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DUST OPACITY AND THE CONTRACTION OF PROTOPLANETARY ATMOSPHERES

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RESUMEN

Presento un modelo del cálculo de opacidad de granos de polvo en una atmósfera protoplanetaria. Se calcula la distribución de tamaños del polvo a diferentes niveles en la atmósfera usando la microfísica de crecimiento de granos de polvo a través de colisiones y su destrucción vía vaporización a altas temperaturas. Se puede calcular la opacidad media de Rosseland de la distribución resultante entonces. Las opacidades del polvo resultantes son mucho menores que aquellas calculadas bajo la suposición de una distribución similar a la del medio interestelar, y es similar a la de baja opacidad usada en los modelos de Hubickyj et al. (2000, 2002).

ABSTRACT

I present a model for the computation of the opacity of grains in a protoplanetary atmosphere. The grain size distribution at different levels in the atmosphere is calculated using the microphysics of grain growth via collisions and destruction via vaporization at high temperatures. The Rosseland mean opacity of the resulting distribution can then be computed. The resulting grain opacities are much smaller than those computed assuming a size distribution similar to that of interstellar grains, and is similar to the low opacities used in the models of Hubickyj et al. (2000, 2002).

Key Words: **ISM: DUST, EXTINCTION — STARS: PLANETARY SYSTEMS: FORMATION**

1. DEDICATION

Celestial objects have spin.
Which remains . . . like the Cheshire Cat's grin.
A solution by Peter
Which couldn't be neater
Explains how the mass still falls in.

2. INTRODUCTION

Recent models of the formation and early evolution of the giant planets (Pollack, *et al.*, 1996; Hubickyj *et al.* 2000; 2002) have studied the contraction of a gaseous envelope formed by imbedding a solid core in the solar nebula. In the region of Jupiter, these models typically require roughly 6×10^6 yr. to reach the stage where the nebular gas will contract rapidly onto the protoplanetary core. This calculated time is uncomfortably close to the upper limit for the observed lifetime of such disks (Hollenbach *et al.*, 2000). In addition, the core mass required to induce the rapid contraction in this region is of the order of 12 $M_\oplus$. This too is uncomfortably close to the upper limit that models set on the Jovian core mass (Wüchterl *et al.* 2001).

As Pollack *et al.* originally showed, it is possible to reduce the critical core mass by reducing the surface density of solids in the region, but this lengthens the time required to reach the rapid contraction stage. Correspondingly, the time to reach the rapid contraction stage can be reduced by increasing the surface density, but this leads to an even larger critical core mass. The contraction rate of the gaseous envelope is very sensitive to its opacity. This, in turn, depends upon the grain size distribution in the atmosphere. Recent work by Hubickyj *et al.* (2000; 2002) has shown that the formation time for Jupiter can be reduced substantially by arbitrarily reducing the opacity in the envelope. The question I wish to address here is whether such a reduction is justified on physical grounds.

A large contribution to the opacity in the cooler regions of the atmosphere is due to solid grains. These grains have at least three sources: The first is the infalling gas. As the protoplanet contracts, additional gas is accreted from the surrounding disk. This gas will contain some solid material as grains, and these grains will be incorporated into the growing planet. A second source is the vaporization of material off of a planetesimal as it plunges through the protoplanetary atmosphere. This vaporized material will quickly cool and recondense into grains. The third source is the breakup of planetesimals when the ram pressure of the gas exceeds their intrinsic strength (see Podolak *et al.* 1988). Pollack *et al.*

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followed the trajectories of planetesimals through the protoplanetary envelope, and determined the rate of mass deposition into the protoplanet at each level in the atmosphere. They did not, however, attempt to determine the size distribution of these grains.

