ASSESSING MILLISECOND PROTO-MAGNETARS AS GRB CENTRAL ENGINES

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Magnetars are a sizable subclass of the neutron star census. Their very high magnetic field strengths are thought to be a consequence of rapid (millisecond) rotation at birth in a successful core-collapse supernova. In their first tens of seconds of existence, magnetars transition from hot, extended "proto-"magnetars to the cooled and magnetically-dominated objects we identify \textsuperscript{1}\textsuperscript{10} years later as Soft Gamma-ray Repeaters (SGRs) and Anomalous X-ray Pulsars (AXPs). Millisecond proto-magnetar winds during this cooling phase likewise transition from non-relativistic and thermally-driven to magneto-centrifugally-driven, and finally to relativistic and Poynting-flux dominated. Here we review the basic considerations associated with that transition. In particular, we discuss the spindown of millisecond proto-magnetars throughout the Kelvin-Helmholtz cooling epoch. Because of their large reservoir of rotational energy, their association with supernovae, and the fact that their winds are expected to become highly relativistic in the seconds after their birth, proto-magnetars have been suggested as the central engine of long-duration gamma ray bursts. We discuss some of the issues and outstanding questions in assessing them as such.

Key Words: GAMMA RAYS: BURSTS — MAGNETOHYDRODYNAMICS — STARS: NEUTRON — STARS: WINDS, OUTFLOWS — supernovae: general

1. INTRODUCTION

Core-collapse supernovae (SNe) leave behind hot proto-neutron stars that cool on the Kelvin-Helmholtz timescale (\(\tau_{KH} \approx 10 - 100\) s) by radiating their gravitational binding energy (\(\sim 10^{53}\) ergs) in neutrinos (Burrows & Lattimer 1986; Pons et al. 1999). A fraction of these neutrinos deposit their energy in the tenuous and extended atmosphere of the PNS.\textsuperscript{3}\textsuperscript{1} In the standard picture, net neutrino heating drives a thermal wind that emerges into the post-supernova shock environment, blowing a wind-driven bubble into the exploding and expanding supernova cavity (Woosley et al. 1994; Burrows, Hayes, & Fryxell 1995). For most massive stellar progenitors with extended hydrogen envelopes (Type-II), the cooling phase is over well before shock breakout at the pro-
genitor’s surface (∼1 hour after collapse). For compact Type-IIc supernovae, the supernova shockwave traverses the progenitor on a timescale comparable to \(\tau_{KH}\).

For typical non-rotating non-magnetic (NRNM) neutron stars the wind/cooling epoch is in some sense a small perturbation to the supernova event as a whole. Depending on how one defines the start of the wind phase, the total wind kinetic energy over \(\tau_{KH}\) is of order \(10^{48} - 10^{49}\) ergs, small on on the scale of the supernova explosion energy, \(\varepsilon_{SN} \approx 10^{51}\) ergs.\(^4\) In addition, the total amount of mass ejected is a mere \(\sim 10^{-4} - 10^{-3}\) \(M_{\odot}\), a minor addition to the few solar masses ejected from the massive stellar progenitor during the explosion. Finally, because of inefficient neutrino heating, the asymptotic wind speed does not exceed \(\sim 0.1c\) (Duncan, Shapiro, & Wasserman 1986; Qian & Woosley 1996).

The primary focus of many previous efforts to understand proto-neutron star winds — particularly in the NRNM limit — has been to assess these outflows as the astrophysical site for production of the r-process nuclides (Woosley et al. 1994; Takahashi, Witti, & Janka 1994; Qian & Woosley 1996; Cardall & Fuller 1997; Otsuki et al. 2000; Sumiyoshi et al. 2000; Wanajo et al. 2001; Thompson, Burrows, & Meyer 2001; for a review, see Thompson 2003).

1.1. Rotation & Magnetic Fields

Simple estimates imply that \(\gtrsim 10\%\) of all supernovae produce magnetars, a class of young neutron stars (the Anamolous X-ray Pulsars and Soft Gamma-ray Repeater) with inferred large-scale surface magnetic dipole fields in the range \(10^{14} - 10^{15}\) G (Duncan & Thompson 1992; Thompson & Duncan 1993; Kouveliotou et al. 1999; see also Woods & Thompson 2006; Kaspi & Helfand 2002). The high magnetic fields of magnetars may result from the collapse of ultra-magnetized iron cores in massive stars or white dwarfs in accretion-induced collapse, or they may be generated by rapid rotation and efficient dynamo action during the Kelvin-Helmholtz cooling epoch of proto-neutron stars (see below, Duncan & Thompson 1992). In this latter scenario, it is millisecond spin periods at birth that are thought to distinguish magnetars from normal neutron stars, with their characteristically lower field strengths.

