RevMexAA (Serie de Conferencias), **15**, 211–216 (2003)

THE PARTITION BETWEEN TERMINAL SPEED AND MASS LOSS: THIN, THICK, AND ROTATING LINE-DRIVEN WINDS

K. G. Gayley and A. J. Onifer

University of Iowa, USA

RESUMEN

Los vientos supersónicos en estado estacionario impulsados por líneas contribuyen a la formación de las burbujas en las regiones de formación estelar. La aportación clave a la burbuja en su fase de conservación de energía es el flujo de energía cinética del viento, el cual involucra tanto la tasa de pérdida de masa como la velocidad terminal del viento. Sin embargo, estas cantidades son parámetros autoconsistentes del proceso de impulso por líneas y por lo tanto se relacionan entre sí y con la profundidad óptica del viento resultante. Esta interrelación compleja entre la profundidad óptica, la pérdida de masa y la velocidad del viento reside en el meollo de la teoría de los vientos impulsados por líneas. Recurriendo a los éxitos y perspicacias de la teoría "CAK", transmitiré una visión simplificada de como unir estos procesos por medio del concepto de la opacidad efectiva, con atención a las consecuencias para las estructuras asféricas y formadas por vientos. Se discuten también las extensiones recientes a los ambientes ópticamente gruesos no grises tales como los vientos de las estrellas Wolf-Rayet y las supernovas.

ABSTRACT

Steady-state supersonic line-driven winds are important contributors to wind-blown bubbles in star forming regions. The key input to the bubble in the energy-conserving phase is the wind kinetic-energy flux, which involves both the mass-loss rate and the terminal speed. However, these quantities are themselves self-consistent parameters of the line-driving process, so relate to each other and to the resulting wind optical depth. This complex interrelation between optical depth, mass-loss, and wind speed lies at the heart of line-driven wind theory. Drawing on the successes and insights of "CAK" theory, I will convey a simplified view of how to unite these processes using the concept of effective opacity, with attention to the ramifications for nonspherical nebular and wind-blown structures. Recent extensions to nongray optically thick environments such as Wolf-Rayet winds and supernovae are also discussed.

Key Words: **ISM: JETS AND OUTFLOWS — STARS: MASS LOSS — STARS: POST-MAIN SEQUENCE**

1. INTRODUCTION

In the energy-conserving phase, the key input to a wind-blown bubble is the kinetic energy flux of the wind from the stellar engine. The purpose of this theory review is to convey an understanding of the processes that self-consistently determine the wind mass-loss rate and its terminal speed. This will be done essentially by repackaging the celebrated breakthroughs of CAK (Castor, Abbott, & Klein 1975) into a more intuitive format that is suitable for extension to optically thick and rotating winds.

Specifically, since spherical symmetry will not in general be assumed here, the key parameters to be found are $\dot{m}$, the mass flux per unit area at the subsonic surface of the star, and a, the local acceleration of the wind. Both in general may depend on θ, the angle from the axis of rotation. The global mass-loss rate $\dot{M}$ is then given by the integral

$$\dot{M} \;=\; \int_S dA\; \dot{m} \;, \tag{1}$$

where S denotes the subsonic surface, and the terminal speed v_∞ can be found from

$$v_\infty \;=\; \int dr\; \frac{a(r,\theta)}{v(r,\theta)} \;, \tag{2}$$

integrated along the flow trajectory. Together, equations (1) and (2) may be used to determine the wind kinetic energy flux.

The key assumptions are that the wind is driven by radiative momentum intercepted by metals via

UV line scattering, and that the Sobolev approximation applies. These assumptions are interrelated because of the extreme Doppler shifting in the supersonic wind, which allows line opacity to be more important than continuum opacity, and implies that photon encounters with each line occur over such a narrow region that the entire line encounter may be treated as a single scattering event.

