NEUTRON DIFFUSION AND NUCLEOSYNTHESIS IN AN INHOMOGENEOUS BIG BANG MODEL

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RESUMEN

Este artículo describe la producción primordial de los núcleos de ^{4}He en un universo inhomogéneo. La distribución bariónica es esféricamente simétrica y consta de una región interior de alta densidad y una región exterior de baja densidad. A medida que la temperature decrece, los neutrones se difunden a la región exterior hasta que están distribuidos homogéneamente y los protones se pueden redistribuir dependiendo de la velocidad de difusión. La nucleosíntesis ocurre primero en las regiones interiores y los neutrones se difunden a esa región. La velocidad de difusión determina la cantidad de ^{4}He que se produce finalmente.

ABSTRACT

This article describes the production of primordial ^{4}He nuclei in an inhomogeneous universe. The baryon distribution is spherically symmetric and consists of a high density inner region and a low density outer region. As the temperature decreases neutrons diffuse to the outer region until they are homogeneously distributed, and protons may be redistributed depending on how fast diffusion occurs. Nucleosynthesis occurs earlier in the inner region and neutrons diffuse back to that region. The rapidity of diffusion determines how much ^{4}He is ultimately produced.

Key Words: **COSMOLOGY — DIFFUSION — NUCLEAR REACTIONS, NUCLEOSYNTHESIS, ABUNDANCES**

1. INTRODUCTION

In 1999 the author wrote a big bang nucleosynthesis code corresponding to a universe with an inhomogeneous baryon distribution. The code envisions the universe as a lattice of spheres and treats one sphere as characteristic of the rest of the universe. The sphere has an initial radius (the **distance scale** r_i) of 25000 cm at temperature T =100 GK. The sphere consists of a high density inner region and a low density outer region. The **boundary radius** r_b between the regions is set to $r_i/2$ and the **density contrast** ratio $R_\rho = 800{:}1$. The baryon–to–photon ratio η in this model is 3×10^{-10} ($\eta_{10} = 3$). The sphere is divided into a core and 63 spherical shells.

The code solve for the number density $n(i,s)$ of isotope species i contained in spherical shell s. The evolution of $n(i,s)$ obeys the equation (Kainulainen, Jurki–Suonio & Sihvola 1999)

$$
\frac{dn(i,s)}{dt} = \frac{1}{n_b(s)} \sum_{j,k,l} N_i \left(-\frac{n^{N_i}(i,s)n^{N_j}(j,s)}{N_i!N_j!}[ij] + \frac{n^{N_k}(k,s)n^{N_l}(l,s)}{N_k!N_l!}[kl] \right)
$$
$$
-3\dot{\alpha}n(i,s) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D_n \frac{\partial n(i,s)}{\partial r} \right).
$$

The first term accounts for nuclear and weak reactions occuring between isotopes within shell s. The second term corresponds to the expansion of the universe. And the last term accounts for diffusion of isotope i between shells, though in the model only neutrons diffuse.
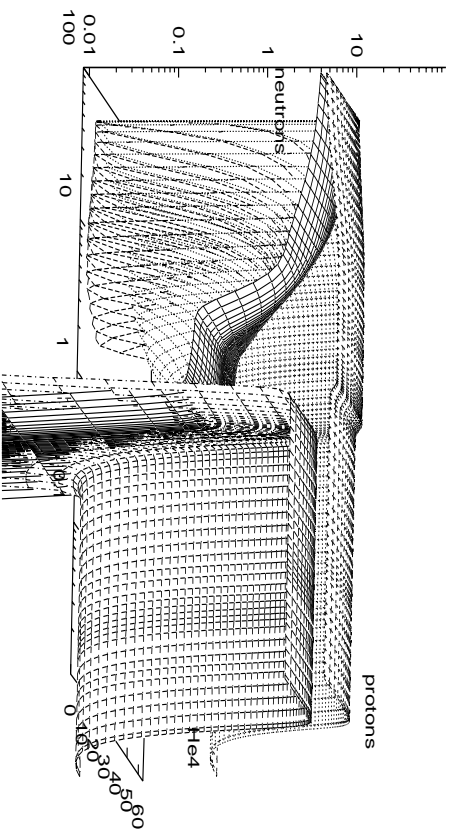
NEUTRON DIFFUSION AND NUCLEOSYNTHESIS IN IBBN

