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STATISTICS OF MULTIPLE STARS

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RESUMEN

Se reseñan las propiedades estadísticas de los sistemas estelares de multiplicidad tres o mayor. Comprenden el 0.15 – 0.25 del total de los sistemas. Se espera que existan 700 múltiples entre las 3383 estrellas de tipos espectrales F, G y K más cercanas que 50 pc, pero sólo se conocen 76. Muchas de las binarias cerradas (y quizás todas) tienen componentes terciarias distantes, lo que indica que la transferencia de momento angular en los sistemas múltiples fue probablemente decisiva para la formación de binarias de corto período. El cociente entre el período largo y el corto en las múltiples mejor estudiadas y en las múltiples de baja masa pre-secuencia principal no excede 10^4 en la época de formación estelar; los cocientes mayores que 10^4 se producen por la evolución orbital subsecuente. Todas las múltiples conocidas con órbitas bien definidas son dinámicamente estables, y las excentricidades de las órbitas externas cumplen con el criterio empírico de estabilidad $P_{\rm ext}(1-e_{\rm ext})^3/P_{\rm int} > 5$, el cual es más estricto que los límites teóricos actualmente aceptados. La orientación relativa de las órbitas en las estrellas triples muestra cierto grado de alineamiento, especialmente en los sistemas débilmente jerárquicos. Las estadísticas apoyan la idea de que la mayoría de las múltiples se originaron por interacciones dinámicas en cúmulos pequeños.

ABSTRACT

The statistics of stellar systems of multiplicity three and higher is reviewed. They are frequent, 0.15–0.25 of all stellar systems. Some 700 multiples are expected among the 3383 stars of spectral type F, G, and K within 50 pc, while only 76 of them are actually known. Many (if not all) close binaries have distant tertiary components, indicating that angular momentum exchange within multiple systems was probably critical in forming short-period binaries. The ratio of outer to inner periods in the best-studied nearby multiples and in low-mass pre-main sequence multiples does not exceed 10^4 at the formation epoch; larger ratios are produced by subsequent orbital evolution. All multiples with well-defined orbits are dynamically stable, the eccentricities of outer orbits obey the empirical stability limit $P_{\rm out}(1-e_{\rm out})^3/P_{\rm in} > 5$ that is more strict than current theoretical limits. Relative orientation of orbits in triple stars shows some degree of alignment, especially in weakly-hierarchical systems. The statistics support the idea that most multiple stars originated from dynamical interactions in small clusters.

Key Words: **BINARIES: GENERAL — STARS: FORMATION — STARS: PRE-MAIN SEQUENCE**

1. INTRODUCTION

The reasons to study the statistics of multiple stars are numerous. The most compelling one (at least for me) is to answer the question: "how were multiple and binary stars formed?". Much progress has been achieved in this direction over recent years (Zinnecker & Mathieu 2001). Now it becomes clear that instead of a single act of creation there was a prolonged process of formation and early evolution, and instead of a single dominating formation mechanism, a combination of mechanisms all of which are important and which act jointly to produce the observed population of multiple stars.

To put the following discussion in a context, a sketch illustrating current understanding of multiple star formation is shown in Fig. 1. Stars condense out of a small collapsing cloud which may be part of a larger cloud. The cloud may have some pre-existing structure, e.g. filaments. The protostars accrete and fall to the center of the cloud at the same time because both processes act on the same free-fall time scale, $10^4 - 10^5$ yr. So when stellar embryos meet close to the center of the cloud, the presence of some gas is guaranteed, significantly affecting the first interplay. A high-resolution image of a young multiple system in this stage, Mon R2 IRS3, is given by Preibisch et al. (2002). Embedded young triple stars driving jets and outflows are listed by Reipurth (2000). Some single or binary stars are ejected as a result of dynamical interactions; some hierarchical multiple systems are formed as well among the decay products. Those multiples, although stable or meta-stable, cannot be *very* hierarchical—they

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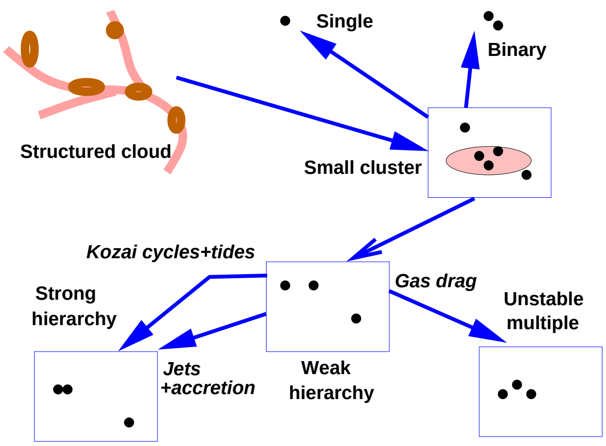

