# STAGNATION KNOTS IN PRECESSING JETS 

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#### Abstract

RESUMEN Se investiga el problema de la formación de nudos de estancamiento en la cabeza de un jet estelar en precesión desde los puntos de vista analítico y numérico. Esta estructura de alta densidad se forma vía un método similar al que propuso Steffen (estas memorias) para la formación de FLIERs en nebulosas planetarias. Resultados iniciales sugieren que la formación de un nudo de estancamiento posiblemente sea una inestabilidad intrínseca de jets en precesión.


#### Abstract

The problem of the formation of a stagnation knot in the head of a precessing stellar jet is investigated from both an analytical and numerical point of view. This dense structure is formed via a similar method to that proposed by Steffen (these proceedings) for the formation of FLIERS in planetary nebulae. Initial results suggest that the formation of a stagnation knot is possibly an intrinsic instability in precessing jets

\section*{Key Words: ISM: MOLECULAR CLOUDS - ISM: SHOCKS - STARS: FORMATION — STARS: JETS AND OUTFLOWS}


## 1. INTRODUCTION

Precessing jets from protostellar sources are becoming more common in the observations (see Palacios, these proceedings). A heavy precessing jet will have a fundamentally different bowshock structure to that of a similar steady jet. We study here the jet propagation and the ring-like structure formed at the head of the bow shock as a result of the impact point of the jet stem having a circular motion in the cone traced out by the precession of the jet velocity vector.

We derive a semi-analytical formula for propagation of a precessing jet and the collapse of the bowshock. Several very strong approximations have been necessary to make headway with this problem, but a comparison with bowshock simulations shows the effect of these to be not too severe. A fuller account of this work is presented in Lim (2001b).

## 2. THE PROPAGATION OF A PRECESSING JET

We give here a brief description of the derivation of the propagation equation for a precessing jet. The advance speed of a steady jet is,

$$
\begin{equation*}
v_{\mathrm{a}}=\frac{\beta v_{\mathrm{j}}}{1+\beta} \tag{1}
\end{equation*}
$$

where, $\beta=\sqrt{n_{\mathrm{j}} / n_{\mathrm{e}}}$. In general this approach is not appropriate for a precessing jet, as the impact point
of the jet stem on the bow shock is not stationary. If the leading edge of the jet stem is coincident with the point A at time, $t$, then the point, A , receives ram pressure support from the jet stem until the following edge of the jet stem just passes A at time, $t+\tau$. Trigonometrical arguments yield the time interval, $\tau$ to be,

$$
\begin{equation*}
\tau=\frac{2}{\omega} \sin ^{-1}\left(\frac{r_{\mathrm{j}}}{2 d}\right) \tag{2}
\end{equation*}
$$

For the rest of the precession period, the bow shock at A coasts into the environment. When the leading edge of the bow shock returns to A we assume that the advance speed at A is "reset" to $v_{a}$. If the momentum, $P_{\tau}$, imparted during $\tau$, is equivalent to the jet pointing directly at A for $\tau$ seconds and results in a cylindrical section of the jet below A. If we neglect sideways ejection of material and the effect of thermal pressures, then the sweeping up of environmental gas results in the jet section, undergoing the deceleration,

$$
\begin{equation*}
\frac{d v(t)}{d t}=-\frac{\pi r_{\mathrm{j}}^{2} v(t)^{3} \rho_{\mathrm{e}}}{P_{\tau}} \tag{3}
\end{equation*}
$$

This equation can be integrated twice to give,

$$
\begin{equation*}
x(t)=\frac{P_{\tau}}{\pi r_{\mathrm{j}}^{2} \rho_{\mathrm{e}}}\left[\left(\frac{2 \pi r_{\mathrm{j}}^{2} \rho_{\mathrm{e}} t}{P_{\tau}}+\frac{1}{v_{\mathrm{a}}^{2}}\right)^{1 / 2}-\frac{1}{v_{\mathrm{a}}}\right] \tag{4}
\end{equation*}
$$

