RevMexAA (Serie de Conferencias), **21**, 147–151 (2004)

THREE-BODY PROBLEM AND MULTIPLE STELLAR SYSTEMS

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RESUMEN

Las interacciones de tres cuerpos ocurren en cúmulos estelares, donde se dan encuentros entre binarias y estrellas sencillas formando temporalmente sistemas triples. Las triples son generalemente inestables y se fragmentan en una nueva binaria y una estrella sencilla. La simple dispersión de una estrella sencilla por una binaria también ocurre. Ambos procesos pueden ser estudiados con la teoría estadística del rompimiento y la dispersión de tres cuerpos. En este trabajo, aplicamos la teoría a las estrellas binarias, suponiendo que éstas han participado en procesos de tres cuerpos. Se discuten las distribuciones de los períodos, las excentricidades y los cocientes de masa de las binarias obtenidos, y se comparan con muestras observacionales.

ABSTRACT

Three-body processes go on in star clusters where binary stars meet single stars and frequently form temporary triple systems. The triples are typically unstable and break up into a new binary and a single star. Also a simple scattering of a single star from a binary may take place. Both processes can be handled by the statistical theories of three-body break-up and scattering. Here we apply the theory to binary stars, assuming that binaries have been involved in the three-body process. The distributions of binary periods, eccentricities and mass ratios are discussed from this point of view and compared with observational samples.

Key Words: **BINARIES: GENERAL**

1. INTRODUCTION

Numerical simulations of star cluster evolution have shown that three-body interactions take place among cluster stars frequently. In the three-body break-up a binary is often expelled out of the cluster and it becomes a binary in the general field of stars of the Galaxy. There may still be further encounters with other stars later on, but on the whole the "hard" binaries probably have their properties more or less frozen since their escape from the star cluster of their origin. We will now study what kind of binary star population we expect from this process and how it compares with the observed binaries. In particular, we are interested to see if Öpik's law follows, i.e. if the orbital periods of binaries are uniformly distributed in the logarithmic scale. Also the distribution of binary mass ratios can be predicted for different types of primary stars.

2. BINARY ENERGY AND PERIOD

A statistical theory for the three-body break-up was derived by Monaghan (1976) assuming that all systems have a constant total energy E_0. He derived the distribution of the binding energy E_B of the final binary after the third body has escaped. The basic principle of the theory is to assume that escapes happen in such a way that the phase space formed by the positions and momenta of the binary and the third body becomes uniformly filled with the representative points. Starting from this principle one calculates the phase space volume σ and derives the final distributions of various quantities. Some assumptions are required on the way, and depending on these assumptions different final distributions are arrived at. Here we follow the calculation of Valtonen and Karttunen (2004) which is slightly different from Monaghan (1976). The distribution of final binary energies is proportional to $E_B^{-4.5}$ rather than $E_B^{-2.5}$ of Monaghan (1976). This modification is in agreement with Heggie (1975). The binary binding energy is normalised to the constant total energy E_0 of the systems.

But in star clusters E_0 may vary greatly from one three-body system to another. Monaghan (1976) calculates the available phase space volume σ which is inversely proportional to $|E_0|$:

$$\sigma(|E_0|)\mathrm{d}|E_0| \propto |E_0|^{-1}\,\mathrm{d}|E_0|. \qquad (1)$$

If for any reason the three-body systems are uniformly distributed in the E_0 space then we expect that the binary energies E_B after the three-body break-up also follow Eq. (1). i.e., Öpik's law should be valid. To what extent this is true can be found out by studying young star clusters observationally as well as by simulating star formation processes the-

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oretically.

Hard binaries in star clusters tend to harden further. At the limit of very hard binaries we may write the average hardening rate

$$\frac{1}{2}\mathcal{M}v_0^2 R_\Delta = \left\langle \frac{\mathrm{d}|E_B|}{\mathrm{d}t} \right\rangle = 3G^2 m_B^3 \frac{\mathcal{M}}{M}\frac{n}{v_3}. \quad (2)$$