In their original work Pollack *et al.* assumed a grain size distribution and composition similar to that observed in the interstellar medium. Although the *composition* of the grains in a protoplanetary atmosphere should be similar, the size distribution, which is influenced by atmospheric microphysics, may well be different from that encountered in interstellar gas. This could lead to a very different wavelength dependent opacity. In this paper I present a model for the growth and sedimentation of grains in a protoplanetary atmosphere. The model is, admittedly, simple by the standards of microphysics used for modeling terrestrial clouds, but the details of protoplanetary atmospheres are sufficiently poorly known, so that a more detailed treatment is probably not justified at this time.

3. SIMPLE MODEL

The underlying physics of the problem can be best understood by considering a very simplified model. The opacity produced by the grains depends on their cross section. For a grain of radius a this consists of the geometric cross section, πa^2 multiplied by some efficiency factor, Q, which depends on the refractive index of the grain and its size relative to the wavelength of the radiation. For grains of a size comparable to the wavelength of the relevant radiation, this efficiency factor is of the order of one, but for much smaller grains it can be orders of magnitude lower. Thus, although smaller grains provide more surface area per unit mass, the reduction of extinction efficiency for very small grains more than compensates for their larger numbers. In such a case, the maximum extinction for a given mass of grain material is achieved when the grains are roughly the same size as the wavelength of the radiation.

Since the grains are in the gravitational field of the protoplanetary core, they will sediment down. Smaller grains have a lower settling speed and so remain in the atmosphere longer, but they also have more time to collide and grow. To first approximation, the time to settle out of a layer of thickness ℓ is $t_{sed} = \ell / v_{sed}$ where v_{sed} is the sedimentation speed. The time between grain collisions is roughly

$$t_{coag} = \frac{\lambda}{v_{th}^{grain}} \,. \tag{1}$$

Here λ is the mean free path between grains, and v_{th}^{grain} is the thermal speed of a grain. If t_{coag} is short compared to t_{sed}, the grains will collide and grow before they sediment out of the region. The grain size can be estimated by setting $t_{sed} \approx t_{coag}$. The mean free path between grain collisions will be $\lambda = 1/2\pi a^2 n$ where n is the number density of grains. In the upper atmosphere, where the gas density is sufficiently low, the sedimentation speed is given by

$$v_{sed} = \frac{g \rho_{grain}}{\rho_{gas} v_{th}^{gas}} a \,,$$

where g is the acceleration of gravity, ρ_{grain} and ρ_{gas} are the densities of the grain and gas, respectively, and v_{th}^{gas} is the thermal speed of a gas molecule. If the mass flux through the layer is F, then

$$F = \frac{4\pi}{3} n \rho_{grain} v_{sed} a^3 \,. \tag{2}$$

The thermal speed of a molecule (or grain) of mass m is given by

$$v_{th} = \sqrt{\frac{8kT}{\pi m}} \,,$$

where T is the temperature and k is Boltzmann's constant. Setting $t_{sed} = t_{coag}$, and using the above relations we find

$$a = \left[\frac{3\sqrt{6kT}}{2\pi} \frac{F \ell \rho_{gas}^2 \left(v_{th}^{gas}\right)^2}{\rho_{grain}^{7/2} g^2} \right]^{2/9} \,. \tag{3}$$

The corresponding optical depth is

$$\tau = \frac{3 F \ell \rho_{gas} v_{th}^{gas} Q}{4 \rho_{grain}^2 g} \left[\frac{2\pi \rho_{grain}^2 g^2}{3\sqrt{6kT} F \ell \rho_{gas}^2 \left(v_{th}^{gas}\right)^2} \right]^{4/9} \,. \tag{4}$$

The opacity per gram of gas is then

$$\sigma = \frac{3 F v_{th}^{gas} Q}{4 \rho_{grain}^2 g} \left[\frac{2\pi \rho_{grain}^2 g^2}{3\sqrt{6kT} F \ell \rho_{gas}^2 \left(v_{th}^{gas}\right)^2} \right]^{4/9} \,. \tag{5}$$

4. THE MODEL

A more careful modeling of these processes, presented below, is based on one originally developed to study Titan's aerosols (Podolak and Podolak, 1980). The code has been adapted to fit the present case (Podolak, 2004; hereafter called paper I), and the details of the relevant physics are summarized below.

