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CONSTRAINTS ON PLANETARY FORMATION SCENARIOS

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RESUMEN

Para entender la variedad de sistemas planetarios extra-solares es necesario comprender mejor el proceso de formación del Sistema Solar. Por esta razón, investigamos la relación entre el origen de la oblicuidad de los planetas gigantes y el origen de sus satélites, a fin de poner límites a las teorías actuales relacionadas con los procesos finales de formación de planetas. Hacemos énfasis en el sistema de Urano.

ABSTRACT

A better understanding of the Solar System formation process is necessary in order to understand the diversity of extra-solar planetary systems. We investigate the problem of the origin of the obliquity of the giant planets in connection with the origin of their satellites in a self-consistent fashion in order to set constraints on processes assumed to occur at the end of the formation of the planets. We pay special attention to the Uranian system.

Key Words: **ORBITS — PLANETS AND SATELLITES: GENERAL — PLANETS AND SATELLITES: INDIVIDUAL (URANUS) — SOLAR SYSTEM: FORMATION**

1. INTRODUCTION

The origin of planetary rotation and obliquity (inclination of the spin axis with respect to the orbital plane) is an open question. Large stochastic impacts at the end of the formation of the planets have been invoked to explain the obliquities of the giant planets of the Solar System. We set constraints on this scenario according to what is know about the physical and dynamical properties of the irregular satellites of the giant planets. This knowledge also allows us to set constraints on the mechanisms of formation (capture) of irregular satellites. The large obliquity of Uranus (98°) is of particular interest. In § 2, we introduce our model of the Uranian system and in § 3 we present the results.

2. MODELLING THE URANIAN SYSTEM

We assume that the present obliquity of Uranus is due to a large tangential and inelastic impact that suffered Uranus with another protoplanet at the end of the accretion process. We model the angular momentum transfer and the impulse transfer to Uranus at impact. From angular momentum considerations at collision we get:

$$v_i = \frac{3(m_U+m_i)}{5m_iR_c}R_U^2\left[\Omega^2+\frac{\Omega_0^2}{\left(1+\frac{m_i}{m_U}\right)^2\left(1+\frac{m_i}{3m_U}\right)^4} - \frac{\Omega\Omega_0\cos\alpha}{\left(1+\frac{m_i}{m_U}\right)\left(1+\frac{m_i}{3m_U}\right)^2}\right]^{1/2}, \qquad (1)$$

where v_i is the impactor incident speed, m_i is the impactor mass and $(m_U + m_i)$ is the present mass of Uranus. R_C is Uranus' core radius at impact (18000 km), R_U the present Uranus' radius and Ω (Ω_0) the angular velocity of Uranus after (before) the impact, α being the angle between $\vec{\Omega}$ and $\vec{\Omega}_0$. We take T_0 ($2\pi/\ \Omega_0$)=20 hs, T ($2\pi/\ \Omega$)=17 hs, α=70^o and m_i=1 $m_\oplus$. Through momentum conservation considerations at collision we have:

$$\vec{v_i}m_i = (m_U+m_i)\Delta\vec{V}, \qquad (2)$$

where $\Delta\vec{V}$ is the orbital velocity change suffered by Uranus' center of mass.

If the large obliquity of Uranus had been the result of a large impact, the impulse imparted at collision would have strongly affected the orbit of any satellite orbiting Uranus at that time. Moreover, any satellite had suffered the same orbital velocity change $\Delta\vec{V}$ with respect to the center of mass of Uranus, being probably unbound (Parisi & Brunini 1997). The discovery of the 9 Uranian irregulars (Gladman *et al*. 1998, 2000; Kavelaars *et al*. 2004) brings valuable clues into this scenario (Brunini *et al*. 2002). Due to the impulse imparted to the system at impact, the orbit of any satellite changes:

$v_1^2 = Av_e^2$, is the square of the satellite velocity before the impact, and

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$v_2^2 = B\big(1+\frac{m_i}{m_U}\big)\ v_e^2$, is the square of the satellite velocity after the impact.

$v_2^2 = v_1^2 + \Delta V^2 + 2 v_1 \Delta V \cos\Psi$,

A and B are coefficients ($0 < A < 1$; $B > 0$), v_e is the escape velocity at the satellite position just before the impact and Ψ is the angle between $\vec{v_1}$ and $\Delta\vec{V}$. For A=0.5 and B=1, the satellite has an initial circular velocity being unbound at collision. The satellite's orbital semiaxis before (a_i) and after (a_f) the impact is:

$$a_i = \frac{r}{2\,(1-A)},\ a_f = \frac{r}{2\,(1-B)}, \qquad (3)$$

being r the satellite position on its orbit at the moment of collision:

$$r = \frac{2\,G\,m_U}{(\Delta V)^2}\left[\frac{B'-A}{\sqrt{A}\cos\Psi \pm \sqrt{(B-A)+A\cos^2\Psi}}\right]^2, \qquad (4)$$

where B'=$B(1+\frac{m_i}{m_U})$. The minimum eccentricity of the orbits before ($e_i m$) and after ($e_f m$) collision is:

$$\begin{aligned} e_{im} &= 2\,(1-A) - 1 \quad if \quad A < 0.5 \\ e_{im} &= 1 - 2\,(1-A) \quad if \quad A > 0.5 \\ e_{fm} &= 2\,(1-B) - 1 \quad if \quad B < 0.5 \\ e_{fm} &= 1 - 2\,(1-B) \quad if \quad B > 0.5 \end{aligned}$$