Within our assumptions, we can now find the advance distance of the point $A$ in a single precession
period. Substituting in the form of $P_{\tau}$ described above and applying a generous amount of manipulation, the total distance advanced is,

$$
\begin{gather*}
D=v_{\mathrm{a}} \tau+u \tau\left[\left(\frac{2 v_{\mathrm{a}}}{u} \frac{T-\tau}{\tau}+1\right)^{1 / 2}-1\right]  \tag{5}\\
u=\frac{\rho_{\mathrm{j}} v_{\mathrm{j}}+\rho_{\mathrm{e}} v_{\mathrm{a}}}{\rho_{e}}
\end{gather*}
$$

This is valid for $\tau<T$, but is not sufficient to define the propagation distance of the jet. A starting point is provided by making the assumption that for $d<r_{\mathrm{j}}$, the jet propagates as a steady jet. We can at least be sure that the point at the centre of the precession cone receives ram pressure support while this condition is satisfied. If we ignore distortion of the jet due to the precession, we can say that a transition occurs at the point $d=r_{\mathrm{j}}$ from the behaviour of a steady jet to that of a precessing jet. This defines a transition height, $h_{\mathrm{t}}=r_{\mathrm{j}} / \tan \theta$ and transition time, $t_{\mathrm{t}}=r_{\mathrm{j}} /\left(v_{\mathrm{a}} \tan \theta\right)$. Thus, for $t<t_{\mathrm{t}}$ we have,

$$
\begin{equation*}
h=\frac{r_{\mathrm{j}}}{\tan \theta}+n D \cos \theta \tag{6}
\end{equation*}
$$

and a distance along the ejection vector,

$$
X=\frac{h_{\mathrm{t}}}{\cos \theta}+n D
$$

Due to the definitions of $t_{t}$ and $h_{t}$ above these expressions refer to the "leading point" of the bowshock ring.

One further complication exists, as the jet propagates in the precession cone it is stretched into a helix of radius $d$. This results in a linear expansion of the jet gas and as the jet moves out from the source its density drops as, $\rho_{\mathrm{j}}(h)=\rho_{\mathrm{j}, 0} r_{\mathrm{j}} /(h \tan \theta)$.

Equation (6) is not a purely analytical expression since the ratio $T / \tau$, and hence $D$ changes with $h$, but the equation can be very simply integrated numerically. Figure 1 shows a series of such integrations for jets with various opening angles as a plot of propagation distance against time. The parameters of these jets are as follows: $\rho_{\mathrm{j}}=1000 m_{\mathrm{H}}, \rho_{\mathrm{e}}=100 m_{\mathrm{H}}$, $r_{\mathrm{j}}=3.0 \times 10^{16}, v_{\mathrm{j}}=302 \mathrm{~km} \mathrm{~s}^{-1}$, and $T=200 \mathrm{yrs}$. The straight line is obviously the propagation of a steady jet $\left(\theta=0^{\circ}\right)$, and the precessing jets show the modified $D \propto t^{1 / 2}$ behaviour of equation (6). One can clearly see a large variation in the propagation distance which increases with opening angle. Even for an opening half-angle of only $1^{\circ}$ a measurable difference in the propagation distance is obtained after the jet has moved out to a distance of the order of parsecs.


Fig. 1. Several integrations of the propagation formula for jets with various precession angles.

## 3. THE FORMATION OF STAGNATION STRUCTURES

Once the impact circle widens to a given radius the central point of the bowshock will collapse and environmental material can flow inward from the, now ring-shaped, bowshock and be trapped in the central region. This is very similar to the formation mechanism of FLIERS in planetary nebulae recently suggested by Steffen (these proceedings).

