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INSTABILITY OF THE GOLDREICH-JULIAN MODEL FOR A PULSAR MAGNETOSPHERE

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RESUMEN

Presentamos simulaciones numéricas de magneto-esferas de pulsar en las cuales el giro y los ejes dipolares magnéticos de la estrella de neutrones están alineados. Demostramos aquí la existencia de distribuciones de carga estables que incluyen espacios vacíos. También demostramos que la distribución de carga usada en el modelo de Goldreich-Julian de pulsar estándar es inherentemente inestable. Este se colapsa en una configuración estable que es muy similar a las otras ilustradas en este artículo.

ABSTRACT

We present numerical simulations of pulsar magnetospheres in which the spin and magnetic dipole axes of the neutron star are aligned. We thereby demonstrate the existence of stable charge distributions that include vacuum gaps. We also demonstrate that the charge distribution used in the "standard" Goldreich-Julian pulsar model is inherently unstable. It collapses to a stable configuration that is very similar to the others illustrated in this paper.

Key words: **PLASMAS — PULSARS: GENERAL**

1. INTRODUCTION

For many years, the "standard" model for pulsar magnetospheres has been the one introduced by Goldreich & Julian (1969: GJ). The arguments leading to their model are simple and quite elegant in their original form. The neutron star is assumed to be an excellent conductor *surrounded everywhere* by a charge-separated plasma. Rotation induces potential differences on the surface of the neutron star, and the high parallel conductivity of the plasma leads to equipotential field lines.

Dipole magnetic field lines in spherical coordinates are given by

$$\sin^2\theta/r = \text{constant} \quad , \tag{1}$$

where θ is the colatitude. For an aligned rotator, the potential V induced by the rotation rate ω is

$$V = A\omega \sin^2\theta/r \quad , \tag{2}$$

where r is measured in units of the stellar radius $R_{\rm NS}$ and $A = B_0 R_{\rm NS}^2/2c$. B_0 is the polar magnetic field strength, and c is the velocity of light. From now on, we will set $R_{\rm NS} = 1$ and $A = 1$.

Close to the neutron star, the GJ plasma corotates rigidly with the star and the space charge density is given by

$$\rho_{\rm GJ} = \omega(1 - 3\cos^2\theta)/2\pi r^3 \quad . \tag{3}$$

However, this cannot hold beyond the light cylinder (where $\omega r \sin\theta = c$), and the GJ model assumes that the plasma then streams away to infinity. This would mean that an aligned rotator could be an active system. However, several studies have shown that this picture cannot be made self-consistent: see Michel (1982) for a review.

One of the implicit assumptions of the GJ model, as shown in eq. (3), is that charge fills the entire region surrounding the neutron star. This has been challenged, and the existence of vacuum gaps has been proposed by numerous authors (Holloway 1973; Ruderman & Sutherland 1975; Michel 1979; Krause-Polstorff & Michel 1985a,b). The question then is whether stable magnetospheric solutions including vacuum gaps can be found, i.e., that satisfy $\mathbf{E}\cdot\mathbf{B} = 0$ everywhere on the stellar surface and in the magnetosphere.

2. MAGNETOSPHERIC MODEL

Stable magnetospheric solutions with vacuum gaps have indeed been found (Krause-Polstorff & Michel 1985a,b: KPM). Associated with the concept of vacuum gaps is the concept of "force-free surfaces". Plasma will tend to congregate behind such surfaces, allowing vacuum gaps to form between them. One such surface forms a pair of linked spheres extending from the center of the star above and below the poles. This surface shapes the formation of polar domes of negatively charged plasma. Positively charged plasma accumulates in the equatorial region of the magnetosphere. The two charged regions are separated by a vacuum gap, and are confined well inside the light cylinder, so no plasma leaves the system.