2. THE SOBOLEV OPTICAL DEPTH

The only parameter needed to determine whether or not such a scattering will occur in line i is called the Sobolev optical depth τ_i, and this is given by the total optical depth encountered by a photon as it redshifts across the line in the comoving frame. If the redshift relative to line center is ν, and ds is the pathlength increment, then

$$\tau_i \ \propto \int ds \ \rho \, \phi(\nu) \ , \tag{3}$$

where ρ is the density and $\phi(\nu)$ gives the frequency dependence of the cross section. Transforming to an integral over Dopper shift and taking ρ and dv/ds to be constant in the Sobolev approximation then gives

$$\tau_i \ \propto \ \rho \left(\frac{dv}{ds}\right)^{-1} \int d\nu \ \phi(\nu) \ \propto \ \left(\frac{a}{\dot{m}}\right)^{-1} \ , \tag{4}$$

assuming a steady-state mass flux so that $\dot{m} = \rho v$. Note the key simplification in the above is that the frequency dependence of the cross section is normalized so is independent of ρ and dv/ds, and the steady-state acceleration along the flow obeys $a = v\, dv/ds$.

The primary significance of the above derivation is that the Sobolev optical depth varies only with the ratio $a/\dot{m}$, the key local parameter pervading all line-driven wind theory. The radiative acceleration g_i due to line i also depends on this parameter, since

$$g_i \ = \ \frac{\Delta \dot{P}}{\Delta m} \ , \tag{5}$$

where $\Delta\dot{P}$ is the rate of momentum deposition into mass Δm. If Δv is the line-of-sight velocity width spanned by Δm, then $\Delta\dot{P} \propto \Delta v$, and furthermore the probability that a given photon will scatter in a line with Sobolev optical depth τ_i is $1 - e^{-\tau_i}$, so $\Delta\dot{P} \ \propto \ (1 - e^{-\tau_i})\Delta v$. Also, $\Delta m \ \propto \ \rho\,\delta s \ \propto (a/\dot{m})^{-1}\Delta v$, so

$$g_i \ \propto \ \left(1 \ - \ e^{-\tau_i}\right)\left(\frac{a}{\dot{m}}\right) \ , \tag{6}$$

from which we see that g_i, like τ_i, depends on wind parameters via the ratio $a/\dot{m}$.

3. THE SENSITIVITY PARAMETER α

The dependence of g_i on $a/\dot{m}$ provides the essential feedback between the wind acceleration and the line forces that provide it, and so it is useful to quantify the sensitivity of this dependence via the parameter

$$\alpha_i \ = \ \frac{d\ln g_i}{d\ln(a/\dot{m})} \ = \ 1 \ - \ \frac{\tau_i e^{-\tau_i}}{(1-e^{-\tau_i})} \ . \tag{7}$$

It is obvious that $0 < \alpha_i < 1$, and larger τ_i yields larger α_i. The total line force is the sum over all the important lines, so we write $g_{\rm lines} = \sum_i g_i$ and define the total α parameter as

$$\alpha \ = \ \frac{d\ln g_{\rm lines}}{d\ln(a/\dot{m})} \ = \ 1 \ - \ \frac{\sum_i \tau_i e^{-\tau_i}}{\sum_i(1-e^{-\tau_i})} \ , \tag{8}$$

which again obeys $0 < \alpha < 1$.

By itself, each α_i has little significance because the acceleration of O-star winds requires contribution from hundreds of lines. Lower density winds require fewer lines and higher density more, but nevertheless in most applications a large enough number of lines contribute that it makes sense to use a statistical treatment of the line distribution. When this is done, the above definition of α yields the same result as the more standard CAK line-list exponent α, where the ionization gradients that give rise to the CAK δ parameter are omitted here for simplicity.

Clearly $\sum_i \tau_i e^{-\tau_i}$ may be interpreted as $\tau_{\rm thin}$, the sum of the Sobolev optical depths of all thin lines, while $\sum_i(1-e^{-\tau_i})$ is in effect the *number* of thick lines. Thus we may supply the physical interpretation

$$\alpha \ = \ 1 \ - \ \frac{\tau_{\rm thin}}{(N_{\rm thick} + \tau_{\rm thin})} \ , \tag{9}$$

which also yields the interpretation $\alpha = g_{\rm thick}/g_{\rm lines}$ if we make the decomposition $g_{\rm lines} = g_{\rm thick} + g_{\rm thin}$, taking $g_{\rm thick} \propto N_{\rm thick}$ and $g_{\rm thin} \propto \tau_{\rm thin}$ as follows from the definition of $g_{\rm lines}$. Thus the sensitivity of $g_{\rm lines}$ to the wind response $a/\dot{m}$ is simply governed by the fraction of $g_{\rm lines}$ provided by *thick* lines, since *thin* lines already provide the maximum force per unit mass independently of variations in their Sobolev optical depths.