Here $\mathcal{M} = (m_a m_b)/(m_a + m_b)$, m_a and m_b are the binary component masses, $m_B = m_a + m_b$, $M = m_B + m_s$, m_s is the mass of the escaper, v_0 is the mean orbital speed of the binary, R_Δ is the rate of energy transfer, n is the number density of single stars, v_3 is the speed of the binary relative to the single stars, and G is the gravitational constant. In a star cluster we may regard the right hand side as a constant in the first approximation, even though in fact the density of stars n and the typical speed of stars do vary during the cluster evolution. But using this assumption, and also putting all stars equal to $1M_\odot$, the equation is easily integrated:

$$\frac{E_B}{(E_B)_0} = \frac{16G^2 M_\odot^2 nT}{v_0^2 v_3}, \quad (3)$$

where $(E_B)_0$ is the initial value of the binary energy $|(E_B)_0| \ll |E_B|$ and v_0 is the corresponding mean orbital speed. T is the time of escape of the binary from the cluster since the birth of the star cluster.

Because of the evolution in the star cluster as well as the effect of the Galactic tides, the cluster is gradually dissolved. The time of dissolution t_d has been estimated at

$$t_d \approx 5.7 \times 10^8 \left(\frac{M_{\rm cluster}}{250 M_\odot}\right)\left(\frac{1\ {\rm pc}}{r_h}\right)^3 {\rm yr} \quad (4)$$

times a factor depending on the structure of the cluster (Binney and Tremaine 1987). Here $M_{\rm cluster}$ is the mass of the cluster and r_h is its median radius. Since 250 solar mass stars within a sphere of 1 pc in radius makes the average number density $n = 250/(\frac{4}{3}\pi\ {\rm pc}^3) \approx 60\ {\rm pc}^{-3}$, the equation may be written by using this mean number density n:

$$t_d \approx 5.7 \times 10^8 \left(n/60\ {\rm pc}^{-3}\right) {\rm yr}. \quad (5)$$

We may take the typical escape time of the binary to be half of t_d, i.e.

$$T \approx 3 \times 10^8 (n/60\ {\rm pc}^{-3})\ {\rm yr}. \quad (6)$$

From here n may be solved and inserted into Eq. (3) above. Then

$$\begin{aligned}\frac{E_B}{(E_B)_0} &\approx \frac{16G^2 M_\odot^2}{v_0^2 v_3}\left(\frac{T}{3\times 10^8\ {\rm yr}}\right) 60\ {\rm pc}^{-3} \times T \\ &= 5.3\left(\frac{T}{3\times 10^8 {\rm yr}}\right)^2 \left[\frac{v_0^2 v_3}{({\rm km/s})^3}\right]^{-1}.\end{aligned} \quad (7)$$

Putting a typical number $v_3 = 0.25$ km/s, and starting from a hard binary with $v_0 = 1$ km/s, we expect to end up with

$$\frac{E_B}{(E_B)_0} \approx 20\left(\frac{T}{3\times 10^8 {\rm yr}}\right)^2. \quad (8)$$

In a typical hardening period of $T = 10^8$ yr we then expect the average binary binding energy to increase by a factor of 2 and the corresponding orbital period to shorten by about a factor of 3.

Since $|E_B|_0 \propto v_0^2$, the final value of $|E_B|$ does not depend on v_0 (i.e. on the initial orbital period) but only on T. Therefore the distribution of final periods P should depend on the distribution of T.

A numerical simulation of the Pleiades star cluster by Kroupa, Aarseth and Hurley (2001) shows that in its assumed 100 million year lifetime the binary period distribution shifts shortward by about a factor of 3 at the end of large periods ($P \gtrsim 30$ yr). This agrees with our simple estimate. At the end of short periods no significant shift is detected in the simulation.

Depending primarily on the cluster star density, clusters live different lengths of time, and provide different periods T for the hardening process. We get an idea of the distribution of T from observations of star clusters. The current age τ of a star cluster is a representative time in the history of a cluster, and may well tell us when a typical binary escape happens. The distribution of τ is observed to be (Wielen 1971)

$$f(\tau) \propto \tau^{-1} \quad (9)$$

in the interval 2×10^7 yr $\lesssim \tau \lesssim 5\times 10^8$ yr, it steepens beyond the upper limit. Let us then suppose that also

$$f(T) \propto T^{-1} \quad (10)$$

in this range.