It is interesting to consider how the standard picture of NRNM neutrino-driven protoneutron star winds is modified by the presence of a strong large-scale magnetic field and rapid rotation. To do so, we first quantify the meaning of “strong magnetic field” and “rapid rotation” in the proto-neutron star context. For this purpose, it useful to consider the physical conditions of the proto-neutron star atmosphere just after birth.

Assuming a successful explosion occurs, the total neutrino luminosity perhaps one second after core collapse is in the range of \(5 \times 10^{52}\) ergs s\(^{-1}\), divided roughly equally between neutrino species. The temperature at the radius of neutrino decoupling, the neutrinosphere, is \(\approx 5\) MeV, so that the pressure scale height is \(h \approx R_{NS}(R_{NS}/GM)(k_{B}T/m_{\nu}) \approx 0.25\) km for a PNS radius and mass of \(R_{NS} \approx 10\) km and \(M \approx 1.4\) \(M_{\odot}\) respectively. The radius where neutrino heating becomes appreciable, the last point in the flow where kinetic equilibrium can be maintained, and where the wind-atmosphere becomes radiation dominated, is characterized by a specific entropy of \(s \approx 4\) (see the detailed discussion of Qian & Woosley 1996). Assuming that neutrino absorption on free nucleons dominates heating, this condition determines a characteristic thermal pressure in the proto-neutron star atmosphere of

\[
P_{s=4} \approx 3.3 \times 10^{28} \frac{L_{\nu_{e},52}^{2/3}}{\varepsilon_{\nu_{e},15} R_{10}^{4/3}} \text{ ergs cm}^{-3},
\]

where we index the total luminosity of the proto-neutron star by the luminosity in \(\bar{\nu}_{e}\) neutrinos and where \(L_{\nu_{e},52} = L_{\nu_{e}}/10^{52}\) ergs s\(^{-1}\), \(\varepsilon_{\nu_{e},15} = \varepsilon_{\nu_{e}}/15\) MeV is the average \(\bar{\nu}_{e}\) energy, and \(R_{10} = R/10\) km.

Although the detailed time evolution of \(L_{\nu_{e}}\) and \(\varepsilon_{\nu_{e}}\) are uncertain, equation (1) shows that as the neutron star cools and \(L_{\nu_{e}}\) and \(\varepsilon_{\nu_{e}}\) decrease, so too does \(P_{s=4}\). Setting \(B^2/8\pi\) equal to \(P_{s=4}\) yields a criterion for the magnetic field strength:

\[
B_{s=4} \approx 9 \times 10^{14} \frac{L_{\nu_{e},52}^{1/3}}{\varepsilon_{\nu_{e},15}^{2/3} R_{10}^{4/3}} \text{ G}.
\]

If \(B \gtrsim B_{s=4}\), then the magnetic energy density is larger than the thermal pressure and it will significantly contribute to or dominate the dynamics of the outflow. This estimate shows that large-scale magnetar-strength magnetic fields of \(\sim 10^{15}\) G are required in order to dominate the pressure at very early times after explosion. Essential to the following discussion, for fixed surface magnetic field strength, the magnetic field becomes increasingly dominant as the proto-neutron star cools. For example, taking \(L_{\nu_{e}} \propto t^{-1}\) and \(\varepsilon_{\nu_{e}} \propto L_{\nu_{e}}^{1/3}\), we find that \(B_{s=4}^2 \propto t^{-1}\);

\(^4\)However, see Burrows et al. (1995) and Scheck et al. (2006) for discussion of how the very early wind is tied to the supernova explosion itself.

\(^5\)See, e.g., Pons et al. (1999) for detailed neutrino luminosity and average energy time profiles.
over the few decades of $\tau_{KH}$, $B_{s=4}$ changes appreciably. Thus, by “strong magnetic field” we mean in this paper that $B_{NS} \gtrsim B_{s=4}(t)$.