4. THE HYPERSONIC FORCE BALANCE

Ignoring gas pressure in the highly supersonic winds of hot stars, the hypersonic force balance may be written simply as

$$a \ = \ g_{\rm lines} \ - \ g \ , \tag{10}$$

where g is the acceleration of effective gravity (i.e., the actual gravity corrected for centrifugal forces and the radiative force on free electrons). When gas pressure is neglected and $g_{\rm lines}$ is taken to depend entirely on $a/\dot{m}$, a simple analysis shows that a maximal $\dot{m}$ exists such that no a solution is possible for larger $\dot{m}$, and multiple a solutions exist for lesser $\dot{m}$. In this simplified limit, the CAK "critical point" analysis reduces simply to the axiom that $\dot{m}$ achieves this maximum; indeed, other values have been found to be unstable in numerous time-dependent simulations.

This limit is easy to find analytically by determining the possible solutions to equation (10) for a given any $\dot{m}$, and noting that the result obeys

$$\frac{d\ln a}{d\ln \dot{m}} \;=\; \frac{\alpha(g+a)}{[\alpha g \;-\; (1-\alpha)a]} \;, \qquad (11)$$

which yields an infinite result when

$$a \;=\; \frac{\alpha}{(1-\alpha)}\, g \;. \qquad (12)$$

Thus this is the acceleration near the surface when $\dot{m}$ achieves its maximum, for any increase in $\dot{m}$ would yield a bifurcation that would leave no accelerating wind solution.

5. THE STEADY-STATE MASS FLUX

This constraint on the acceleration near the surface in turn may be used to specify the value of $\dot{m}$, which then becomes a constraint for finding the steady-state a over the rest of the wind. This may be accomplished by first determining the line-driven acceleration $g_{\dot{m}=0}$ in the limit of small $\dot{m}$, which is a simple limit because then all atoms experience the full stellar continuum. The actual line-driven acceleration for any finite $\dot{m}$ is then given by

$$g_{\rm lines} \;=\; \frac{(g_{\rm thick} \;+\; g_{\rm thin})}{g_{\dot{m}=0}}\, g_{\dot{m}=0} \;=\; \sigma\, g_{\dot{m}=0} \;, \qquad (13)$$

where the "self-shadowing parameter" σ accounts for all the Sobolev optical depth effects and is given by

$$\sigma \;=\; \frac{(N_{\rm thick} \;+\; \tau_{\rm thin})}{\tau_{\rm total}} \;. \qquad (14)$$

The above follows because if $\dot{m} = 0$, all lines are thin and $\tau_{\rm thin}$ may be replaced by $\tau_{\rm total}$, the total Sobolev optical depth of all the lines. Since σ depends only on the Sobolev optical depths of the lines, and this in turn depends on $a/\dot{m}$ with $a = \alpha\, g/(1-\alpha)$, one need merely adjust $\dot{m}$ until σ in equation (13) yields the required $g_{\rm lines} = (1-\alpha)\, g$ from equation (10).

Once $\dot{m}$ is known at the lower boundary of the wind, the geometry of the wind acceleration and the local value of the wind speed constrain the local wind density, and this in turn constrains the Sobolev optical depths and the hypersonic force balance. For example, in spherical symmetry we have simply $\rho = \dot{m}/v$, although stellar rotation complicates this expression considerably.

The reason that $\dot{m}$ is set near the surface is that the line-driven radiative force is least efficient there, owing to the local mismatch between the bright hemispheric radiation field and the relatively slow rate of Doppler shifting along oblique rays when the wind speed is low and the velocity gradient is primarily radial. Farther from the surface, the driving efficiency grows and a decouples from its value at the barely supersonic "choke point", but the steady-state mass-flux constraint continues to apply.