Since $E_B/(E_B)_0 \propto T^2$, the corresponding period ratio $P/P_0 \propto T^{-3}$. Therefore we find

$$f(P/P_0) = \frac{f(T)\,\mathrm{d}T}{\mathrm{d}(P/P_0)} \propto (P/P_0)^{-1}. \quad (11)$$

In a logarithmic scale the distribution of P/P_0 is flat:

$$\frac{f(P/P_0)}{\mathrm{d}\log(P/P_0)} = \text{constant} \quad (12)$$

since $d(P/P_0) = (P/P_0)\, d(\log(P/P_0))$. This should be valid over one and half orders of magnitude in T, which corresponds to over four orders of magnitude in P/P_0.

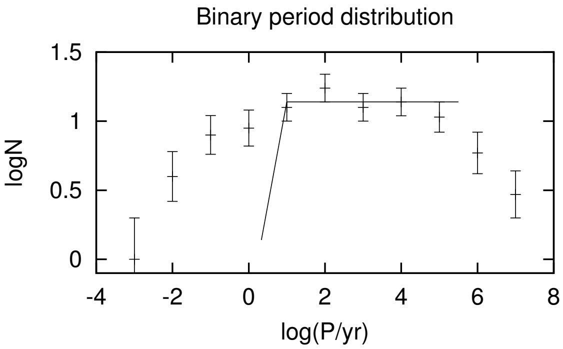

Fig. 1. The period distribution of a sample of nearby binary stars with a solar type primary (Duquennoy & Mayor 1991, Fig. 7). Lines refer to theoretical expectations.

What is the range of validity of this result? At the end of small T, below about $T = 2\times10^7$ yr, there is negligible binary hardening. At the other end, $T \geq 5\times10^8$ yr, the power-law of Eq. (9) steepens and the expected period distribution becomes ($P \lesssim 10$ yr):

$$\frac{f(P/P_0)}{\mathrm{d}\log(P/P_0)} \propto (P/P_0)^{3/2}. \quad (13)$$

These distributions are compared with observations (Duquennoy and Mayor 1991) in Fig. 1. We notice that the predicted break at the end of low values of P/P_0, below the orbital period of ten years, is not borne out by observations. It appears that these short period binaries come from a binary population which have short periods to start with. Such "primordial" binaries are observed in star clusters and they make an important contribution to the short period end of the distribution.

The reason for the relative flatness of the short period binary distribution may be in the star formation process. Apparently, Eq. (1) applies there at least over a limited range of E_0. The scale free property of the distribution for longer periods may result from binary hardening. The steepening of the period distribution beyond $\log(P/\mathrm{yr}) \approx 5$ is well understood by the disruption of long period binaries in the Galactic field. Relative to the stellar background, these binaries are "soft" and tend to become even softer until they break up.

3. BINARY ECCENTRICITIES

The distribution of the eccentricities of binaries leads to the same conclusion: tight binaries, with periods less than 3 yr, have a bell shaped distribution with a peak around $e = 0.3$. Wider binaries, with periods exceeding 3 yr, show a distribution which agrees with $f(e) = 2e$, the distribution expected after three-body evolution (Duquennoy and Mayor 1991, Kroupa 1995).

4. BINARY MASS RATIOS

The three body evolution also modifies the binary mass ratios. Binary pairs where both components are massive are more likely to survive than pairs with unequal masses. This makes the mass ratio distribution evolve towards $m_b/m_a \approx 1$. The mass ratios obtained by picking pairs of stars at random from the initial distribution of stellar masses are therefore subject to later evolution.

Different binaries evolve by different amounts. The most massive binaries tend to settle near cluster centres and they are subject to many strong three-body interactions. As a result, exchanges of binary members take place until the binary is made up of two rather heavy members.

Ordinary binaries are involved in fewer strong three-body interactions. There we may assume that only a single three-body interaction is responsible for the mass ratio distribution. Starting from this assumption, we may pick three mass values at random from the Salpeter initial mass function $f(m)$. Then we use the probability distribution of

$$P_s = \frac{m_s^{-2}}{m_s^{-2} + m_a^{-2} + m_b^{-2}} \quad (14)$$

to decide which star (m_s) escapes and which are the two others (m_a and m_b) that make up the binary pair. The mass ratio $m = m_b/m_a$ ($m_b < m_a$) is thus obtained. Repeat the process many times and the distribution of mass ratios is built up. The procedure is best carried out by computer in Monte Carlo fashion, i.e. by picking out random numbers from suitable distributions.