Fig. 1. Scenario of multiple star formation.

should have moderate ratios of outer to inner periods at adjacent hierarchical levels. Many such multiples remain in this state and are actually observed in both young and old stellar populations. Further evolution toward more hierarchical configurations may occur later through a combination of dynamics and tidal dissipation (§ 3).

The gas remaining after star formation interacts with the outer components of a multiple system and shrinks their orbits until the system becomes again non-hierarchical and unstable (Bate, Bonnell & Bromm 2002). An instability may also occur when new components come close to the multiple and start to interact with it dynamically. A new episode of interplay would likely lead to the disintegration of the multiple, although a more tight multiple may form as well.

2. FREQUENCY OF MULTIPLE STARS

The current version of the Multiple Star Catalog (MSC, Tokovinin 1997a) contains 905 physical multiple (i.e. triple, quadruple etc.) systems. Given that a discovery of a multiple star is, typically, a result of several observational techniques and good luck, there are many reasons to believe that MSC is very incomplete.

In order to estimate the true frequency of multiple stars, I select the best studied sample of nearby dwarfs—stars with $0.5 < B - V < 1.0$ within 50 pc from the Hipparcos (ESA 1997) catalog. When giants are removed from those 3486 stars, 3383 dwarfs remain. Discovery of multiple systems should be most complete for this sample because the stars are brighter than $V = 10$, hence they are screened for visual duplicity and have a good chance of spectroscopic binary detection. Similarly, I select from the MSC the systems with $0.5 < B-V < 1.0$ (this refers

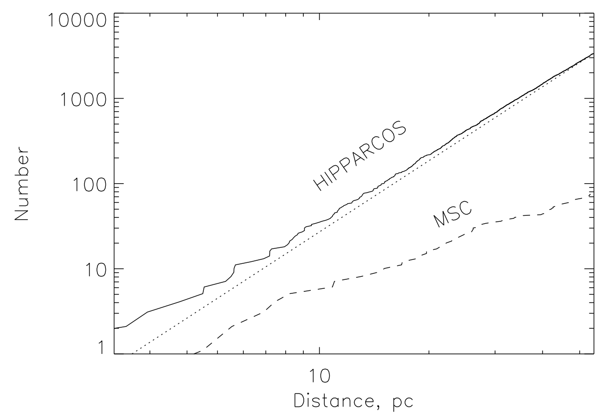

Fig. 2. The number of F, G, K stars within a given distance d. Full line: all stars in the Hipparcos catalog (the dotted line shows a d^3 law), dashed line: multiples from the MSC.

TABLE 1

DWARFS IN HIPPARCOS AND MSC

Distance	MSC	HIP	MSC/HIP
8 pc	5	18	0.28
10 pc	6	36	0.17
15 pc	11	112	0.10
20 pc	18	252	0.07
50 pc	76	3383	0.02

to the combined light of all components). Excluding 3 giants leaves 76 systems. In Fig. 2 and Table 1 I compare the total number of Hipparcos objects with the number of nearby multiples, as a function of distance. It is apparent that the discovery of multiples is very incomplete beyond 10 pc, and that the fraction of systems which are at least triple seems to be as high as 0.2–0.25. It was estimated previously as 0.05 from the sample of nearby G-dwarfs (Tokovinin 2001).

Additional support for the high fraction of multiples comes from the work of Tokovinin & Smekhov (2002). They estimate that the probability to find a spectroscopic component in a visual binary is between 0.12 and 0.24 per component. Let us take 0.18 as a representative number. Then the fraction of visual binaries that contain additional components is $1 - (0.82)^2 = 0.33$. If the fraction of visual binaries among all stars is 0.6, as often assumed, then the fraction of higher-order multiples will be $0.6 \times 0.33 = 0.20$.