It is also interesting to note that the details of the radiation field, its angular character and even the total flux, do *not* appear in the constraint $a = \alpha\, g/(1-\alpha)$; their influence is entirely absorbed into $g_{\dot{m}=0}$. Thus the acceleration scale for a hypersonic CAK-type wind is determined entirely by gravity and the distribution of lines over line strength. In the case of a power-law line list, as is implicit in the CAK parametrization, the value of α depends on neither $\dot{m}$ nor a, because both $\tau_{\rm thin}$ and $N_{\rm thick}$ increase proportionately to each other as $\dot{m}$ increases or a decreases. The only way $\dot{m}$ or a affect the line-list properties is through the ionization balance, and this is often treated by introducing a new parameter δ which usually is of small importance and is not included in this pedagogical description.

A final important point about the magnitude of σ is that, for O stars, it is typically $\sigma \sim 10^{-3}$. This means that each metal atom receives on the average only about 0.1% of the continuum flux that it would receive if it were not shadowed by its neighbors. This in turn implies that if an increase in velocity is given to a small-scale optically thin group of atoms, they will experience reduced shadowing and the velocity perturbation will grow yet more. Since $\sigma \sim 10^{-3}$, this exponential instability will not saturate until the acceleration grows by a factor of order 10^3, yielding a very strong instability indeed. Numerical simulations of this line-driven instability track the formation of rarified regions and dense shells, implying that a smoothly accelerated flow is formally impossible. Fortunately, the scale of the instability is small enough that the global flow properties of the wind continue to be surprisingly well described by smooth-wind CAK theory.

6. FORCE LEVERAGING AND THE DETERMINATION OF THE TERMINAL SPEED

Once $a = \alpha g/(1-\alpha)$ is known at the lower boundary of the wind, it is tempting to expect that this proportion will remain fixed over the wind, since both g and the radiative flux fall off like $1/r^2$. However, as mentioned above, the force efficiency increases with radius, and these increases get further "leveraged" by the feedback inherent in line driving. This leveraging occurs because if the line force is increased a small amount, then the acceleration will also increase, but this will in turn reduce the degree of self-shadowing so increase the σ factor and the resulting line force. Thus increases in $g_{\rm lines}$ seed further increases, and this feedback can substantially "dress" seemingly unimportant changes in the force.

The degree of force leveraging may be quantified by noting that if $g_{\rm lines}$ in equation (10) is increased by an amount dg, then the new solution for a is altered by an amount da, where da/dg is not unity but rather

$$\frac{da}{dg} = \frac{1}{(1-\alpha)}\left[1 - \frac{\alpha g}{(1-\alpha)a}\right]^{-1} . \qquad (15)$$

This is the leveraging factor, and it is always at least as large as $1/(1-\alpha) \sim 3$. Even more significantly, close to the surface where $a \cong \alpha g/(1-\alpha)$, the leveraging factor is *unbounded*. Physically, this is because changes in the driving force at the critical point must increase the *mass flux* but, just above the point where the mass flux is set, force increases will instead increase the local *acceleration*.

Thus the net result of the expected increases in force efficiency with radius is that gravity quickly becomes almost negligible, and the resulting free acceleration achieves kinetic energies that typically exceed the gravitational lifting energy by an order of magnitude or more (Gayley 2000). This "afterburner" effect substantially enhances the kinetic energy input to the wind-blown bubble.

7. OPTICALLY THICK WINDS

When each UV continuum photon encounters many lines as it diffuses through the wind, the radiative environment is more isotropic. However, since it is the *force* we are concerned with, we only need *odd* moments of the radiation field. To lowest order this is quite similar to the radiative flux, which is the same in optically thick and thin winds due simply to the radiative equilibrium constraint. Thus when the opacity in thick winds is treated as effectively gray, there is no fundamental difference between the driving of thick winds and thin winds in CAK theory; all that is needed is additional opacity to explain why Wolf-Rayet winds are so much denser than their O-star progenitors.

On the other hand, gaps in the Wolf-Rayet spectrum, especially in the visible, present a challenge to the driving efficiency because frequency redistribution will tend to channel flux into spectral regions with longer mean-free-paths between line encounters. The impact of this process may be treated approximately in the spirit of a Sobolev-modified Rosseland mean opacity, and it is found that for thick enough winds, the flux as a function of wavelength will be inversely proportional to the line density in the vicinity of that wavelength. Thus to support efficient driving in optically thick winds, it is necessary to maintain fairly close spacing between optically thick lines over the majority of the stellar continuum. Again, this is a challenge for opacity modelers to make sure that not only are enough strong lines present, but that they are suitably spread over the spectrum. Recent advances suggest that Wolf-Rayet winds may soon be explained with essentially the same CAK-type line-driving formalism that is so successful for O stars.