In order to derive a condition for the collapse we require the radial bowshock shape, Masson \& Chernin (1993) derive an approximate shape from momentum balance arguments and Raga \& Cabrit (1993) calculate the shape of bowshocks internal to a jet cocoon which result from sideways ejection of material from a velocity variable jet. These last authors also state that the calculation of a leading bow shock shape is a difficult problem-we will attempt to make it easier.

Considering the radial structure (from the point at the centre of the bowshock ring) and neglecting azimuthal motion in the ring, we see that environmental material which encounters the bowshock can move only in one of 2 directions. Namely, outwards and down the outer face of the (global) bowshock structure or inwards towards the centre, where we assume it will eventually become part of the stagnation knot. If we also ignore radial variations in the shape of the ring bowshock, the problem can be reduced to one dimension.

Firstly, we approximate the speed, $v_{\mathrm{ej}}$, at which material is ejected sideways out from the stagnation
region of the bowshock. Applying the formulae from Raymond (1994), we can obtain temperatures for the material behind the both the jet and bowshock. Our sideways ejection will thus have 2 components, in keeping with our strong (some might say sickening) simplifying assumptions, we consider this to be a single outflow at a single velocity determined by the mean temperature,

$$
\begin{equation*}
T=\frac{1}{2}\left\{\frac{3 m}{16 k}\left[\left(v_{\mathrm{j}}-v_{\mathrm{a}}\right)^{2}+v_{\mathrm{a}}^{2}\right]\right\} . \tag{7}
\end{equation*}
$$

If we assume that the material is ejected sideways sonically then the ejection speed, $v_{\mathrm{ej}}$, will be the sound speed determined by $T$, and we can only hope that the effect of our averaging assumption is in some way reduced by the fact that $v_{\mathrm{ej}}$ depends upon $\sqrt{T}$. For strong shocks, we can (with some manipulation) obtain,

$$
\begin{equation*}
v_{\mathrm{ej}}=\frac{v_{\mathrm{j}}}{4(1+\beta)} \sqrt{3\left(1+\beta^{2}\right)} \tag{8}
\end{equation*}
$$

This equation neglects changes in the advance speed from the push/drift cycle of the precession. We can also obtain the mass loss rate, from the stagnation region as being, by definition, the inflow rate from the jet and environment. Under the assumptions above (and with more manipulation),

$$
\begin{equation*}
\dot{M}=\frac{0.5 \pi r_{\mathrm{j}}^{2} v_{j}}{1+\beta}\left(\rho_{\mathrm{j}}+\sqrt{\rho_{\mathrm{j}} \rho_{\mathrm{e}}}\right) \tag{9}
\end{equation*}
$$

So our problem has been reduced to finding the curved path of a planar outflow, velocity $v_{\text {ej }}$ and mass loss rate $\dot{M}$ which is moving relative to an environment of density, $\rho_{\mathrm{e}}$ with speed $v_{\mathrm{a}}$. Fortuitously, a very similar problem has recently been solved analytically by Cantó \& Raga (1995), and we apply their adiabatic path, which is a simple parabola given by,

$$
\begin{equation*}
z=\frac{1}{2 \lambda} y^{2} \tag{10}
\end{equation*}
$$

where, $y$ and $z$ are coordinates measured from the stagnation point of the path. $\lambda$ is a constant denoting the radius of curvature of the jet path at the stagnation point. Substituting the expressions above into the definition of $\lambda$ and manipulating as far as possible, yields,

$$
\begin{equation*}
\lambda=\frac{r_{\mathrm{j}}}{4}\left[\left(1+\beta^{-1}\right) \sqrt{3\left(1+\beta^{2}\right)}\right]^{1 / 2} \tag{11}
\end{equation*}
$$

Figure 2 shows a schematic diagram of the propagation of the precessing jet. At time, $t$, the bowshock on the right hand side of the precession cone is at point, A this is also the impact point of the


Fig. 2. Schematic illustration of the propagation of a precessing jet over half a period.
jet stem at time, $t$. After half of one precession period the impact point of the jet stem has moved to B , and the bow shock on the right has drifted to C . Also shown is the coordinate system, $y-z$, which pertains to equation (10).

