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POTENTIAL–DENSITY PAIRS FOR SPHERICAL GALAXIES AND BULGES IN SCALAR–TENSOR THEORIES OF GRAVITY

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A family of potential–density pairs has been found for spherical halos and bulges of galaxies within the Newtonian limit of scalar–tensor theories of gravity. The scalar field is described by a modified Helmholtz equation with a source that is coupled to the standard Poisson equation of Newtonian gravity. Then, the net gravitational force is given by two contributions: the standard Newtonian potential plus a term stemming from massive scalar fields. General solutions have been found for spherical systems. Hence, we compute potential–density pairs of spherical, galactic systems, and their associated circular velocities

Taking the Newtonian limit of general scalar–tensor theories of gravity, in which scalar field perturbations deviate from the background, defined here as $\langle\phi\rangle = 1$, then the field equations for the perturbation $\bar{\phi} \equiv \phi - 1$, obey (Helbig 1991)

$$R_{00} = \frac{1}{2}\nabla^2 h_{00} = 4\pi\rho - \frac{1}{2}\nabla^2\bar{\phi} \quad (1)$$

$$\nabla^2\bar{\phi} - m^2\bar{\phi} = -8\pi\alpha\rho \quad (2)$$

where $\alpha \equiv 1/(3+2\omega)$, ω being a coupling function of the theory. Eq. (2) is the modified Helmholtz equation with a source.

General solutions can be found in terms of the corresponding Green's functions

$$\psi = -\int d\mathbf{r}_s \frac{\rho(\mathbf{r}_s)}{|\mathbf{r}-\mathbf{r}_s|} + \text{B.C.}\,, \quad (3)$$

$$\bar{\phi} = 2\alpha \int d\mathbf{r}_s \frac{\rho(\mathbf{r}_s)\exp(-m|\mathbf{r}-\mathbf{r}_s|)}{|\mathbf{r}-\mathbf{r}_s|} + \text{B.C.} \quad (4)$$

where $\psi \equiv (1/2)(h_{00} + \bar{\phi})$ and the new Newtonian potential is

$$\Phi_N \equiv \frac{1}{2}h_{00} = \psi - \frac{1}{2}\bar{\phi} = -\int d\mathbf{r}_s \frac{\rho(\mathbf{r}_s)}{|\mathbf{r}-\mathbf{r}_s|} - \alpha\int d\mathbf{r}_s \frac{\rho(\mathbf{r}_s)\exp(-m|\mathbf{r}-\mathbf{r}_s|)}{|\mathbf{r}-\mathbf{r}_s|} + \text{B.C} \quad (5)$$

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The first term, given by ψ, is the contribution of normal Newtonian gravitation (without scalar fields), whereas the information of the scalar field is contained in the second term of Eq. (5).

We use the family of density profiles for spherical halos and bulges of galaxies proposed by Dehnen (1993)

$$\rho(r) = \frac{(3-\gamma)M}{4\pi}\frac{a}{r^\gamma(r+a)^{4-\gamma}} \quad (6)$$

where a is a scaling radius and M is the total mass.

Solving the modified Helmholtz equation, using Eq. (6) for the density, we obtain

$$\Phi_N = -(3-\gamma)\frac{M}{a}\left\{\int_{r_a}^{\infty} dy \frac{\bar{m}(y)}{y^2} + \eta\left[\frac{\exp(-r_a/\lambda_a)}{r_a}s(r_a) + \frac{\sinh(r_a/\lambda_a)}{r_a}t(r_a)\right]\right\} \quad (7)$$

where r_a and λ_a are given in units of a, $\eta = \alpha\lambda_a$ and $\bar{m}(y) = \int_0^y dx\,\frac{x^{2-\gamma}}{(x+1)^{4-\gamma}}$, $s(r_a) = \int_0^{r_a} dx\,\frac{x^{1-\gamma}}{(x+1)^{4-\gamma}}\sinh(x/\lambda_a)$, and $t(r_a) = \int_{r_a}^{\infty} dx\,\frac{x^{1-\gamma}}{(x+1)^{4-\gamma}}\exp(-x/\lambda_a)$

The circular velocity is

$$v_c^2 = (3-\gamma)\frac{M}{a}\frac{\bar{m}(r_a)}{r_a} + \eta(3-\gamma)\frac{M}{a}\left[\left(\frac{1}{r_a}+\frac{1}{\lambda_a}\right)\exp(-r_a/\lambda_a)\,s(r_a) + \left(\frac{\sinh(r_a/\lambda_a)}{r_a} - \frac{\cosh(r_a/\lambda_a)}{\lambda_a}\right)t(r_a)\right] \quad (8)$$

The first right-hand-side term is the contribution from ψ and the second term is the contribution from the scalar field. The latter term depend on $\frac{\eta M}{a} = \frac{\alpha M}{m a^2}$, rather than α, which is typical for forces exerted on punctual systems. One sees that at sufficiently distances larger than the size of the galaxy, the scalar field influence decays exponentially, recovering the standard Newtonian result.

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