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COMPACT STAR COOLING BY MEANS OF HEAT WAVES

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RESUMEN

La teoría de enfriamiento de las estrellas compactas es revisada usando la ley de Cattaneo para el flujo de calor. Este formalismo predice cambios en la ecuación de transporte de energía, insinua pulsos cuasiperiódicos en la luminosidad y que la energía es propagada por ondas térmicas modificando el tiempo de enfriamiento. Se sugieren aplicaciones en el estudio de las variaciones rápidas de luminosidad en enanas blancas y en la de emisión de pulsos por estrellas neutrónicas.

ABSTRACT

Compact star cooling theory is revised using the Cattaneo law for the heat flux. It is shown changes in the energy transport equation, insinuates quasiperiodic pulses in the luminosity and predicts that the energy is spread by heat waves changing the cooling time. Applications in rapid variations in single white-dwarf oscillators and quasi periodic luminosity pulses of neutron stars are suggested.

Key Words: **STARS: COMPACT – STARS: COOLING — STARS: NEUTRON**

1. INTRODUCTION

Cooling of white dwarfs (WDs) and neutron stars is determined by the rate of energy flow through internal layers. In general the possibility of heat wave propagation is obviated, a simplification that can be spurious in degenerate material where relaxation is not necessarily negligible. This article describes the cooling time and luminosity in WD and NS stars when heat waves are considered, replacing the Maxwell-Fourier equation by the Cattaneo causal law in the energy transport equation (§2). An outline of cooling theory of NS and WDs is presented in §3, with a short discussion of astrophysical applications in the conclusions.

2. THE CATTANEO LAW AND THE ENERGY TRANSPORT EQUATION

In energy transport theory the temperature gradient is given in terms of local values of opacity κ, density ρ and energy flux F by (Shapiro & Teukolsky 1983, ST83) $dT/dr = -\left(3\kappa\rho/4acT\right)F$, where a is the radiation-density constant and c the speed of light. This is the Fourier-Maxwell law for energy flux due to thermal conductivity and/or radiative diffusion, $\vec{F}(\vec{x},t) = -\kappa\nabla T(\vec{x},t)$, which leads to a parabolic equation for T, according to which perturbations propagate with infinite speed (Joseph & Preziosi 1989). The origin of this non-causal behavior is the implicit assumption that the energy flux appears at the same time as the temperature gradient. Neglecting the relaxation time, τ, is, in general, a sensible thing to do because for most materials τ is very small ($\sim 10^{-11}$ s for phonon-electron interactions and $\sim 10^{-13}$ s for phonon-phonon and free electron interactions, at room temperature). There are, however, situations where τ may not be negligible: for superfluid He II, at $T \sim 1-2°$K, $\tau \sim 10^{-3}$ s, and in NS interiors $\tau \sim 10^{2}$ s for $T \sim 10^{6}$ °K (Herrera & Falcón 1995). The proper heat flux equation, which leads to a hyperbolic equation, is the Cattaneo law (Joseph & Preziosi 1989):

$$\vec{F}(\vec{x},t) = -\frac{\kappa}{\tau}\int_{-\infty}^{t} e^{-(t-t')/\tau}\,\nabla T(\vec{x},t')\,dt' \,. \qquad (1)$$

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3. COOLING OF NEUTRON STARS AND WHITE DWARFS

Following ST83, and using $\nabla T \approx DT/R \equiv (T_{center} - T_{surroundings})/R$, the total luminosity $L = -4\pi R^2 F$, is given by $L = C_v\,(d(DT)/dt) = -(4\pi R\kappa/\tau)\int_{-\infty}^{t} DT.\exp\left[-(t-t')/\tau\right]dt'$. This can be Laplace transformed to lead to: $L = \left[4\pi R\kappa\, DT(0)\exp\left(-t/\tau\right)\right] f(x,w) = L_0 f(x,w)$, with $x \equiv t/2\tau$ and $w \equiv \sqrt{4\tau/\xi - 1}$ and where

$$f(x,\omega) = \frac{1}{\omega^2+1}\left[(5+\omega^2)\cos(\omega x) - \frac{(3-\omega^2)}{\omega}\sin(\omega x)\right]\exp\left[\frac{x}{2}\left(\omega^2-1\right)\right]. \tag{2}$$