We also require the distance, $Z$, from D to C . Geometrically, this is,

$$
Z=X_{1}(1-\cos 2 \theta)+D(T / 2)
$$

For a knot to be formed, the distance from the line, $X_{2}$, to the bowshock formed at C must be less than $l=X_{1} \sin 2 \theta$, Our knot formation condition is therefore,

$$
\begin{equation*}
X_{1} \sin 2 \theta>\sqrt{2 \lambda Z}+r_{\mathrm{j}} \tag{12}
\end{equation*}
$$

where $r_{\mathrm{j}}$ is added since we measure $y$ from the edge of our jet section.

## 4. 3D SIMULATIONS OF STAGNATION KNOT FORMATION

We use the 3D version of the Reefa linked adaptive grid code which solves the well known Euler fluid


Fig. 3. Several timeslices of the 3D simulation. The dotted line represents the predicted collapse distance and the solid bars the predicted propagation distances
dynamic equations on a 3-dimensional binary adaptive grid. The technical aspects of the code are described in detail in Lim \& Steffen (2001) and Lim (2001a).

A several simulations performed to with which the analytical results were compared, only one is shown here. The parameters of this simulation are: $r_{\mathrm{j}}=1.2 \times 10^{17} \mathrm{~cm}, \rho_{\mathrm{j}}=1000 \mathrm{~cm}^{-3}$, $v_{\mathrm{j}}=307.7 \mathrm{~km} \mathrm{~s}^{-1}, \theta=12.5^{\circ}$, and $T=400$ yrs. This jet propagates into a stationary, quiescent medium of density, $\rho_{\mathrm{e}}=100 \mathrm{~cm}^{-3}$, and the initial jet and environment temperatures are $10,000 \mathrm{~K}$. Timeslices from this simulation are shown in Figure 3. It can be seen that the estimated propagation distance (shown
by solid bars) matches well with the numerical simulation. We also see a good match for the predicted height of the bow shock collapse (dotted line in Fig. 1).

## 5. CONCLUSION

We have derived from a semi-analytical point of view the propagation of a precessing jet and the formation of stagnation structures in the bowshock of such a jet. It is found that precession results in the advance of the leading point of the bowshock takes place according to a modified $t^{0.5}$, law as opposed to the linear propagation off a steady jet. For opening angles larger than a few degrees, a precessing jet will have propagated a much smaller distance in a given time than an otherwise identical steady jet.

We have also derived a condition for the collapse of the central point of the bowshock due to a lack of ram pressure from the jetstem which impacts in a circle around the this point. This is also seen in simulations, matching well with the distances predicted in the analytic model. The formation of stagnation features in concave sections of the bowshock may leave a tracer in jets which have undergone episodic precession. Alternatively, in continuously precessing jets, this process will result in one or more dense features in the head of the jet. Given strong molecular cooling, these stagnation features may be long lived and even drift ahead of the decelerating bowshock of a large-angle precession jet. Precessing jets are, therefore, a possible source of high velocity clumps of material in star-forming regions. Such ejecta are likely to be chaotically structured due to the time dependent nature of their formation, giving rise to the idea of an "interstellar blunderbuss".

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## REFERENCES

Cantó, J ., \& Raga, A. C. 1995, MMNRAS, 277, 1120
Lim, A. J. 2001a, MNRAS, 321, 306
Lim, A. J. 2001b, MNRAS, 327, 507
Lim, A. J. \& Steffen, W. 2001, MNRAS, 322, 166
Masson, L., \& Chernin C. 1993, ApJ, 414, 230
Raga, A. C.,\& Cabrit, S. 1993, A\&A, 278, 267
Raymond, J. C. 1995, PASP, 103, 781
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