If multiples are indeed frequent, some 700 systems which are at least triple are expected among the 3383 nearby dwarfs, whereas only 76 are cataloged at present. The undiscovered multiples hide among the visual binaries that were not surveyed spectroscopically and among faint and distant tertiary components to known binaries. Four of the 5 multiples within 8 pc have such distant components (e.g. α Cen + Proxima), the remaining one is the visual-spectroscopic quadruple ζ UMa.

It is instructive to compare the frequency of multiple stars with the simulations of Sterzik & Durisen (1998). They predict that the decay of small clusters can produce a significant number of hierarchical multiples and that this fraction increases for higher-mass primaries. Averaging the results of simulations for $N = 4$ and $N = 5$ clusters leads to a multiple fraction of 0.21 for primaries of spectral type G and K, with a binary fraction of 0.43. Thus there is a plausible match between dynamical decay simulations and the real fraction of multiple systems.

3. ARE ALL CLOSE BINARIES TRIPLE?

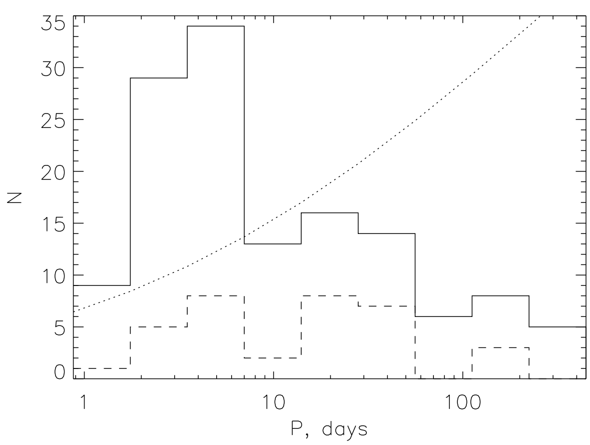

Fig. 3. Distribution of periods of inner sub-systems of solar-type multiple stars in the MSC (full line) and in the dedicated spectroscopic survey (dash) from Tokovinin & Smekhov (2002). The dotted curve traces the model distribution of G-dwarf periods from Duquennoy & Mayor (1991).

The distribution of periods in the inner subsystems of multiple stars with low-mass dwarf primaries is shown in Fig. 3. It is remarkable by a sharp drop in the number of systems at $P_{\text{in}} > 7$ d. Such a sharp feature cannot result from observational selection; it is real.

A likely explanation for the formation of close binaries within multiples has been suggested by Kiseleva, Eggleton & Mikkola (1998) who studied Algol. In the case when the orbit of the inner binary is almost perpendicular to the orbital plane of the outer system, the inner binary can become very eccentric as a result of Kozai cycles. When the eccentricity grows to the point where the components start to interact tidally at periastron, the Kozai cycles are perturbed and the inner binary is then slowly circularized by tidal dissipation of orbital energy. The final period is determined by the distance where tides become effective—around 7–10 days for Main Sequence (MS) stars or longer for Pre-Main Sequence (PMS) stars. This process increases the period ratio $P_{\text{out}}/P_{\text{in}}$, leading to a more hierarchical and stable system.

The relation between close binaries and higher-order multiplicity has been noted by Tokovinin (1997b): 43% (26 out of 61) of the nearby (within 100 pc), low-mass (0.5 to 1.5 $M_{\odot}$), spectroscopic binaries with $P < 10$ d cataloged by Batten, Fletcher & MacCarthy (1989) have known tertiary components. The real proportion of multiples must be higher; for example, it is 100% for the Duquennoy & Mayor (1991) sample of G-dwarfs (all 5 systems with $P < 10$ d are triple). The presence of a tertiary component provides a natural sink for the angular momentum that needs to be removed from a binary in order to make it close. There may be several mechanisms of such interaction, of which the combination of Kozai cycles and tides is just one possibility.

A statement that *all close binaries are triple* is probably too extreme and strong; it will be very difficult to test. As a counter-example, I suggest a young nearby active binary HD 17433 = VY Ari ($P = 13.19$ d) which does not seem to be triple according to Hipparcos and speckle-interferometry. Of course, it is difficult to exclude a low-mass distant tertiary like the 16^m star found around BY Dra by Zuckerman et al. (1997).

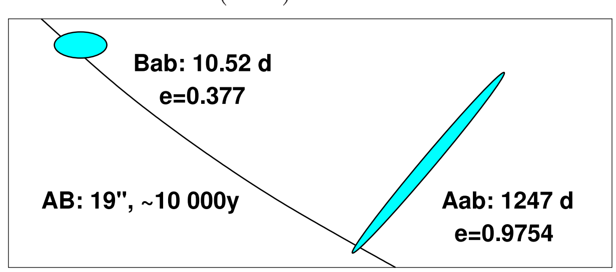

Fig. 4. The quadruple system 41 and 40 Dra (Tokovinin et al. 2003).