The grains are assumed to be formed of monomers of a given size. I have experimented with sizes in the range between 0.01 and 10 μm. These monomers can agglomerate into larger grains. Both laboratory experiments (Bar-Nun, *et al.*, 1988) and numerical simulations (Weidenschilling, *et al.*, 1989) show that these larger grains will have a fractal structure. Following Weidenschilling and Cuzzi (1993), I assume a fractal dimension of 2.11. This has two important consequences: First, if we approximate the fractal grains by equivalent spherical grains, the density of such a grain will depend on its radius. Second, the scattering properties of the grains will differ from those of solid grains.

The monomers are assumed to have a density of $\rho_0 = 2.8$ g cm^{-3}, a value typical of the density of silicate material. The grains become more porous as they grow because of their fractal nature. When they are large enough, collisions cause them to undergo compaction, and for sufficiently large grains, the density returns to the density of the monomer. The detailed algorithm for computing the density as a function of radius is described in paper I. The grain size distribution is modeled by dividing it up among a number of discrete size bins with radii given by $a_n = 2^{\alpha n} a_0$, where a_0 is some scale factor. In the model presented below, α was chosen to be 0.5. This was based on the desire to have both good size resolution and a relatively small number of bins. Numerical tests showed that the results were not sensitive to reasonable changes in this value.

The transport of grains as a result of sedimentation as well as by convection was computed. Grain growth was calculated due to collisions as a result of Brownian motion, motion in convective eddies, and differences in sedimentation speed between large and small particles. In the region of the atmosphere where the temperature is sufficiently high, mass loss by vaporization was taken into account. The details for computing these processes are given in paper I.

For the baseline model, I have assumed that the grains are composed of rock (silicates). If the partial pressure of the rock vapor reaches the saturation value at any point in the atmosphere, it will condense onto any existing grains. This has not been included in the current model, but will be treated in future work. Here I assume that the mass fraction of vapor is sufficiently low so that saturation is never reached.

The opacity of the dust grains is calculated by computing the Rosseland mean using a code kindly supplied by J. Cuzzi of the NASA Ames Research Center. This code assumes spherical grains and computes the extinction cross sections using an approximation to Mie scattering given by Draine and Lee (1984). The fractal nature of the particles is approximately taken into account by computing the scattering for the whole sphere, and assuming that the voids are filled with a "material" that has a real refractive index of 1.0 and an imaginary refractive index of 0. The effective refractive index of the actual material (solid plus void) is then computed by using Maxwell-Garnett theory (1904). The grain material itself is assumed to have a real index of 1.5 and an imaginary index of 10^{-2}. The sensitivity of the results to these choices is not great, and is discussed in paper I.

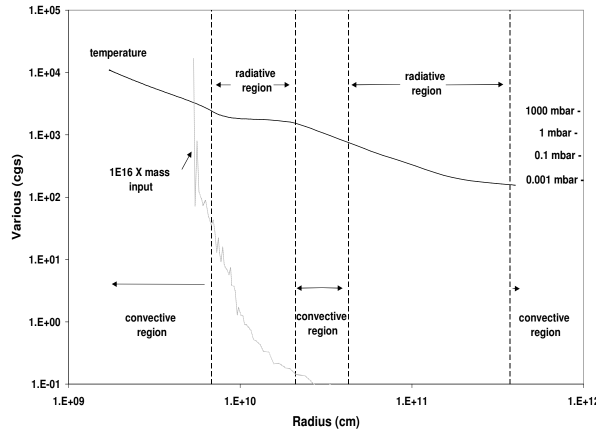

Fig. 1. Temperature, pressure, density, and grain input for a the atmosphere of a protoplanet just before the slow contraction stage. Also shown are the convective and radiative zones.