The result of this operation is shown in Fig. 2 as a dashed line. A comparison of the data points for a sample of binaries with B-type primaries (where the Salpeter mass function is applicable) shows good agreement. It thus appears that these binaries (of typical orbital period 3 yr) have had at least one three-body interaction in the past.

In the case of solar type (spectral class G) primaries the Salpeter mass function for single stars is not suitable. However, a flatter power-law, with index $\alpha = 1.25$ in

$$f(m) \propto m^{-\alpha} \quad (15)$$

may be used. Then the same process as described above leads to the distribution of Fig. 3 (dashed line). The observations by Duquennoy and Mayor

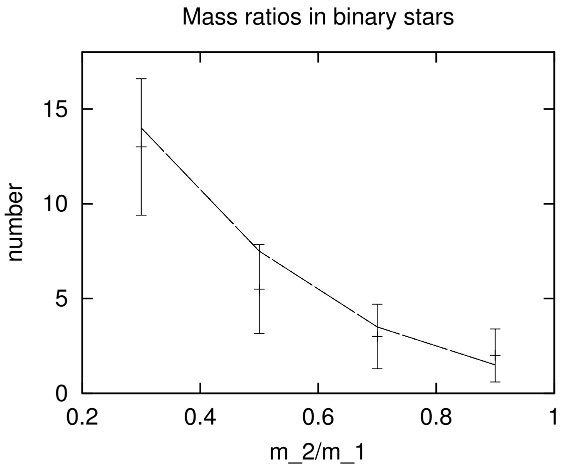

Fig. 2. The mass ratio of binary star components in an observational sample with a B spectral type primary (points with error bars; Evans 1995). The dashed line is based on a theory where two lower mass companions for the B-type star have been picked at random, and one of the companions has escaped.

(1991) are well described except at the low values of m_b/m_a where both the observations and the power-law assumption are very uncertain.

For the most massive O-type stars this procedure is not reasonable since numerous three-body encounters have in fact truncated from below the distribution of the possible mass values. Now we may pick three mass values from the power-law distribution with $\alpha = 3.2$ (applicable to the upper end of the mass range), all of which are above a given lower limit. Then we again ask which one of the three stars escapes, which ones make the binary and what is their mass ratio. The mass ratio distribution built up in this way is shown as a continuous line in Fig. 4. It agrees well with the observed O-star primaries sample (Abt 1983).

The rather puzzling situation with the mass ratio distribution varying as a function of the spectral type of the primary is therefore explained as a result of three-body interactions among stars (Valtonen 1997).

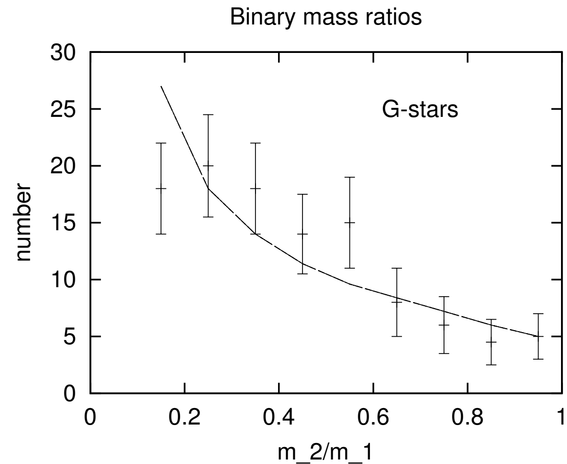

Fig. 3. The observed distribution of binary star mass ratios when the primary is a solar type star (points with error bars; Duquennoy and Mayor 1991). It is compared with the three-body theory with $\alpha = 1.25$ (dashed line).