The words “rapid rotation” can be quantified in several ways. First, a neutron star with a millisecond spin period has a reservoir of rotational energy of

$$E_{\text{rot}} \approx 2 \times 10^{52} M_{1.4} R_{10}^2 P_{-1}^{-2} \text{ ergs},$$

(3)

where $P_1$ is the spin period in units of 1 ms. Therefore, from the standpoint of the evolution of the supernova remnant, an initial spin period of $P \lesssim 5$ ms implies that the rotational energy of the neutron star is $E_{\text{rot}} \gtrsim E_{SN} \approx 10^{51}$ ergs. That is, if there exists a mechanism for spindown of the neutron star on any timescale shorter than the remnant age — some way to efficiently communicate the energy of rotation to the remnant material — a spin period of $\lesssim 5$ ms implies that this rotational energy will modify the remnant dynamics at order unity or larger.

A second estimate of what constitutes “rapid rotation”, comes from comparing the spin period with the characteristic convective eddy turn-over timescale ($\tau_{\text{con}}$) within the proto-neutron star during the cooling epoch. As argued by Duncan & Thompson (1992), the Rossby number $R = P/\tau_{\text{con}}$ determines whether or not an efficient dynamo operates in the proto-neutron star; for $P \sim 1$ ms, the Rossby number is less than unity and conditions are ripe for dynamo action. Duncan & Thompson (1992) argue that it is by virtue of their millisecond rotation at birth that magnetars develop $10^{14} - 10^{15}$ G magnetic fields.

A last and physically distinct estimate of “rapid rotation” comes from noting that the breakup spin period of a neutron star is order of 0.5 ms.

In summary, “strong magnetic field” means $B \gtrsim 10^{14}$ G at early times after core collapse and the criterion for magnetic field domination in the proto-neutron star atmosphere decreases as a function of time. Additionally, to order of magnitude, “rapid rotation” means millisecond spin periods. An essential point is that both conditions may be met by all magnetars at birth. Indeed, Duncan & Thompson (1992) argue that $P$ and $B$ are inseparably linked.

6The $s \approx 4$ point is generally close enough to the proto-neutron star neutrinosphere that it is unnecessary at the level of the present discussion to distinguish the surface magnetic field strength from $B_{s=4}$.

7However, it should be born in mind that both the total energy stored in rotation and the properties of the proto-neutron star atmosphere as breakup is approached are very strong functions of the spin period ($P^{-2}$ and exponential in $P^{-2}$, respectively).

1.2. Millisecond Proto-Magnetars & Gamma Ray Bursts

The combination of rapid rotation and high magnetic fields has dramatic consequences for the dynamics of any outflow that accompanies proto-magnetar cooling. As in studies of magnetic winds from rotating stars, the winds from proto-magnetars are dominated by magneto-centrifugal forces. Like beads on a wire, the magnetic field lines force the wind material into corotation with the stellar surface out to the Alfvén point, where the magnetic energy density equals the kinetic energy density of the outflow. This provides an efficient mechanism for spindown (Schatzman 1962; Weber & Davis 1967; Mestel 1968a,b; Pneuman & Kopp 1971; Belcher & Macgregor 1976; Mestel & Spruit 1987). Thus, a proto-magnetar’s rotational energy can be tapped and communicated, in the form of an energetic outflow, to the surrounding medium.

Although the problem of magnetar birth, and how it differs from the birth of more typical neutron stars, is interesting in its own right, there are several reasons for considering millisecond proto-magnetars as the central engine of long-duration GRBs: (1) the reservoir of rotation energy is in the range required to powers GRBs, (2) proto-magnetar winds, as for probably all neutron stars, should become relativistic on the $10 - 100$ second Kelvin-Helmholtz cooling timescale, and (3) the strong observational connection between core-collapse supernovae and long-duration GRBs is easy to understand in such a model. Of course, because magnetars are thought to constitute a relatively large fraction of all supernovae, not every magnetar can produce a canonical long-duration GRB. As in the collapsar model for GRBs (Woosley 1993; Macfadyen & Woosley 1999), it may be that only millisecond magnetars born within compact Type-Ic progenitors produce energetic GRBs with $\sim 10^{51}$ ergs. The relativistic outflow from millisecond magnetars probably cannot sustain high kinetic luminosity for the $\sim$ hours required in an extended Type-II progenitor. Alternatively, it may be that only those magnetars born with the highest fields and most rapid rotation yield conditions favorable for producing a GRB.