Eq. (2) relates the standard luminosity, L_0, (without heat waves) and the "true" luminosity (in presence of heat waves). For a NS of mass ratio $N \equiv M/M_\odot$, density ρ and internal temperature T_9 (in units of 10^9 °K), the total thermal energy is $U = 6\times10^{47}\,N\,(\rho/\rho_{nuc})^{2/3}\,T_9^2 \equiv AT_9^2$. The luminosity in eq. (2) is time dependent and the cooling time is defined as e-folding time through the cooling equation $\tau_v = A\,\left[T^2(fi) - DT^2(0)\right]/ < L >$ where $< L >$ is the time-averaged luminosity. For oscillatory funtions $< L >$ is very different to L_0. The cooling time might be larger or smaller than the usual cooling time depending on the relaxation time and specific heat.

Simple radiative models of WDs ignore convection completely. If the core is degenerate (or with layers superfluid He) then τ might not be negligible and the WD luminosity can admit quasi-periodic variations on time-scales smaller than τ. Using the convection theory for the energy transport in Cattaneo's régime, one finds that the luminosity has the form of equation (2) (Herrera & Falcón 1995). The condition of hidrostatic equilibrium becomes $dT/dP = (3\kappa/64\pi\sigma)(1/GM\,T^3)\,(\tau(\partial L/\partial t) + L)$. Note that for the "classical" equations of energy transport, the presence of a sub-surface convection zone can highly affect the WD rate cooling and its age. According to our energy transport equation, the WD cooling time is given by (Falcón 1997):

$$\int_0^{\tau_v}\left(1 + \frac{\tau}{\tau_d} + \frac{\tau}{f(x,\omega)}\frac{\partial f(x,\omega)}{\partial x}\right)^{-1} dt = \frac{2}{5}\left(\frac{M_\odot}{L_\odot}\right) C_v \vartheta T^{-5/2}. \tag{3}$$

When $\tau \to 0$ the cooling time has the expression found in the literature; introducing heat waves it increases.

4. CONCLUSIONS

The superfluid interior of NS facilitates the propagation of heat waves making the luminosity dependent on the time behaviour of the temperature gradient and on the envelope composition, affecting the cooling time, depending on the values assumed for relaxation time and specific heat. On the other hand, "the lattice nuclei in the crust leads to an increase in the calculated neutronic specific heat" (Lazzari & De Blazio 1997). Although beyond the scope of this work, it is pertinent to ask to what extent those estimates would change if a Cattaneo equation is used. The relaxation time for WD and quasi-periodic variation of luminosity could model the rapid variation of single WD oscillation (and/or ZZ Ceti stars) because the typical period for well studied variable WD are a few hundred seconds and the observed oscillations can not be acoustic, but may be due to a second sound model (heat waves in superfluids). Research in this way will be undertaken. In the special (critical) case of the cooling time, indicates that the age of WD will be two times greater, when the causal propagation of heat is considerated. Also the WD cooling time (coolest objects) is very sensitive to luminosity and this can charge the results for the oldest stellar ages and the galactic disk. However, useless to go deeper into the explanation of these fluctuations until uncertainties pertaining the numerical values of relaxation time is dissipated.

REFERENCES

Falcón, N., 1997, Proceedings of the I Congreso Venezolano de Física. Ciencia (in Press).
Herrera, L., & Falcón, N. 1995, Astrophys. Space Sci. 234, 139.
Joseph, D., & Preziosi, L. 1989, Rev. Mod. Phys 61, 41.
Lazzari, G., & De Blasio, F. V. 1997, Astro. Lett and Comm. 35,339
Shapiro, S., & Teukolsky, S. 1983, Black Holes, White Dwarfs and Neutron Stars , Wiley eds