In Fig. 4 the structure of a noteworthy quadruple system, 41+40 Dra, is depicted (Tokovinin et al. 2003). The high-eccentricity ($e = 0.9754$) sub-system Aab has now a good-quality speckle-spectroscopic orbit and the age is established as 2.5 ± 0.1 Gyr. It is likely that the sub-system Bab acquired its short period by Kozai cycles and tidal dissipation, but why then did the other pair Aab not follow the same path despite its extreme eccentricity? Why did it survive for so long? The most plausible answer is that the components Aa and Ab are slightly more massive than Ba and Bb and had very thin convective zones while on the MS, so tides were less effective than in Bab. Now Aa and Ab are leaving the MS, and violent tidal interaction at each periastron will soon circularize the orbit of Aab. Further evolution may lead to a merging of these stars, leaving an unusual triple with a giant primary A and a MS binary Bab. This example is given to illustrate the complex interplay between stellar evolution and dynamics.

4. PERIOD RATIO

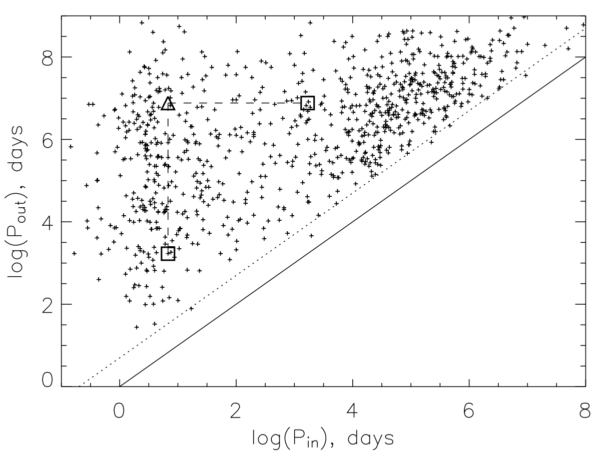

Fig. 5. The relation between the inner period $P_{\rm in}$ and the outer period $P_{\rm out}$ for all physical multiple systems in MSC. The full line shows period equality and the dotted line $P_{\rm out}/P_{\rm in} = 5$. Two squares and the triangle mark the true and wrong locations of HD 7119.

Previous statistics based on the MSC indicated that all period ratios $P_{\rm out}/P_{\rm in}$ at adjacent hierarchy levels are possible as long as the system is dynamically stable, i.e. $P_{\rm out}/P_{\rm in} \geq 10$. However, there is a problem of undiscovered intermediate levels, which I illustrate by an example. HD 7119 was studied by Carquillat et al. (2002) and found to be a spectroscopic triple. In addition, there is a physical visual companion, so this quadruple system contains two hierarchical pairs as plotted in Fig. 5. The amplitude of the radial velocity variation caused by the spectroscopic tertiary is only 3 km/s. If those authors had not been attentive enough, or if the precision of the radial velocities had been lower, the tertiary would have been missed and the system would be

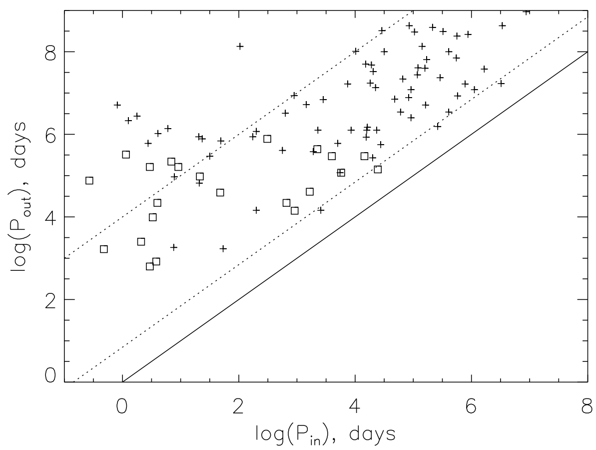

Fig. 6. The relation between $P_{\rm in}$ and $P_{\rm out}$ for low-mass multiples within 50 pc. The full line shows period equality and the dashed lines $P_{\rm out}/P_{\rm in} = 5$ and $P_{\rm out}/P_{\rm in} = 10^4$. Cases where both periods are known from computed orbits are plotted as squares, otherwise (crosses) at least one of the periods is estimated from the apparent separation between the components.

considered only as triple. Its wrong position in the $P_{\rm in} - P_{\rm out}$ diagram is shown in Fig 5 by a triangle. Are all those points in the upper left corner also wrongly placed because their intermediate hierarchical levels have not yet been discovered?