Assessing millisecond proto-magnetars as GRB central engines requires answering a number of questions: (1) What is the spindown timescale, and how much energy is extracted as a function of time as the proto-magnetar cools? (2) How relativistic is the outflow and how is the asymptotic Lorentz factor connected with the energy loss rate? (3) What is the wind geometry, and specifically, how might that
geometry change as a function of time as the wind transitions from non-relativistic to highly Poynting-flux dominated? Finally, (4) how are the relativistic outflow and the supernova coupled? Are there potential nucleosynthetic signatures of magnetar spin-down? And what might we expect generically from the remnants of supernovae that birthed millisecond proto-magnetars? In this paper we address (1) in detail (§2), drawing on analytic estimates and recent exploratory numerical models. We emphasize connections with magnetic stellar winds on one hand, and force-free models of pulsar spindown on the other — each representing the separate limits in the life of a millisecond magnetar. We discuss (2), (3), and (4) as well as the phases of PNS spin-down in §3.

2. MILLISECOND PROTO-MAGNETAR SPINDOWN

If the surface magnetic field becomes larger than $B_{\text{surf}}=4$, either because $L_\nu(t)$ decreases sufficiently, or $B_{\text{NS}}$ increases sufficiently as a result of dynamo action, the previously solely neutrino-driven outflow begins to be accelerated by the action of magneto-centrifugal forces. The wind begins to efficiently extract rotational energy from the newborn neutron star. At this stage the outflow is non-relativistic and the timescale for a millisecond magnetar’s spin period to $e$-fold — the spindown timescale — is

$$\tau_d = \frac{\Omega}{\dot{\Omega}} \approx 2 \left( \frac{M}{\dot{M}} \right) \left( \frac{R}{R_A} \right)^2,$$

where $\dot{M}$ is the mass loss rate, $R$ is the stellar radius, $\dot{M}$ is the mass loss rate, and $R_A$ is the radial position of the Alfvén point.

Under the assumption that $P \approx 1$ ms, because $E_{\text{Rot}} \gg E_{\text{SN}}$ (eq. 3), just one $e$-folding of $\Omega$ is sufficient to modify the dynamics of the supernova remnant significantly and potentially power a GRB. If $\tau_d$ is small with respect to the time for the supernova shock to traverse the progenitor (~tens of seconds for type-Ib, -Ic progenitors) we also expect this extra energy source to modify the supernova nucleosynthesis (Thompson, Chang, & Quataert 2004).

2.1. The Dominance of Magneto-Centrifugal Forces

The asymptotic velocity of a wind accelerated predominantly by magneto-centrifugal forces is $V_\infty \sim R_A \Omega$. Therefore, a rough criterion for the dominance of magneto-centrifugal forces in proto-magnetar winds is that $R_A \Omega \gtrsim V_\nu$, where $V_\nu$ is the asymptotic velocity of an outflow accelerated solely by neutrino heating ($\lesssim 0.1c$). For $P \sim 1$ ms and $R_A \gtrsim 15$ km this criterion is satisfied, the wind is driven primarily by magneto-centrifugal slinging, neutrino heating is relatively unimportant in determining the asymptotic wind velocity, and rotational energy is transferred efficiently from the proto-magnetar to the outflow.

In order to estimate the spindown timescale and the asymptotic velocity of the wind, we must first estimate the Alfvén radius. In what follows, we discuss the non-relativistic limit and then the transition to the relativistic regime. We then discuss these estimates in light of recent numerical models, which more fully capture the essential physics.

2.2. The Non-Relativistic Regime: Magnetic Stellar Winds

Angular momentum conservation implies that $\dot{J} = d/dt(\Omega I) \approx -\dot{M} R_A^2 \Omega$. The location of the Alfvén point depends on the radial dependence of the poloidal magnetic field. For the purposes of making a simple estimate, we assume that the field is strictly monopolar so that $B(r) = B_0(R_{\text{NS}}/r)^2$. Using $\rho = M/(4\pi r^2 v_r$ and the fact that the Alfvén speed is $v_A = v_r(R_A) \sim v_\phi(R_A) \sim R_A \Omega$, the location of $R_A$ is simply

$$R_A = B_1^{2/3} R_4^{4/3} (M\Omega)^{-1/3},$$

$$\sim 40 B_1^{2/3} R_4^{4/3} M_3^{-1/3} P_1^{1/3} \text{ km},$$

(5)

where $v_r$ is the radial velocity, $v_\phi$ is the azimuthal velocity, $\rho$ is the mass density, and $B$ is the radial magnetic field. In equation (5) $B_1 = B/10^{15}$ G and $M$ is scaled in units of $10^{-3} M_\odot$ s$^{-1}$.