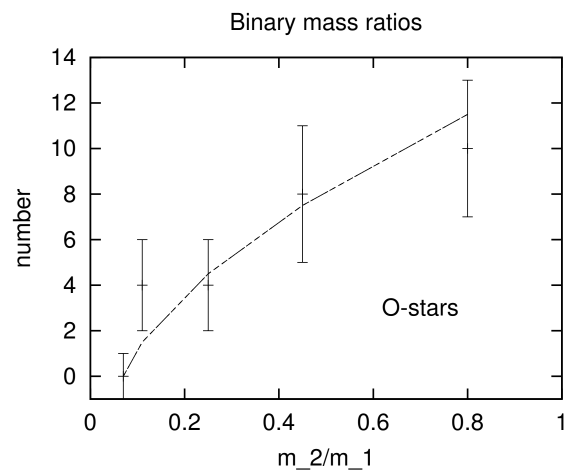

Fig. 4. The observed distribution of binary star mass ratios when the primary is an O-type star (points with error bars; Abt 1983). It is compared with the three-body theory with $\alpha = 3.2$ and single star mass distribution truncated from below.

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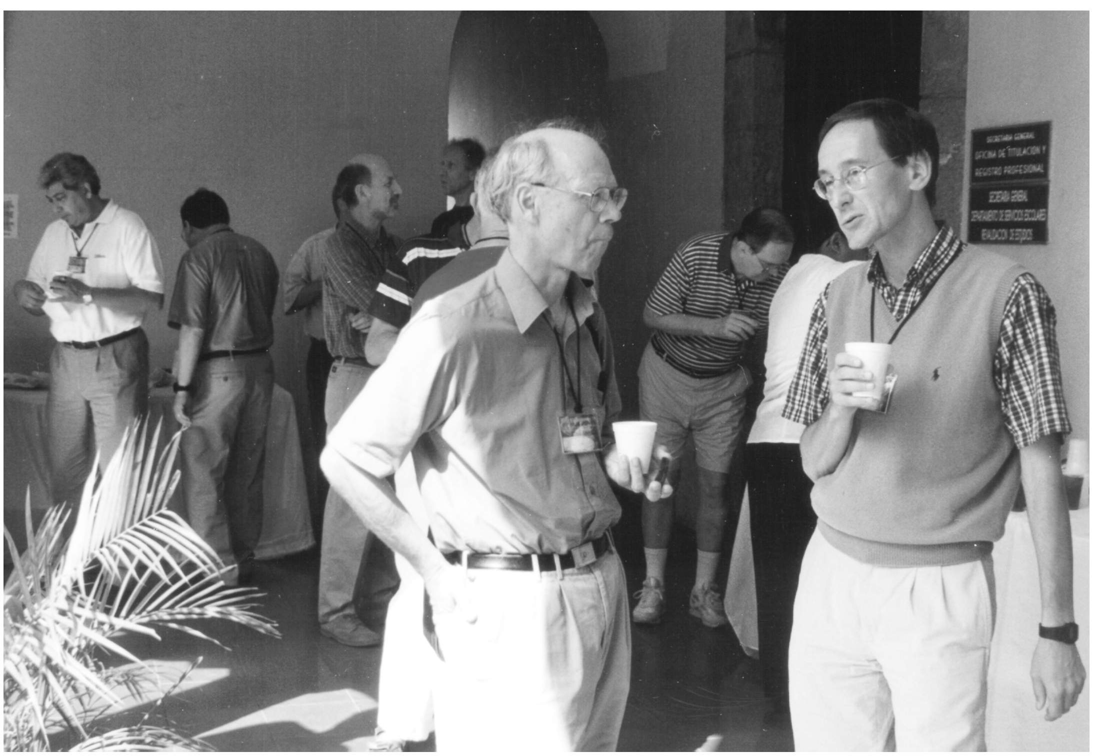
DISCUSSION

Zinnecker – Where has this discussion on the dynamical origin of the binary period distribution been published before? More specifically, is there a 3-body dynamical explanation of Poveda's and Öpik's law that $f(\log\ P) =$ constant between $\log\ P_{\rm min}$ and $\log\ P_{\rm max}$?

Valtonen – As far as I know, this type of mechanism has not been discussed before.

Sterzik – Broadening of the period distribution due to the binary hardening process by passing through star clusters is an evolutionary process that lasts a long time. Broad period distributions and very short-period (spectroscopic) binaries are, however, already frequent in the pre-main-sequence phase. Could you please comment?

Valtonen – This mechanism assumes a broad range of environments from which binaries came from. Therefore, one should not expect Öpik's law to apply in individual clusters over the same wide range as among the field binaries. Short-period binaries probably require a different mechanism.



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