Trying to answer this question, I consider now only nearby (within 50 pc) low-mass ($< 1.5\ M_\odot$ primary) multiple stars—a total of 173 systems selected from the MSC. I hope that in this sample the discovery of multiplicity is nearly complete; thus the "hidden" intermediate levels do not distort the $P_{\rm in} - P_{\rm out}$ diagram.

In Fig. 6 such a diagram is shown. Compared to the full MSC (Fig. 5), the upper left corner is cleared, as expected. For $P_{\rm in}$ shorter than 10 d (leftmost points), the inner orbits were likely modified by the dissipative Kozai evolution which decreased $P_{\rm in}$. For longer $P_{\rm in}$, the points fall in the band that corresponds to a limited period ratio range, $5 < P_{\rm out}/P_{\rm in} < 10^4$. The lone point in the upper part of the diagram belongs to Capella—a system with evolved components which most likely experienced the Kozai evolution in its inner 100-day circular orbit when the components left the MS and expanded. The initial orbit of Capella must have had a much longer period like the 390-yr visual dwarf pair HL in this quadruple system.

So *multiple stars form with only a limited range of period ratios at adjacent hierarchical levels; ratios larger than* 10^4 *result from subsequent orbit evolu-*

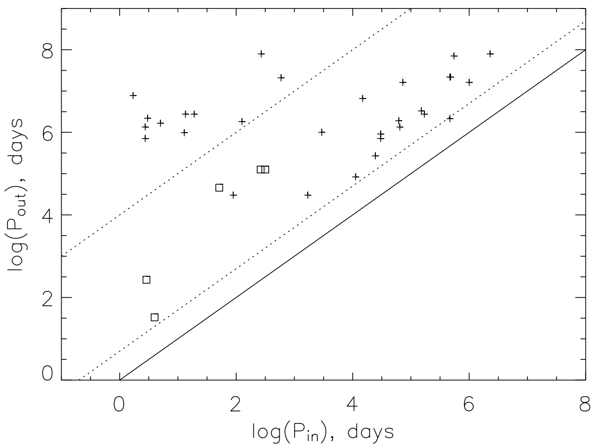

Fig. 7. The relation between $P_{\rm in}$ and $P_{\rm out}$ for young multiples in nearby star-forming regions (same symbols as in Fig. 6).

tion. I expect that all dwarf multiple systems with $P_{\rm out}/P_{\rm in} > 10^4$ and $P_{\rm in} > 30$ d contain yet undiscovered intermediate-level components.

The number of known multiple systems among PMS stars in nearby star-forming regions grows steadily as a result of systematic imaging and spectroscopic surveys (e.g. Covino et al. 2001; Köhler et al. 2000). Of course, the multiplicity statistics are less complete than for nearby MS stars. Nevertheless, I show in Fig. 7 the first $P_{\rm in} - P_{\rm out}$ diagram for the known low-mass PMS multiples. The wide (visual) multiples are on the average less hierarchical than their MS counterparts, but it is not clear whether this is not a selection effect (distant companions are not recognized as physical until information on their proper motions and radial velocities becomes available). A "cluster" of visual-spectroscopic triples is apparent, but with only large $P_{\rm out}/P_{\rm in}$ ratios.

In constructing Fig. 7 I avoided systems with massive primaries. Only two such stars, located in the lower left corner of the diagram, were included – the AeBe star TY CrA and the spectroscopic triple λ Tau, with periods of 33.1 and 3.95 days. They are only weakly hierarchical. These systems could have been formed by dynamical interactions in very dense stellar groups. Interestingly, no such systems are found among low-mass PMS stars and low-mass MS stars in the solar neighborhood.