Upper bounds on a_i and a_f as functions of A and B are obtained by setting in equation (4) $\Psi = 180^o$ and taking only the positive sign in the square root. The assumed parameters lead to the highest value of a_i and a_f, defined as a_{iM} and a_{fM}. From equations (1) and (2), we obtain ΔV. Then, from equations (3) and (4) for each A, we obtain the value of B corresponding to the transfer to $a_{fM} = a$, where a is the present orbital semiaxis of each of the known irregulars of Uranus. This value of B provides the minimum eccentricity $e_f m$ acquired by the orbit of each satellite. The existence of the Uranian irregulars before collision, or even more, the collision hypothesis itself imply that this theoretical minimum eccentricity e_{fm} after collision must be less than the present maximum eccentricity e_{Max} of each satellite.

3. RESULTS AND CONCLUSIONS

We computed the orbital evolution of the Uranian irregulars by means of a numerical integration of their equations of motion that includes the perturbations of the Sun, Jupiter, Saturn and Neptune. We integrated the orbits of these satellites over 10^5 yrs. to obtain e_{Max}.

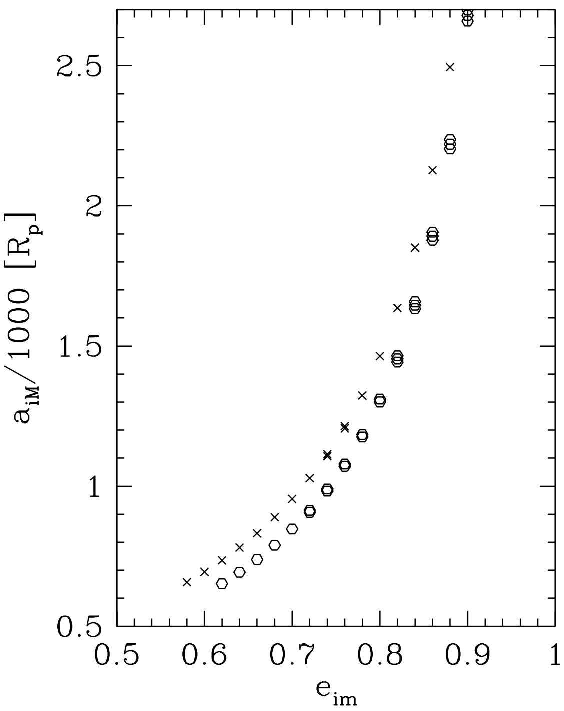

Fig. 1. Initial conditions of the orbits able to be transferred to the present orbits of the Uranian irregulars Prospero (crosses) and Trinculo (open hexagons).a_{iM} (in units of the planet' radius R_p.), is the satellite's orbital semiaxis before collision and e_{im} is the minimum eccentricity of the satellite's orbit before collision.

In Figure 1, we show the permitted initial conditions able to produce the present orbits of S/1999U3 (Prospero) and S/2001U1 (Trinculo). The value of a for Prospero and Trinculo is 658 planetary radii and 337 planetary radii, respectively. The value of e_{Max} is 0.571 for Prospero and 0.237 for Trinculo. The only allowed transfers for Prospero and Trinculo are those arising very close to the pericenter of a very eccentric orbit (*i.e.*, $e_i m > 0.58$ for Prospero and $e_i m > 0.62$ for Trinculo). The theoretical minimum eccentricity $e_f m$ is very close to e_{Max} for both satellites ($e_f m$ in the range [0.52-0.57] for Prospero and [0.16-0.23] for Trinculo). These restrictions are severe since one would expect a wide range of allowed transfers. Therefore, we hardly expect the existence of these satellites before collision. Then, these objects had to be captured after collision. Most capturing theories for irregulars act before the end of the accretion process (Pollack *et al*. 1979). Since the impact is assumed to have occurred very late in the stage of formation of the planet, the single capturing mechanism capable of producing the permanent capture of these objects is a disruptive mechanism. We then conclude either that the origin of these irregulars must be a disruptive mechanism or that the giant collision at the end of the formation of the planet did not occur; if that is the case, another mechanism to tilt Uranus would be required.

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