In NRNM protoneutron star winds the mass loss rate is (Qian & Woosley 1996)

$$\dot{M}_{\text{NRNM}} \approx 4 \times 10^{-5} L_5^{5/3} L_{p,52}^{10/3} B_1^{2/3} M_3^{1/3} \Omega^{10/3} \text{ km s}^{-1},$$

(6)

for a 1.4 $M_\odot$ neutron star, considerably less than that implied by the scaling of equation (5). When the magnetic field is strong, centrifugal forces extend the scale height of the proto-magnetar atmosphere. Because the mass outflow rate is essentially determined by the density profile at the sonic point ($R_s$), this effect can increase $\dot{M}$ significantly if $R_A$ is larger than the nominal $R_s$. For a wind dominated by magneto-centrifugal acceleration, one finds that

$$R_s \approx (GM/\Omega^2)^{1/3} \approx 17 P_1^{2/3} \text{ km}.$$  

(7)

Because equation (5) implies that $R_s > R_A$ for fiducial values of $B$ and $M$, the larger scaling is in part justified. Indeed, more detailed calculations show that for $P = 1$ ms and $L_{\nu,52} = 1$, $\dot{M} \approx 10^{-3} M_\odot$
s\(^{-1}\) (Thompson et al. 2004; Metzger, Thompson, & Quataert 2006).

Using equation (5) we estimate that the absolute value of the rotational energy loss rate is

\[
\dot{E}_{\text{NR}} \sim B^{4/3} R^{8/3} M^{1/3} \Omega^{4/3} \\
\sim 10^{51} B^{4/3} R^{8/3} M^{1/3} P_1^{-4/3} \text{ ergs} \text{s}^{-1}. \quad (8)
\]

The subscript ‘NR’ is added to emphasize that when the flow is non-relativistic, \(E\) depends explicitly on \(M\). The spindown timescale \(\Omega/\dot{\Omega}\) in the non-relativistic limit is

\[
\tau_{\text{JNR}} \simeq 30 \text{s} \ M_{1.4} M^{-1/3} R_{10}^{-2/3} B_{15}^{4/3} P_1^{-2/3}. \quad (9)
\]

Note that for slower rotation, larger spin periods, the spindown timescale decreases at fixed \(M\).

There are several uncertainties in these simple estimates. First, the wind is not isotropic, so that the factor of \(4\pi\) that appears in the relation between \(M\) and \(\rho\) is incorrect. Second, the radial velocity, even in the idealized problem presented here is not \(R_A\), but can differ at the factor of two level. Most importantly, the field is assumed monopolar, when in reality the surface field must be dipolar at lowest order. However, one may argue that even if the field has a dipole field it is the open field lines that carry the energy and angular momentum, and perhaps these may be — to first approximation — considered monopolar. For this reason, it turns out that the assumption of a monopole field is remarkably good, as long as the surface field strength is suitably renormalized to reflect only the field lines that are opened to infinity by the wind. We return to this issue below in discussing numerical models.

### 2.3. The Non-Relativistic to Relativistic Transition

As the neutrino luminosity decreases, the characteristic thermal pressure in the atmosphere of a proto-magnetar decreases (eq. 1). For fixed surface magnetic field strength, we thus expect millisecond magnetar winds to become increasingly magnetically-dominated. In particular, \(M\) in equation (5) should decrease as \(L_\nu\) decreases (see eq. 6) and therefore \(R_A\) should increase. Although \(L_\nu\) can decrease arbitrarily as \(t \to \infty\), \(R_A\) cannot increase indefinitely; instead, it approaches the radius of the light cylinder \(R_L = c/\Omega \simeq 48 P_1\) km asymptotically. As it does so, the flow becomes increasingly relativistic. This is the transition between non-relativistic magnetically-dominated mass-loaded outflow and relativistic Poynting-flux dominated wind. All neutron stars likely pass through such a transition, regardless of their initial spin period and magnetic field strength. Millisecond proto-magnetars are particularly interesting as a candidate central engine for GRBs because this transition to relativistic flow occurs on the Kelvin-Helmholtz timescale (~10 to 100 s) and at high wind kinetic luminosity, and because magnetars are born in supernovae.