5. DYNAMICAL STABILITY

The lower limit of $P_{\rm out}/P_{\rm in}$ is clearly related to dynamical stability constraints. This consideration can be further refined by studying the multiples where the orbits at two adjacent hierarchy levels are known. I selected 120 such systems from the MSC, mostly with outer visual and inner spectroscopic orbits but also with two visual or two spectroscopic orbits. All spectral types and primary masses are included in this sample. The periods come from orbital solutions; the eccentricities of inner and outer orbits are known. The ratio $P_{\rm out}/P_{\rm in}$ that corresponds to the stability limit depends on the eccentricity of the outer orbit. The criterion of Mardling & Aarseth (2002, MA02) for coplanar prograde orbits is

$$\begin{aligned}(P_{\rm out}/P_{\rm in})^{2/3} &\geq 2.8(1+q_{\rm out})^{1/15} \\ &\times (1+e_{\rm out})^{0.4}(1-e_{\rm out})^{-s}, \quad (1)\end{aligned}$$

where $q_{\rm out}$ is the mass ratio in the outer system (unimportant and taken here to be 0.3) and $s = 1.2$ is the exponent. In Fig. 8 I plot the eccentricity of the outer orbit $e_{\rm out}$ as a function of the period ratio $P_{\rm out}/P_{\rm in}$. The MA02 stability criterion is also plotted; it can be seen that some points fall in the forbidden zone. However, those points come from multiple systems with very long outer periods ($P_{\rm out} > 300$ yr) and hence insecure orbital elements. The highest eccentricity $e = 0.98$ belongs to the visual and eclipsing system Kui 93 = QS Aql with an uncertain visual orbit.

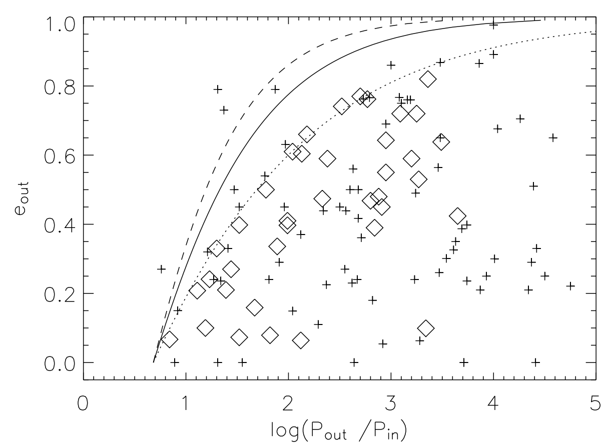

Fig. 8. The eccentricity of the outer orbit $e_{\rm out}$ as a function of the period ratio $P_{\rm out}/P_{\rm in}$ for multiple systems with two known orbits. Diamonds correspond to $P_{\rm out} < 10^5$ d ≈ 300 yr (reliable outer orbits) and $P_{\rm in} > 10$ d (not tidally evolved), and crosses to the remaining systems. The full line shows the stability criterion of MA02 ($s = 1.2$), the dashed line its modification by ST02 ($s = 0.9$), and the dotted line, $s = 2$.

Keeping in mind that long-period visual orbits are of poor quality and that the $P_{\rm out}/P_{\rm in}$ ratio could

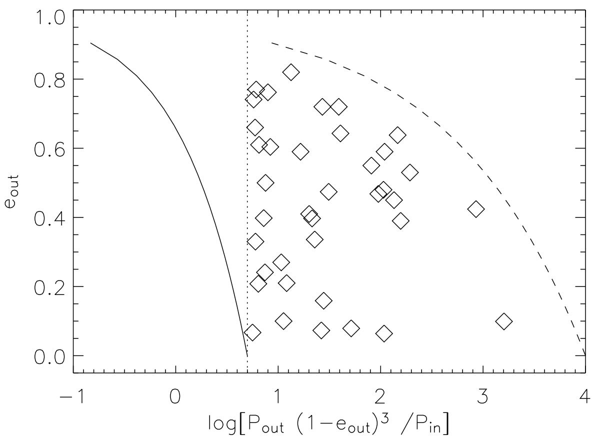

Fig. 9. Same as Fig. 8 plotted in different coordinates. The dotted line is an empirical stability limit, the full line corresponds to a fixed ratio of the outer-orbit periastron distance to the inner-orbit semi-major axis, the dashed line shows the limit $P_{\rm out}/P_{\rm in} < 10^4$.