8. STELLAR ROTATION AND RADIATIVE TORQUE

The inclusion of stellar rotation breaks the radial nature of the line driving process, since both the flux and the opacity become nonradial, and so radiative torques appear. These torques exist in both the axial and poleward directions. The increased axial opacity is in the retrograde direction due to the combination of radial acceleration with azimuthal quasi-orbital motion, and this causes an estimated 30% spindown of the wind angular momentum (Gayley & Owocki 2000). Meanwhile, the increased poleward opacity, due to the more rapid acceleration over the poles where the effective gravity g is stronger, causes a poleward deflection of the line force (Owocki, Cranmer, & Gayley 1998) that inhibits the formation of rotationally induced equatorial disks called wind-compressed disks (Bjorkman & Cassinelli 1993).

Other processes that also favor poleward wind enhancements are the oblate shape of the rotating star, which tilts poleward the surface normal and the radiative flux, and von Zeipel-type gravity darkening, which increases the radiative flux over the poles at the expense of the bloated equatorial region (Maeder & Meynet 2000). Since the winds are radiatively driven, the combination of increased flux at the poles and the poleward turning of the flux, along with poleward line-opacity increases, favor *prolate* winds rather than disk formation (Owocki et al. 1998). This leaves as a puzzle the disk-like structures observed around B and B[e] stars, and although variations of opacity with temperature may help recover oblate structures (Lamers, Snow, & Lindholm 1995), the problem of disk formation may require consideration of interior physics quite separate from the wind driving.

On the other hand, prolate structures are often seen in the nebulae around evolved stars. The celebrated *HST* images of η Carinae, for example, show clearly bipolar ejecta along with evidence for an equatorial skirt. One popular model to explain this involves interactions between the changing winds emitted by the star as it evolves over time (Frank, Ryu, & Davidson 1998; Langer, García-Segura, & Mac Low 1999). As the stellar radius bloats up in an LBV phase, the wind is expected to be slow, and this should be followed by a much faster wind as the star contracts and gets hotter. If the LBV wind is equatorially enhanced, perhaps by opacity effects, then the subsequent fast wind may be channeled poleward by the circumstellar material, yielding a bipolar nebula. Simulations support the plausibility of this model, yet it is interesting to point out that bipolar enhancements might also occur as a natural consequence of line driving from rapidly rotating stars.

9. RADIATIVELY ASSISTED ROTATIONAL BREAKUP

The most direct way to yield equatorial enhancements is via critical rotation, at which point the rapidly rotating star experiences a strong enough centrifugal force to eject equatorial gas into orbit. Interestingly, for extreme stellar luminosities, such equatorial breakup may be *preceded* at lower rotation rates by a wind-driven breakup that occurs primarily at the *poles*, due to polar brightness enhancements from gravity darkening. The potential for this type of mechanism to yield bipolar nebulae has only recently begun to be explored (Maeder & Meynet 2000).

A key issue in the analysis of the mass loss from gravity darkened stars is how the volume of the star responds to the rotation and the radiation. Generally speaking, the more the equator-to-pole aspect ratio approaches its maximal value of 3/2 due to centrifugal forces, the closer the star gets to equatorial breakup, whereas the more the polar radius puffs out due to the enhanced polar radiation, the closer the star gets to radiatively driven bipolar breakup. Advances in interior models are now constraining the volume, shape, and total luminosity of rapidly rotating stars, and Figure 1 gives the global line-driven mass-loss ratio between a critically rotating rigid star and a nonrotating star with the same luminosity and polar radius. Note that the rapidly rotating mass-loss rate may become extremely large if $\Gamma > 0.64$, in agreement with Maeder (1999).