Setting \(R_A = R_L\) in equation (5) we can estimate the critical mass loss rate below which the wind becomes relativistic:

\[
\dot{M}_{\text{crit}} = B^2 R^4 \Omega^2 c^3 \\
\sim 7 \times 10^{-4} B_{15}^2 R_{10}^4 P_1^{-2} M_\odot \text{ s}^{-1}, \quad (10)
\]

where \(B\) is the magnetic field strength. For lower effective field strengths, \(\dot{M}_{\text{crit}}\) decreases — that is, the mass flux must decrease further in order to enter the relativistic regime. For a neutron star born with a 10 ms spin period and a 10\(^12\) G surface field, \(\dot{M}_{\text{crit}} \sim 7 \times 10^{-12} M_\odot \text{ s}^{-1}\). Thus, for weaker fields, the transition occurs at lower \(M\), lower \(L_\nu\), and at a time longer after collapse and explosion, a time later in the Kelvin-Helmholtz cooling epoch.

### 2.4. Relativistic Winds & The Force-Free Limit

Once \(R_A\) becomes close to \(R_L\), the degree of Poynting-flux domination is quantified by the parameter (Michel 1969; Goldreich & Julian 1970)

\[
\sigma(R_L) = \frac{B^2}{4\pi \gamma \rho c^2} \bigg|_{R_L}, \quad (11)
\]

where \(\gamma\) is the Lorentz factor and \(B = B(R_L)\). If the outflow is driven primarily by magneto-centrifugal forces, \(\gamma(R_L) \sim 1\). Roughly speaking, if energy transfer from the electromagnetic field to the matter is efficient, \(\sigma(R_L)\) measures the maximum achievable asymptotic Lorentz factor of the outflow as \(r \to \infty\). Assuming that \(\gamma(R_L) \sim 1\), \(\sigma(R_L)\) can be written simply in terms of the mass loss rate:

\[
\sigma(R_L) \approx B^2 R_{15}^4 \Omega^2 c^3 M^{-1} \\
\sim 70 B_{15}^2 R_{10}^4 P_1^{-2} M_\odot^{-1} \quad (12)
\]

where \(M_{\odot} = M/10^{-5} M_\odot \text{ s}^{-1}\) and we have again assumed a monopole field geometry. Compare equation (12) with equation (10); setting \(M = M_{\text{crit}}\) in the former, yields \(\sigma(R_L) = 1\). Thus, as \(L_\nu\) continues to decrease as the cooling epoch progresses, we expect the flow to become increasingly relativistic and Poynting-flux dominated (\(\sigma\) increases).
In the limiting case of force-free electrodynamics one neglects the inertia of the matter completely. This limit corresponds formally to $\sigma \to \infty$. Recent numerical models have calculated the spin-down luminosity for a neutron star with aligned magnetic and rotational axes in the force-free limit (Contopoulos, Kazanas, & Fendt 1999; Gruzinov 2005; Komissarov 2006; Spitkovsky 2006; McKinney 2006); they find that

$$
\dot{E}_{\text{FF}} = B^2 R_{\text{NS}}^6 \Omega^4 c^{-3}
\sim 5.8 \times 10^{-10} B_{15}^2 R_{10}^6 P_{15}^{-4} \text{ ergs s}^{-1}, \quad (13)
$$

where $B$ here corresponds to the surface dipole field strength. In the case of an orthogonal rotator — with the magnetic axis perpendicular to the rotation axis — $\dot{E}_{\text{FF}}$ is larger than that in equation (13) by precisely a factor of two (Spitkovsky 2006).

In the force-free limit, the spin-down timescale for the aligned rotator is

$$
\tau_{\text{FF}} \approx 760 \text{ s } M_{1.4} R_{10}^{-4} B_{15}^{-2} P_{15}^2. \quad (14)
$$