be modified by Kozai evolution for $P_{\rm in} < 10$ d, the systems that avoid those conditions are highlighted in Fig. 8. The diamonds concentrate more closely to the stability limit. The left envelope of the points can be described by changing the exponent in the MA02 formula to $s = 2$, opposite to the modification suggested by Sterzik & Tokovinin (2002, ST02). The $(1 - e)^{-s}$ term in Eq. 1 is the most important one. To show this, I plot the eccentricities vs. modified period ratio $P_{\rm out}(1 - e_{\rm out})^3/P_{\rm in}$ in Fig. 9. Now the empirical stability limit $P_{\rm out}(1 - e_{\rm out})^3/P_{\rm in} > 5$ is represented by a vertical line. For most systems the modified period ratio does not depart from this limit by more than 2 orders of magnitude. If dynamical stability were governed by the ratio of the periastron distance in the outer orbit to the semi-major axis of the inner orbit, as in all existing criteria, this would translate to the limit on $P_{\rm out}(1-e_{\rm out})^{3/2}/P_{\rm in}$ and the location of points would be very different (cf. the full line in Fig. 9).

The crosses in Fig. 8 that fall to the left of the dotted line are most likely explained by wrong visual orbits. I predict that *those orbits will be revised.*

Why do real multiples deviate from the dynamical stability limit in the systematic way described by $s = 2$? It is possible that the currently established dynamical stability limit does not take into account some secular terms or resonances that slowly destroy systems with eccentric outer orbits (R. Mardling, private communication). Processes other than stability that restrict $e_{\rm out}$ can not be excluded, but the coincidence of empirical and theoretical limits for *circular* orbits (in Eq. 1, $2.8^{1.5} = 4.7 \approx 5$) speaks strongly against this conjecture.

It has been noted by Shatsky (2002) that the outer orbits in multiple stars tend to have moderate eccentricities and do not follow the $f(e) = 2e$ distribution established for wide visual binaries. The absence of large $e_{\rm out}$ could be explained as a combination of the limited $P_{\rm out}/P_{\rm in}$ range (cf. the dashed line in Fig. 9) with the stability constraint, as shown by Sterzik et al. (2003) and reinforced by the modified stability limit suggested here.

6. MASS RATIOS

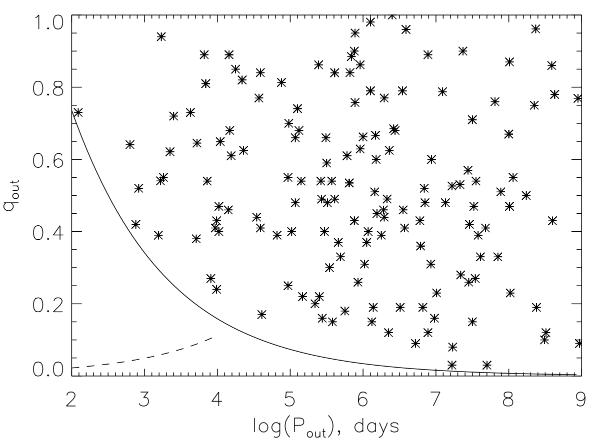

Fig. 10. Mass ratio of outer sub-systems as a function of their period for systems with primary mass less than 1.5 $M_\odot$ within 50 pc. The full line shows a suggested cut-off $q_{\rm out} \propto P_{\rm out}^{-1/3}$, the dashed line is an indicative 1 km/s spectroscopic detection limit.

Statistics of the mass ratios in multiple stars contain valuable information on the formation mechanisms. Valtonen (1998) used the MSC to argue that dynamical interactions are important. He ignored the bias due to discovery incompleteness, however. Such studies should be repeated when more-or-less complete samples of multiple stars become available.

It has been noted (Tokovinin 2001) that the mass ratios in the outer sub-systems of multiple stars show some correlation with the periods of those systems. Namely, for short $P_{\rm out}$ only systems with high $q_{\rm out}$ are found. This effect, shown in Fig. 10, can result from observational selection, as it is difficult to detect low-mass companions at close separations. However, in Fig. 10 only the best studied sub-sample of MSC (within 50 pc with primary mass $< 1.5\ M_\odot$) is plotted, making this argument questionable. Short-period tertiary components are discovered almost exclusively through radial velocity variation superimposed on the orbital motion in the inner sub-system.

The detection limit should be proportional to $P_{\rm out}^{1/3}$, as shown by the dashed line in Fig. 10.