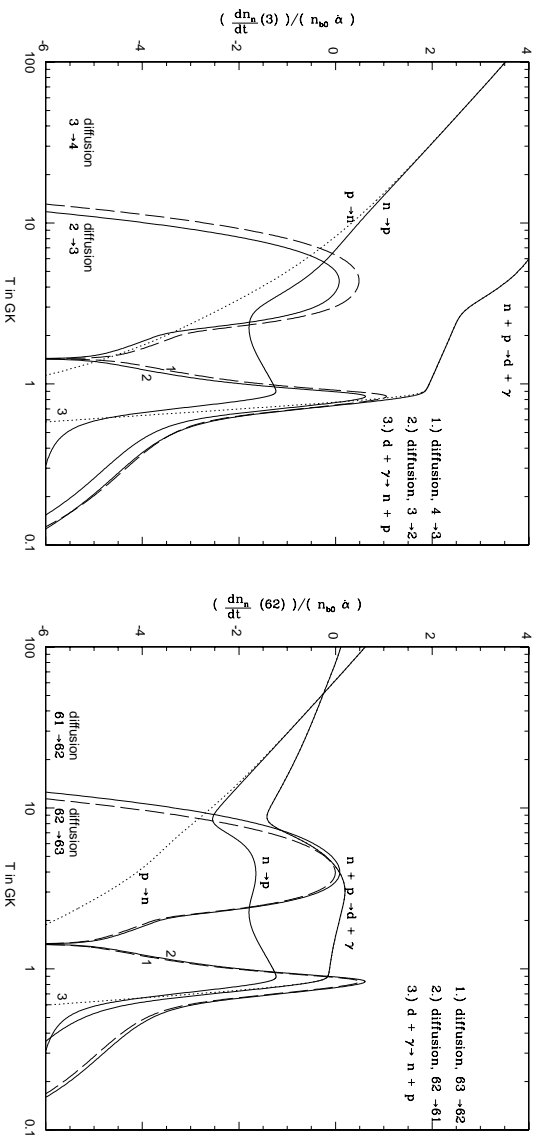Fig. 1. Number densities of neutrons, protons and $4 \times {}^4\mathrm{He}$

Fig. 2. Neutron Reaction Rates (shell 3 and shell 62).

TABLE 1

^{4}HE, WHERE $\eta = 3.0 \times 10^{-10}$.

r_i in cm.	$X_{^4\text{H}e}$
10^2	0.239029657
1.58489×10^3	0.239369667
2.51189×10^4	0.243663794
10^5	0.241847338
7.94328×10^5	0.229781933
6.30957×10^6	0.250711334

2. RESULTS FOR $R_I = 25000$ CM, $\eta_{10} = 3$

Figure 1 shows the number densities of free neutrons, $n(\text{n}, s)$, protons, $n(\text{p}, s)$, and $4 \times ^4$He, $4n(^4\text{He}, s)$. The horizontal axes correspond to the temperature T from 100 GK down to 0.01 GK and the distance scale r_i normalized to 64 dimensionless units, and the vertical axis is number density $n(i, s)$ normalized to the initial baryon density (n_{b0}). At $T = 100$ GK number densities in the inner shells versus the outer shells are large. Weak reactions convert neutrons to protons until they no longer become significant at around $T = 13$ GK. Figure 2a shows the neutron–proton conversion rates for shell 3, a high density inner shell (normalized to n_{b0} and to the expansion rate of the universe $\dot{\alpha}$). Figure 2a also shows that around $T = 6$ GK neutrons start to diffuse significantly into the lower density outer shells. But since the conversion rates are no longer effective at this temperature, the protons in the inner shells remain there. At around $T = 2.5$ GK the neutrons are homogeneously distributed throughout the whole sphere.

Nucleosynthesis occurs at about $T = 0.9$ GK, when the nuclear reaction n + p $\leftrightarrow$ d + γ falls out of equilibrium. Other nuclear reactions then build heavier nuclei, but most baryons wind up as part of ^{4}He nuclei. Because of higher density, nucleosynthesis occurs earlier in the inner shells, depleting neutrons. So neutrons diffuse back into the inner regions as shown on the right of Figure 2a. Figure 2b shows the same reaction rates for shell 62, a low density outer shell. The rate for n + p $\leftrightarrow$ d + γ is lower and flatter than in Figure 2a. Figure 1 shows that most neutrons back diffuse into the inner shells and then undergo nucleosynthesis, leading to ^{4}He production overwhelmingly concentrated in the inner shells.