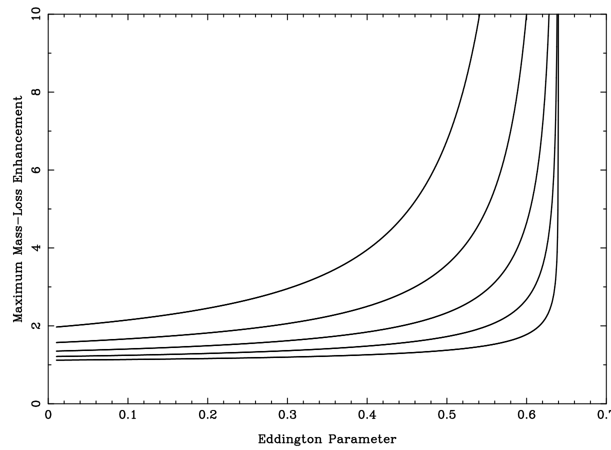

Fig. 1. The factor by which the line-driven mass-loss rate is enhanced for a star approaching critical rotation, as a function of the Eddington parameter Γ. Results are shown for $\alpha = 0.4$, 0.5, 0.6, 0.7, and 0.8, where the smaller the α the greater the mass-loss enhancement.

10. CONTINUUM-DRIVEN WINDS

LBV models often invoke super-Eddington luminosities to yield extremely dense winds driven by continuum opacity. Continuum opacity can only drive a wind at all when the Eddington parameter exceeds unity, since it lacks the self-regulation of line driving. For continuum driving, if the force exceeds gravity at some density, it will exceed gravity at all densities until the continuum opacity itself changes. Thus very high density winds, such as LBVs, may be driven, and the only ultimate limit is the energy content of the driving radiation.

Since continuum-driven winds have no self-consistent feedback mechanism to determine the mass-loss rate, the only constraint is that the radiative flux must be capable of lifting the mass through the gravitational potential. This constrains their response to gravity darkening and centrifugal forces in the presence of rapid stellar rotation.

Even near critical rotation, the surface kinetic energy is at most half the gravitational escape energy, so substantial energy input will still be necessary to drive the wind to infinity, and to within a factor of two the maximum allowable local mass flux density will be proportional to the local radiative flux density. Since the local radiative flux is also proportional to the effective gravity, this yields nearly the same dependence as for CAK-type winds, and again prolate flows are to be expected. On the other hand, when rapid rotation induces the required "super-Eddington" radiative flux needed for continuum driving, it may be very efficient at lifting critically rotating material into *orbit* around, as opposed to escaping from, the star.

However, it has been argued by Shaviv (2001) that such super-Eddington winds are unstable to clumping, and the radiative flux will then flow through gaps between the clumps, reducing the average driving efficiency to sub-Eddington levels. Thus the Eddington luminosity may be exceeded without breaking apart the star, and the inherently inhomogeneous winds driven in this situation would be efficient at converting radiative momentum but not radiative energy to the bulk wind flow. Thus if stellar rotation and gravity darkening create locally super-Eddington radiation fields, the response of the star may be quite complicated and self-consistent models may need to resolve small-scale inhomogeneities. Clearly, the combination of stellar rotation and mass loss yields theoretical challenges that we have only begun to address, but the current revolution in observational imaging and spectroscopy makes this a timely topic for sweeping advances.

I would like to acknowledge scientific assistance from S. P. Owocki and financial assistance from NSF grant AST-0098155.

REFERENCES

Bjorkman, J. F., & Cassinelli, J. P. 1993, ApJ, 409, 429
Castor, J. I., Abbott, D. C., & Klein, R. I. 1975, ApJ, 195, 157
Frank, A., Ryu, D., & Davidson, K. 1998, ApJ, 500, 291
Gayley, K. G. 2000, ApJ, 529, 1019
Gayley, K. G., & Owocki, S. P. 2000, ApJ, 537, 461
Lamers, H. J. G. L. M., Snow, T. P., & Lindholm, D. M. 1995, ApJ, 455, 269
Langer, N., Garcia-Segura, G., & MacLow, M.-M. 1999, ApJ, 520, L49
Maeder, A. 1999, A&A, 347, 185
Maeder, A., & Meynet, G. 2000, A&A, 361, 159
Owocki, S. P., Cranmer, S. R., & Gayley, K. G. 1998, Ap&SS, 260, 149
Shaviv, N. J. 2001, MNRAS, 326, 126

K. G. Gayley and A. J. Onifer: 203 Van Allen Hall, University of Iowa, Iowa City, IA 52242, USA (kgg, ajo@astro.physics.uiowa.edu).