It is tempting to compare $\dot{E}_{\text{FF}}$ and $\tau_{\text{FF}}$ with equations (8) and (9) directly. Unfortunately, such a comparison is complicated by the fact that the former are written assuming a dipole surface field strength, while the latter assume a monopole field strength. One requires knowledge of the amount of open magnetic flux at a given $M$ to make a fair comparison. As we discuss in more detail below, the results of Bucciantini et al. (2006) show that for typical millisecond proto-magnetar parameters the equivalent monopole field strength is a factor of $\sim 3$ less than the dipole field strength. Therefore, to compare $\dot{E}_{\text{NR}}$ and $\dot{E}_{\text{FF}}$, the former should be decreased by a factor of $\sim 4$, implying that $\dot{E}_{\text{NR}} / \dot{E}_{\text{FF}} \approx 4 R_{10}^{-10/3} M_{-3}^{1/3} P_1^{8/3}$. This expression has a strong dependence on $P$ and a rather weak dependence on $M$. For example, taking $P = 3$ ms one finds that the ratio $\dot{E}_{\text{NR}} / \dot{E}_{\text{R}}$ is nearly 20 times larger. Thus, the monopole scalings derived in §2.2 suggest that the early spin-down of proto-magnetars is rapid because the flow is non-relativistic, but magnetocentrifugally driven. The spindown luminosity is significantly larger in this phase than an application of the force-free spindown law would suggest.

The original suggestion that rapidly rotating, highly magnetic neutron stars might power GRBs in Usov (1992) essentially employed the force-free approximation. Similar, but more fully conceived, models were developed by Thompson (1994) and Wheeler et al. (2000). The latter addressed the non-relativistic wind phase, but with a dipole-like scaling for the magnetic field strength. Thompson et al. (2004), whose analytic estimates we have so far in essence summarized also constructed approximate numerical solutions of magnetocentrifugal winds from proto-magnetars in order to assess the importance of these forces in setting the mass loss rate and the spindown luminosity.

### 2.5. Numerical Results

Two sets of results for the spindown luminosity have been presented in the preceding discussion: (1) the non-relativistic, but magneto-centrifugally dominated monopole (eq. 8) and (2) the aligned force-free rotator (eq. 13). At face value, the latter is more secure, coming directly from recent numerical calculations. However, the applicability of the force-free result is suspect because it is only formally valid in the limit $\sigma \to \infty$. We expect quantitative and potentially qualitative differences between this limit and the case where $\sigma$ is merely, say, 2, 10, or 100. We wish to understand the applicability and limitations of the two limits in the millisecond proto-magnetar context. A set of recent numerical calculations address these issues directly.

Metzger et al. (2006) solve the time-dependent one-dimensional Weber & Davis (1967) problem of a non-relativistic magnetically-dominated flow in the equatorial plane, including neutrino heating and cooling and an appropriate equation of state. This approach necessarily employs a monopole magnetic field configuration. They provide a detailed account of the physics of millisecond proto-magnetar spin-down from thermal-magneto-centrifugal winds. We emphasize just two results from the study of Metzger et al. (2006): (1) because $R_A \Omega$ overestimates the radial velocity at the Alfvén point, the spindown timescale in the monopole limit is generally a factor of $1.5 - 2$ lower than that given in equation (9) and (2) the mass loss rate increases exponentially when $R_A$ is larger than the sonic point (eq. 7) so that we do expect an epoch of enhanced mass loss as the proto-magnetar wind becomes increasingly magnetically-dominated, even as $L_v$ decreases.

Bucciantini et al. (2006, hereafter B06) solve the two-dimensional axisymmetric time-dependent wind problem with both monopole and dipole surface magnetic fields, in general relativity. The mass outflow rate is determined self-consistently by imposing a finite thermal pressure at the neutron star surface, as appropriate in the proto-magnetar context. They include the effects of neutrino heating and cooling in a parameterized way by employing a $\Gamma$-law equation of state. B06 explore both low magnetiza-
tion non-relativistic winds and relativistic Poynting-flux dominated outflows with $\sigma > 1$. For the non-relativistic, but magnetically-dominated monopole, they find good agreement with equation (8). In the high-$\sigma$ regime, they confirm the analytic force-free monopole limit (Michel 1991; Beskin, Kuznetsova, & Rafikov 1998).

B06 find very interesting results for the aligned dipole. For a strong surface dipole field a region of closed magnetic field lines (the “closed zone”) develops at mid-latitudes around the equator in a helmet streamer-like configuration. At higher latitudes, the field lines are opened to infinity by the outflow. The last closed field line meets the equatorial plane at a $Y$-type point and its radial position in that plane is denoted $R_Y$. In the force-free limit one expects $R_Y = R_L$ at the equator (e.g., Contopolous et al. 1999). In the dipole case, there is not a one-to-one correspondence between the magnitude of the surface magnetic field strength and the open magnetic flux because as $B$ is increased, the closed zone becomes larger. For this reason, B06 find it convenient to write an approximate relation, derived from the simulations, that relates the dipole surface field strength at the pole to an equivalent monopole surface field strength:

$$B_r(R_{NS}, \theta = 0) \approx B_{r,eq-m}(R_{NS}) \times \left( \frac{1.6R_Y}{R_{NS}} \right).$$