The final mass fraction $X_{^4\text{H}e}$ of the whole model is 0.243664. This is the result for $r_i = 25000$ cm and $\eta_{10} = 3$ Table 1 shown below lists $X_{^4\text{H}e}$ for $r_i = 100$ cm up to 6.3×10^6 cm. For r_i less than 25000 cm neutron diffusion occurs early enough to coincide at least partly with the time when the conversion reactions are significant. In the inner regions the reactions convert protons into neutrons that then diffuse to the outer regions where they get converted back to protons. But as r_i gets larger more protons remain in the inner regions and $X_{^4\text{H}e}$ increases. $r_i = 25000$ cm is the turning point when all the protons in the innermost regions remain there. For larger r_i the back diffusing neutrons cannnot reach the innermost regions before nucleosynthesis is completed. So with fewer neutrons in the inner regions $X_{^4\text{H}e}$ decreases. At $r_i = 7.9 \times 10^5$ cm though diffusion becomes significant at $T = 0.9$ GK, right during nucleosynthesis. For larger r_i nucleosynthesis occurs before neutron diffusion and the inner regions, with high proton and neutron densities, produce enough ^{4}He to raise $X_{^4\text{H}e}$ significantly.

3. CONCLUSIONS AND FUTURE RESEARCH

Figure 3a shows $X_{^4\text{H}e}$ for a range of η from 10^{-10} to 3×10^{-9} as well as the range of r_i listed in Table 1 For all values of η shown one can see the turning points at which $X_{^4\text{H}e}$ increases, then decreases and then increases again. The code can generate contour maps for 67 other isotope species as well. Comparing the contour maps of $X_{^4\text{H}e}$, $Y(\text{d})/Y(\text{p})$ ($Y(\text{d})$ is the abundance of deuterium), and $Y(^7\text{L}i +^7 \text{B}e)/Y(\text{p})$ to the 2σ ranges of recent observations (Izotov & Thuan 1998; Burles & Tytler 1998; Pinnsoneault et al. 1998):
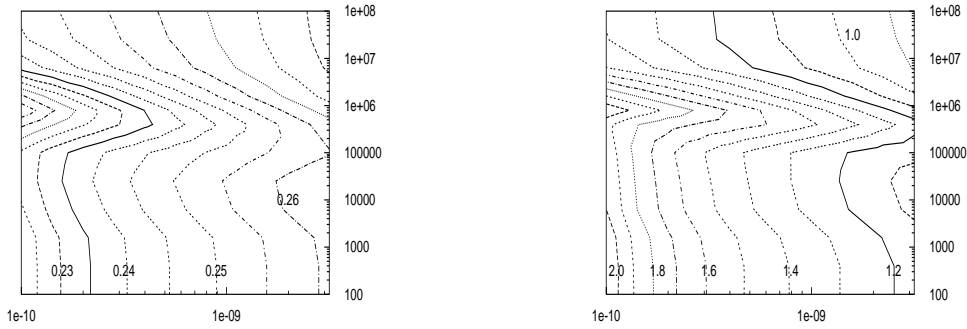
Fig. 3. $X_{^4\mathrm{H}e}$ and $10^4 \times \Delta X_{^4\mathrm{H}e}$

$$
\begin{aligned}
X_{^4\mathrm{H}e} &= 0.244 \pm 0.002, \\
Y(\mathrm{d})/Y(\mathrm{d}) &= (3.4 \pm 0.3) \times 10^{-5}, \\
\log[Y(^7\mathrm{L}i)/Y(\mathrm{d})] &= -9.45 \pm 0.20,
\end{aligned}
$$

the author determined an observational range for η from 4.7×10^{-10} to 6×10^{-10}.

A planned article will describe ^{4}He production in greater detail, describing results for values of r_i other than 25000 cm. R_ρ, r_b and the geometry of the model can be varied in the code. In the future the author will look at results with different values for these parameters. Also, the code has written into it subroutines to calculate the neutrino heating effect. Electrons and positrons annihilate each other from $T = 5$ GK to $T = 1$ GK. Through annihilation and scattering they can transfer some energy to neutrinos, which in turn can alter the neutron–proton conversion reaction rates and ultimately ^{4}He production. Figure 3b shows the change $\Delta X_{^4\mathrm{H}e}$ of the mass fraction of ^{4}He for the same ranges of η and r_i as in Figure 3a. $\Delta X_{^4\mathrm{H}e}$ imitates $X_{^4\mathrm{H}e}$, but not exactly. The change also remains within an order of 10^{-4}. The author will determine if the change is equivalent to a constant shift of Figure 3a to the left, and if the coding for the neutrino heating effect is correctly represented in the code.

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