To find particular magnetospheric solutions, we use a numerical simulation based on that of KPM. The central charge of the neutron star is fixed by the rotation rate, and for the work shown here is set to a value of +10. All the units are made dimensionless, and the solutions are equally valid for a wide range of physical situations. Because of the symmetry of the aligned axes, we can use infinitesimal rings as the quanta of charge in the magnetosphere. The axisymmetric charge distribution can then be easily displayed in a two-dimensional cross-section. The total charge of the system, star and plasma, can be altered by the addition of a uniform density surface charge.

The KPM code has been significantly re-written and improved, and can now run with orders of magnitude more particles than before. However, the qualitative results found by KPM have remained robust. The code follows an iterative procedure, starting with a given charge distribution in the magnetosphere. The main steps are:

1. If there are areas of the surface of the star where $\mathbf{E} \cdot \mathbf{B} \neq 0$, rings of charge are separated and launched into the magnetosphere.

2. The newly launched rings are considered to be "frozen" to their field lines, and are moved to equilibrium positions based on the electric fields in the magnetosphere.

3. All the rings in the magnetosphere are moved to new equilibrium positions, because of the perturbations from the new rings.

The code then returns to step 1. A solution is reached when all the rings are sufficiently close to equilibrium positions at the end of step 3.

3. RESULTS

3.1. Stable Magnetosphere Solutions

Figure 1 shows sample simulation results when the code is started with a vacuum in the magnetosphere. In all three cases, the charge of each ring is ± 0.02. The only difference between the three cases is the amount of surface charge Q_S added.

Self-consistent and stable charge configurations containing vacuum gaps are found in each case. The configurations are all characterized by negatively charged polar domes and positively charged equatorial belts. It should be noted that the V-shaped region above the pole is not a vacuum gap: it slowly fills in as the charge quantization is reduced.

Figure 1(a) shows the case when no surface charge is added. Adding a large positive surface charge greatly shrinks the negatively charged polar dome, as shown in Figure 1(b). Adding a large negative surface charge greatly expands the polar dome, as shown in Figure 1(c). The shape of the positively charged equatorial belt is little changed in the three runs, though the fraction of all the rings that are positively charged varies appropriately.

The general dome, belt, and vacuum gap geometry always seem to appear, even if we do not start the simulation with a vacuum in the magnetosphere, or if we choose different methods for launching the particles from the surface of the star. For a given surface charge, there is no unique solution, because different stable solutions can be reached depending on the geometry of the plasma in the magnetosphere at the start of the run. However, we have not found any other stable class of configuration.

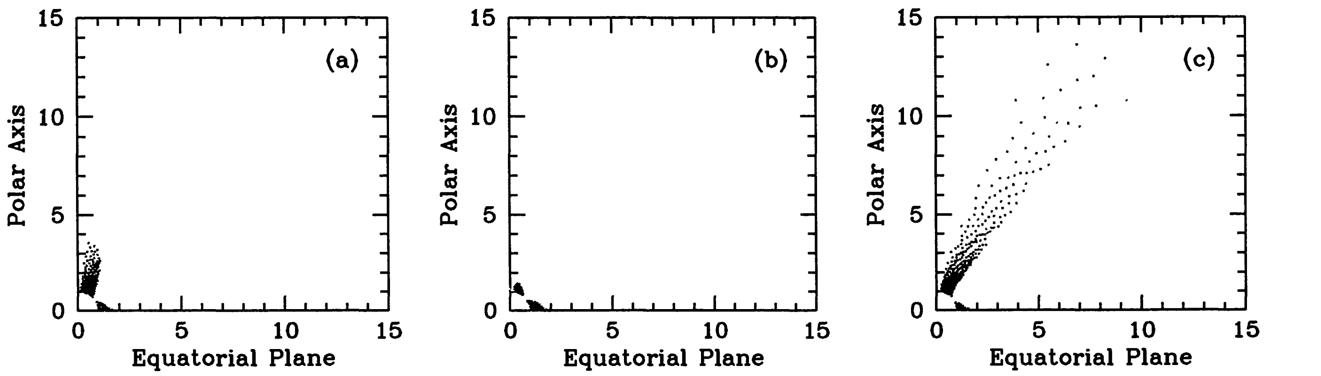

Fig. 1. Sample simulation results. For all three cases, the central charge of the neutron star is +10, and the rings have charges of ± 0.02. (*a*) Surface charge $Q_S = 0$. (*b*) $Q_S = +12$. (*c*) $Q_S = -8$. Polar dome particles are negatively charged. Equatorial particles are positively charged.

