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ON THE LIMITS OF APPLICABILITY OF CHANDRASEKHAR'S DYNAMICAL FRICTION FORMULA

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RESUMEN

En este trabajo se discuten algunas limitaciones de las ventajas de introducir el concepto de fricción dinámica, así como algunos factores que pueden hacer que la fórmula de Chandrasekhar produzca resultados incorrectos.

ABSTRACT

In this work some limitations on the advantage of introducing the concept of dynamical friction and some facts that can drive Chandrasekhar's formula to yield misleading results are discussed.

Key Words: **CELESTIAL MECHANICS — STELLAR DYNAMICS**

1. DISCUSSION

Much of the work done in stellar dynamics lies upon the assumption that stellar systems can be modeled as ensembles of point masses interacting by means of the Newtonian law. In most cases of interest in Stellar Dynamics, the time interval during which the interaction due to an encounter is considerable, is very short in comparison with the time elapsed between two successive events. This fact justifies the assumption of individual binary encounters from which Chandrasekhar (1943) derived his famous formula. However, the introduction of the concept of dynamical friction as a force that an individual particle suffers during a shorter timescale than the relaxation time is meaningless in several cases of astrophysical interest.

In the classical derivation of the dynamical friction formula, a Maxwellian velocity distribution for field particles $f(v_f, \theta, \varphi)$ is assumed and the distribution of encounters

$$f_E = f(v_f, \theta, \varphi) 2\pi D W(v_f, v_t, \theta),$$

is obtained. Here W is the relative velocity of an encounter, v_t is the velocity of the test particle, and D is the impact parameter. Thus, after a time interval Δt, the total velocity change in the direction of movement of the test particle $\sum \Delta v_{//}$ is evaluated as the integral of $\Delta v_{//} \cdot f_E$ over the parameters of encounters.

The caveat is that, due to the discreteness of encounters, this integral is not necessarily equal to the sum over individual events, except when, as in Kalnajs (1972) approach, it is valid to treat the media as a fluid. Let us suppose that the closest encounter occurs at an impact parameter value D_1. Then, because of the logarithmic dependence on the impact parameter cutoff value of Chandrasekhar's formula, half the contribution to the integral over impact parameters will come from values between D_1 and $D_2 = \sqrt{D_1 D_{max}}$, being D_{max} the maximum impact parameter to be considered. Then, we can infer that almost fifty percent of the contribution to the dynamical friction comes from the few events with $D < D_2$, which is equivalent to the square root of the total number of encounters. Moreover, the velocity change due to the closest interaction is not negligible when compared to the whole sum.

For the Chandrasekhar's formula to make sense, the sum of the velocity increments $\Delta \mathbf{v}$ of the scarce closest events must be negligible in comparison with the sum of $\Delta v_{//}$ due to the whole rest of encounters. Otherwise, the statistical noise arising from the small amount of strong interactions will be larger than any systematic cumulative effect of all the rest.

As Chandrasekhar (1943) points out, the time interval Δt must be "long compared to the periods of elementary fluctuations but short compared to the time interval during which $\mathbf{v_t}$ may be expected to change appreciably". However, this does not occur in most of realistic cases.

We call T_S the time interval after which the systematic cumulative effect of encounters equals its standard deviation and T_D the time of relaxation (Chandrasekhar 1942). Within a reasonable approximation we have

$$T_S/T_D \sim 4\,(\sigma/v_t)^2\,\left(m_f/m_f + m_t\right)^2,$$

where σ is the velocity dispersion of field particles and m_t and m_f are the masses of the test particle and field particles. We need $T_S \ll \Delta t \ll T_D$ for the dynamical friction formula to hold. In cases in which $v_t \gg \sigma$, the dynamic drag diminishes drastically while T_S grows, that is, T_S could become longer than the age of the universe. Now, we could well think that if, for instance, $m_t \approx 10 m_f$ and $v_t \approx 3\sigma$, we could trustfully use dynamical friction formula predictions after some few times T_S.

In order to test how the statistical noise due to the discreteness of encounters affects the predictions of the dynamical friction formula, a series of Monte Carlo experiments has been performed. The results showed that the systematic effect of dynamical friction is appreciable only when $m_t > 50 m_f$. Nevertheless, the noise is excessively high even in such cases.

2. CONCLUSIONS

Chandrasekhar's formula computes the systematic cumulative effect of the large amount of interactions that a test particle suffers. However, as shown in this work, the discreteness of encounters introduces such a high noise level that, in most cases, makes analytical predictions inapplicable to an individual particle. The only exceptions correspond to the cases where $m_t \gg m_f$ or $v_t \gg \sigma$.

