted by our ME can be generated by sampling n and m values arbitrarily.

As pulsars with $n \leq 2$ evolve to higher $\dot{P}$ values, the multipole equation predicts, from the position of the four youngest pulsars, a population of pulsars with $\log \dot{P} \approx -11$ and $\log P \approx 0.5$. This has not been observed. We propose three scenarios:

- Observational biases have not allowed to detect these radio objects. This is likely to be testable.
- Such a population exists in the form of radio quiet pulsars. Because of the slow rotation, these should be searched for among unidentified X-ray or γ-ray sources.
- More likely, the applicability of the ME is limited. While the backwards evolutionary paths seem reasonable, evolution to the future might not be properly described. The model can be refined to include processes like glitches and evolution of physical parameters, like the magnetic field, through physically meaningfull temporal behaviour of the coefficients g, r, s.

As $n \rightarrow 1$ in the ME, the dynamical age tends to be constant and does not serve as age indicator anymore. In particular, the dynamical clocks of the four youngest pulsars will stop before reaching 1.5×10^4 yrs. This is in conflict with the dynamical ages of all other pulsars and is likely to represent a limitation of the ME. It also stresses the general limitation of the dynamical time as age indicator, although t_{dyn} might still be measuring other physical property. In general, if $n < 1$ dynamical age decreases. Therefore, in practice $n \leq 1$ will no be reached, because t_{dyn} would run backwards and there would have been more than four pulsars with $t_{dyn} \leq 10^4$ years. Note that the future evolution of the Vela pulsar is already very much constrained by its braking index being so close to unity. Finally, we note that a ME can be seen as a Taylor expansion of a more general antisymmetric spin-down evolution, $\dot{\nu}$=f(ν), with condition f(ν)=$-f(-\nu)$, consistent with assigning a sign to the direction of rotation. The limited applicability of a ME can be investigated in further studies addressing the question: is there a universal spin-down equation for pulsars?

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