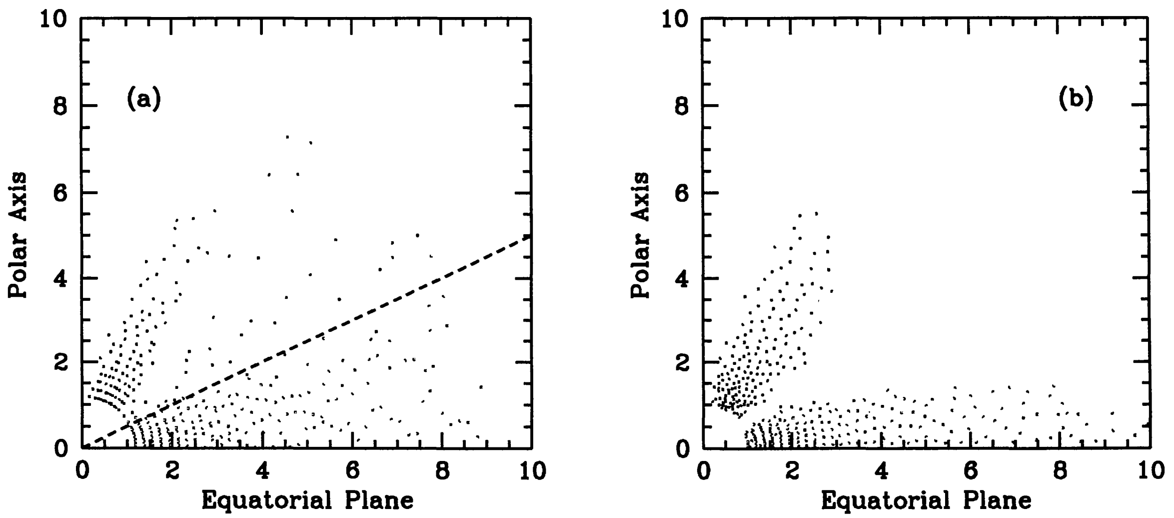

Fig. 2. Instability of the GJ solution. (*a*) A GJ charge density used as the starting point of our simulation. Dashed line shows the boundary between the negatively and positively charged regions. (*b*) The final stable solution. The central charge of the neutron star is +10, the rings have charges of ± 0.06, and the surface charge is zero. Polar dome particles are negatively charged. Equatorial particles are positively charged.

3.2. Goldreich-Julian Instability

Figure 2 illustrates what happens if we start our simulation with a GJ charge density in the magnetosphere. Note that the real GJ plasma fills the magnetosphere out to the light cylinder: to reduce the computational burden, we have only started with plasma out to $9R_{\rm NS}$, though we have found from using a larger charge quantization that the results do not depend on this radius.

It is clear that the GJ configuration is unstable. It collapses to form a dome, belt and vacuum gap configuration similar to those shown in Figure 1. These results illustrate the point we made in section 3.1: the general dome, belt, and vacuum gap geometry always seem to appear, no matter how we start the simulation running.

4. ONGOING WORK

Since stable magnetospheric solutions have been found, and no plasma leaves the system, the aligned rotator is a "dead" pulsar.

It might be thought that including electron-positron pair creation in the vacuum gap might greatly alter the situation. However, our studies so far show that the only effect is to add plasma to the charge-separated regions, with the vacuum gap shrinking until the electric field becomes low enough that pair production turns off.

To create a real pulsar, it will be necessary to break the alignment of the spin and magnetic field axes. To study this, we are currently developing a code that uses discrete particles, rather than rings of charge.

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