

UNIVERSE EVOLUTION WITH SCALAR FIELDS

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RESUMEN

Diferentes consideraciones teóricas y observaciones recientes apuntan hacia la existencia de campos escalares cosmológicos, con presión negativa y densidad de energía variable en el tiempo y con fluctuaciones espaciales – este tipo de componente del Universo ha recibido el nombre de quintaesencia. Presentamos aquí un panorama de la evolución cosmológica de campos escalares con diferentes potenciales, en presencia de un fluido barotrópico, y de su efecto sobre la expansión del universo.

ABSTRACT

Cosmological scalar fields with negative pressure and a time-varying, spatially fluctuating energy density – quintessence – have recently obtained wide support from different observations and theoretical considerations. We present here an overview of the cosmological evolution of scalar fields with arbitrary potentials, in the presence of a barotropic fluid, and their influence on the universe expansion.

Key Words: **COSMOLOGY: THEORY**

A few years ago, the Hubble diagram for type Ia supernovae gave the first serious evidence for an accelerating universe (Riess *et al.*, 1998, and Perlmutter *et al.*, 1999). The two major teams investigating high-redshift SNe Ia obtained almost identical results: a systematic dimming of SNe Ia relative to that expected in the standard Einstein–de Sitter model, best explained by the presence of a cosmological constant-like component. At present, various combinations of constraints from CMB, supernovae and galaxy redshift survey favor the region $(\Omega_m, \Omega_\Lambda) \approx (0.3, 0.7)$ (see e.g. Lineweaver (2001)).

As the theoretical understanding of the origin of quintessence remains incomplete, a large variety of effective potentials can be conceived for the scalar field. In de la Macorra & Piccinelli (2000), we have studied the behaviour of Friedmann–Robertson–Walker spatially flat cosmologies containing a barotropic fluid (either matter or radiation) and a scalar field with a self-interaction potential, without making any hypothesis on which energy density dominates. We determine the equation of state dynamically for each case. The parameter ω_ϕ for the effective equation of state for the scalar field will in some cases oscillate between -1 and 1 , before settling to an asymptotical value. Some of these potentials lead to an interpretation of dynamical cosmological constant. It must be stressed that recent CMB data open the way for discriminating between different quintessence models (Wetterich 2001).

The equations to be solved, for a spatially flat FRW Universe, are:

$$\begin{aligned} \dot{H} &= -\frac{1}{2}(\rho_f + p_f + \dot{\phi}^2) \\ \dot{\rho} &= -3H(\rho + p) \\ \ddot{\phi} &= -3H\dot{\phi} - \frac{dV(\phi)}{d\phi}, \end{aligned}$$

where H is the Hubble parameter, $V(\phi)$ is the scalar field potential, $8\pi G = 1$, ρ_f and p_f are the barotropic energy density and pressure respectively, with a standard equation of state $p_f = (\gamma_f - 1)\rho_f$; $\rho = \rho_f + \rho_\phi$ and $p = p_f + p_\phi$.

As the universe evolves, the scalar field will go to its minimum (ϕ_{min}), rolling to infinity or oscillating around its vacuum expectation value, depending on whether ϕ_{min} is finite or not. The oscillating behaviour of ϕ or λ is important in determining the evolution of the cosmological parameters. We find, in fact, that all model dependence is given in terms of $\lambda \equiv -V'/V$ only. In terms of this parameter, and using a variety of analytic and numerical techniques, we were able to generalize to other cases the expressions for the attracting solutions found in the literature for the simple case of an exponential potential. We have worked with the most usual forms for the potential, which are also the basic blocks of composite potentials constructed for fitting all the cosmological requirements of different epochs.

We classify all the possible scenarios containing a scalar field depending on the asymptotic value of λ and its oscillating behaviour. This classification is valid also for composite potentials which can have different dominant terms in different epochs, but of course the epoch when the scalar field starts to roll down its potential has a cosmological relevance, leading to distinct possible evolution histories before the

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TABLE 1
ASYMPTOTIC BEHAVIOUR OF Ω_ϕ AND γ_ϕ FOR DIFFERENT LIMITS OF λ AND AN EXAMPLE OF POTENTIAL WHICH SATISFIES THIS LIMIT

$\lambda(\phi) = -V'/V$	$\Omega_\phi = \rho_\phi/\rho$	γ_ϕ	<i>e.g.</i> $V(\phi)$	cosmological behaviour
$const. > \sqrt{3}\gamma_f$	$\frac{3\gamma_f}{c^2}$	γ_f	$V_0 e^{-c\phi}$	scaling solution, ρ_ϕ redshifts as ρ_f
$const. < \sqrt{6}$	1	$\frac{c^2}{3}$	$V_0 e^{-c\phi}$	power-law inflation
∞ (no oscil.)	0	γ_f	$V_0 e^{-ce^\phi}$	same attractors as for $\lambda = const. > \sqrt{3}\gamma_f$ but with a time varying λ
∞ (oscil.)	0	$\frac{2n}{2+n} (> \gamma_f)$	$V_0 \phi^n, n > 0$ even	ρ_ϕ rapidly decays
	cte	$\frac{2n}{2+n} (= \gamma_f)$		scaling solution
	1	$\frac{2n}{2+n} (< \gamma_f)$		cosmological constant
0	1	0	$V_0 \phi^{-n}, n > 0$	cosmological constant

final behaviour.

We present in Table 1 a summary of the asymptotic values of the cosmological relevant quantities for all different limits of λ . Some of these models have been largely studied in the literature, see Sahni & Starobinsky (1999) or Padmanabhan (2002) for a review. The asymptotic behaviour starts when the field has reached the bottom of the potential.

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