The distribution of pressure, density, temperature, and convective motion in the atmosphere are taken from the models of Pollack *et al.* (1996). I present here the case of a protoplanet with a $11.5 M_{\oplus}$. This corresponds to the beginning of phase 2 in the planet's evolution. This is by far the longest part of the evolution, and if it can be significantly reduced, then the total time to contraction will be reduced correspondingly.

The temperature profile through the atmosphere is shown by the solid curve in Fig. 1. The boundaries between convective and radiative regions are indicated by dashed vertical lines. Note that temperatures near the center are well in excess of 2000 K, so that the grains will all be vaporized in that region.

The dotted curve in Fig. 1 shows the mass input in units of 10^{-16} g cm^{-3} s^{-1}in order to use the same scale as the temperature. The size distribution of the grains is unknown, so two limiting cases were examined: either all the mass was deposited as grains in the smallest size bin, or the mass was distributed equally over all of the size bins. The grains carried by the infalling gas were always put into the smallest size bin, and were deposited in the uppermost atmospheric layer.

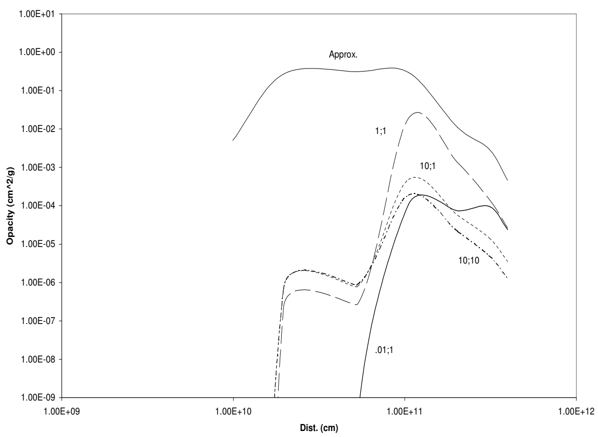

Fig. 2. Opacities for different initial grain distributions. See text.

5. MODEL RESULTS

The Rosseland mean opacities for the model are shown in Fig. 2. The four lower curves are each characterized by two numbers. The first is a_0 in microns, and the second is the number of bins over which the mass is input. As can be seen, the opacity increases as we go deeper into the atmosphere. This is because the mass input of grains increases with depth, and the sedimentation rate of the grains decreases. The curves all display a sharp downturn where convection draws the grains towards high temperatures and evaporation. In some cases there is a second peak where the mass input is so high that some grains survive temporarily despite the high local temperature.

As expected, the opacities are higher if all the grains are put into the first bin (case 10,1) rather than spread over 10 bins (case 10,10). When a_0 is small enough there is grain growth and the opacity curve looks somewhat different (case .01,1). The simple model, for comparison, is given by the uppermost curve. What is especially interesting is that there seems to be a maximum value to the opacity for a given atmosphere and source function. This can be understood in terms of the simple model presented in the first section: If the source puts all of the mass into large grains, the opacity will be small. As a_0 decreases, the area per unit mass increases and the opacity increases as well. But if the grains become small enough, grain growth by collisions ensues, and mass is shifted back to larger grains. This, in turn, reduces the surface area per unit mass, so that the increase in opacity as the grains get smaller is self-limiting.

In all the cases examined, the opacity is well below 0.1 cm^2 g^{-1}. This is considerably lower than the opacity used by Pollack *et al.* (1996), and is also lower than the low opacity assumed by Hubickyj *et al.* (2000). As Hubickyj *et al.* showed, these lower opacities can dramatically reduce the time to form a giant planet. Models for core mass of $21.1 M_{\oplus}$, representing an atmosphere at the end of the slow accretion stage, give similar results, with opacities less that ~ 0.1 cm^2 g^{-1}.

6. CONCLUSIONS

Although there is still much work to be done. The model presented here used silicate grains. In fact the grains will be a combination of silicates and other less refractory substances. The vaporization of these species at lower temperatures will lower the opacity even further. Thus, there is good reason to expect that the grain opacity can be reduced still further. This, in turn, will lead to a significant reduction of the formation time predicted by the core-instability model.

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