On the other hand, the dynamical friction formula has been widely used in numerical simulations to model the interaction between different stellar systems. As a typical example we could recall the problem of a globular cluster traveling across a galaxy halo. In such case, the amount of encounters is indeed very high, and it is possible to argue that the media can be treated as a fluid since the statistical noise vanishes.

Nevertheless, there still remain some problems listed below:

(i) When $m_t \gg m_f$ the gravitationally disturbed region around m_t grows up. Therefore, we certainly should reconsider the right choice of D_{max}. Even more important is the fact that very massive stars die young, without having the opportunity to experience dynamical friction.

(ii) If the disturbed region becomes comparable in size with the host system, then the hypothesis of "an infinite, uniform medium" will turn inappropriate and the utilization of Chandrasekhar's formula lacks physical justification.

(iii) If $v_t \gg \sigma$, we should take care of replacing v_{typ}^2 by v_t^2 in the Coulomb logarithm (see Chandrasekhar, 1942). This is because the approximation used to justify the choice of v_{typ} is no longer valid when $v_t \gg \sigma$. Anyway, in these cases the frictional force vanishes and the time scale of the process could become longer than the age of the universe.

(iv) The dynamical friction formula as been derived assuming the test object to be a point mass. White (1976) calculated how to deal with extended objects (he introduced a minimum impact parameter). However, when the test object itself is not a rigid body but a small stellar system sensitive to tidal forces, the blind use of the dynamical friction formula can be a dangerous extrapolation.

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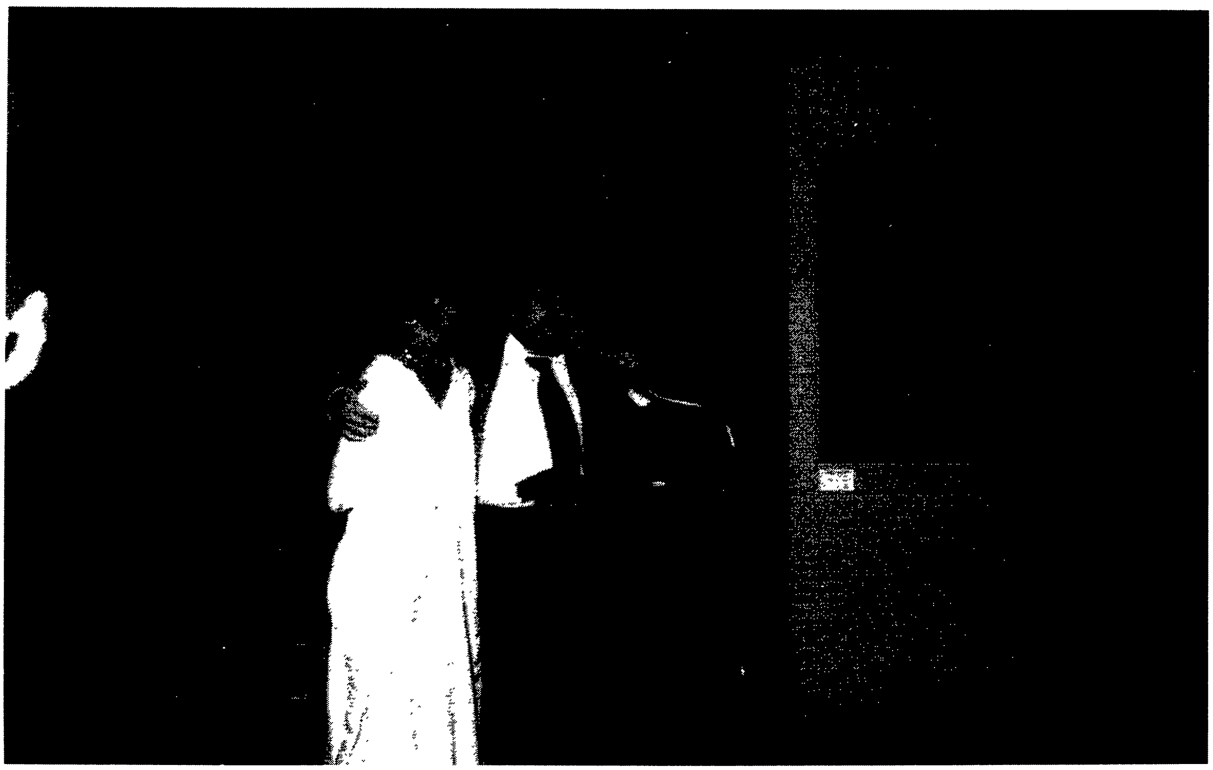

Alfonso Serrano presenting Edna García, author of the paintings of the exhibition “Oteando el Universo”.