It is generally accepted that formation of close binaries proceeds through the loss of angular momentum which requires some mass to be carried away. This consideration leads to a lower limit on the mass ratio if we suppose that a distant tertiary component absorbs the angular momentum of the close binary. The more distant the tertiary, the less massive it can be to hold a given angular momentum. The $q_{\rm out} \propto P_{\rm out}^{-1/3}$ relation that follows from this argument is shown in Fig. 10 with an arbitrary proportionality coefficient that was adjusted to describe the lower envelope of the points.

The empty lower left corner in Fig. 10 is intriguing. This tendency persists when *all* low-mass systems in MSC are considered. On the other hand, some short-period systems with massive primaries, e.g. λ Tau, do have low-mass tertiaries. Apparently, the formation mechanisms for massive multiples could be different.

7. RELATIVE ORIENTATION OF ORBITS

Relative orientation of orbital angular momentum vectors in triple stars has been studied recently in ST02. Compared to an earlier study of 1993, a somewhat larger number of systems was used. More importantly, however, the results were confronted with simulations of small-cluster decay. It turns out that, contrary to the naive expectation, decay may result in partially correlated momenta if the initial cluster was flattened and/or had some rotation. A set of reasonable cluster parameters can reproduce the observed weak correlation of momenta. I repeated the analysis for the sample of 10 quadruple systems of the ϵ Lyr type from the MSC where both visual orbits are known, and have not found any correlation in the orientations of inner pairs.

The statistics of apparently co- and counter-rotating visual triples (135 systems with measurable motions) has been studied. It is directly related to the average angle between the angular momenta. A new result of ST02 is that the correlation of angular momenta is higher in weakly-hierarchical multiples than in highly-hierarchical ones. The simulations were analyzed in the same way and show a similar tendency.

8. CONCLUSIONS

The main conclusion of this work is that systems of multiplicity three and higher are frequent, representing $\sim 1/5$ of the total stellar population. This shows that formation of hierarchical multiples is an important part of general star formation rather than a rare exception.

A relation between close binaries and higher-order multiples is established empirically and substantiated by the physical argument that a tertiary companion plays an important role in the close-binary formation through an exchange of angular momentum. One mechanism of such interaction – Kozai cycles coupled with tidal dissipation – seems to be confirmed by current theories and by multiple-star statistics, showing up as a peak in the period distribution of low-mass inner sub-systems at periods from 2 to 7 days, with a sharp drop at $P_{\rm in} > 7$ d.

The observed distribution of the period ratio $P_{\rm out}/P_{\rm in}$ at adjacent hierarchical levels is corrupted by incomplete knowledge of stellar multiplicity. For the nearest, best studied multiples, this ratio does not exceed 10^4 (except for the short $P_{\rm in}$ that were modified by tides) and supports the idea that most multiple stars originate from dynamical interactions in small clusters. Stellar dynamics alone cannot produce very high $P_{\rm out}/P_{\rm in}$ ratios, matching in this respect the observed statistics of both MS and PMS multiples. The observed weak correlation between the orientations of inner and outer orbits can be explained through modeling of small disintegrating clusters with some initial rotation and flattening (ST02).

A small cluster where several stellar embryos interact dynamically and accrete at the same time seems to be a critical stage in multiple star formation. Theoretical modeling and description of this "melting pot", producing single, binary, and multiple stars, is needed to explain multiple-star statistics.

The lower limit of $P_{\rm out}/P_{\rm in}$ is set by dynamical stability. Closer examination of triple stars with known orbital elements reveals that current stability criteria work well for circular orbits but do not match the more strict *empirical stability limit* $P_{\rm out}(1 - e_{\rm out})^3/P_{\rm in} > 5$, which remains to be explained.

The insights into stellar formation and dynamics gained from multiple-star statistics are valuable and can not be obtained in any other way. This justifies continued efforts in multiple-star observations that benefit from new techniques like adaptive optics and precise radial velocities. Only 10% of all multiples within 50 pc from the Sun are actually known, calling for dedicated and systematic multiplicity surveys of nearby stars and other stellar populations.

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DISCUSSION

Clarke – I would like to know the discovery limits for SBs as the primaries to visual binaries (for which you found that about 1/3 VBs contain SBs). What are the velocities and periods?

Tokovinin – Our discovery limit is $K \approx 1$ km/s for periods less than few years.

Mardling – Stars with shallow convective zones and stars with radiative envelopes can dissipate tidal energy efficiently.

Tokovinin – I would be happy to learn more about this, talking to theoreticians.

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