(15)

B06 find that the spindown luminosity of the aligned dipole with surface field $B_r(R_{NS}, \theta = 0)$ is the same as that for the 2D monopole if they normalize in terms of the open magnetic flux. Thus, the aligned dipole with $B_r(R_{NS}, \theta = 0)$ spins down like a monopole with $B_{r,eq-m}(R_{NS})$ given by equation (15). Importantly, for $\sigma$ as large as $\sim 20$, B06 find that $R_Y$ is considerably less than $R_L$ and that the ratio $R_Y/R_L$ is a weak function of the magnetic field strength. For parameters typical of millisecond proto-magnetars, they find that $R_Y/R_L \sim 1/4-1/2$. The direct implication of this result is that the monopole scalings for spindown derived in §2.2 are applicable to spindown with dipole fields, but that the magnetic field strength should be decreased in, for example, equation (8) by a factor given by equation (15). Thus, if one assumes a dipole field strength of $10^{15}$ G and that $R_Y/R_{NS} = 2$ (compare with B06), then a field strength of $10^{15}/3.2$ should appear in equation (8). Indeed, B06 find that even as $\sigma$ increases to values as large as $\sim 20$, spindown is more efficient than an application of the pure dipole force-free limit would predict (eq. 13) because $R_Y$ is significantly less than $R_L$. These results suggest that only at significantly higher $\sigma$ does $R_Y$ approach $R_L$ and the force-free limit obtain. In the context of proto-magnetar spindown, this means that the transition to the force-free limit occurs at smaller $L_{\nu}(t)$, later in the Kelvin-Helmholtz cooling epoch.

3. DISCUSSION & CONCLUSIONS:
ARE MILLISECOND PROTO-MAGNETARS
GRB CENTRAL ENGINES?

3.1. Phases of Spindown Evolution

There are five phases of spindown in any very young rotating neutron star’s life (see also B06): (1) a pressure-dominated phase in which the wind is driven by neutrino-heating ($B < B_{s=4}$; eq. 2), (2) a phase in which magnetic field effects are present, but not dominant so that $R_{A\Phi} < 0.1c \approx c_T$, where $c_T$ is the isothermal sound speed at the proto-neutron star surface, (3) a non-relativistically magnetically-dominated phase when $R_A$ is greater than $R$ and $R_s$, but less than $R_L$, (4) a relativistic phase in which $R_A \sim R_L$, but $R_Y < R_L$ (as in B06), and lastly (5) an epoch when the force-free limit is (presumably) applicable and $R_Y \simeq R_A \simeq R_L$. Roughly speaking, for monotonically decreasing $L_{\nu}$, phases (1)–(5) represent a time evolution starting immediately after the supernova explosion commences.

The transition from phase (1) to phase (2) is dictated by equation (1). When $B^2/8\pi$ exceeds $P_{s=4}$ near the proto-magnetar surface, phase (2) begins. As the magnetic field becomes increasingly important, the sonic point will move to smaller radii and the radial scale over which the magnetic field dominates will move to larger radii. When $R_A$ is of order $R_s$, phase (3) begins. This transition will be complicated by the fact that $M$ may increase as a result of increased magneto-centrifugal support in the quasi-hydrostatic atmosphere (see Metzger et al. 2006 for details). Throughout phase (3) $R_A \Omega > c_T$, $\sigma < 1$, and $R_A$ increases from of order the slow magnetosonic radius to $R_L$. The characteristic spindown luminosity is

$$\dot{E} \approx 4 \times 10^{50} B_{14.5}^2 R_{10}^4 P_1^{-5/3} M_{1.4}^{-1/3} \text{ ergs s}^{-1},$$

(16)

where $B$ is the equivalent monopole field (see eq. 15). Because we expect this phase to last of order the Kelvin-Helmholtz timescale, the total amount of energy extracted is comparable to the asymptotic supernova energy, $\sim 10^{51}$ ergs.

The transition from phase (3) to phase (4) occurs at $\sigma \approx 1$ ($R_A \approx R_L$) and for a critical $M$ given by equation (10). Throughout phase (4) the flow is relativistic and Poynting-flux dominated ($\sigma > 1$).
The total energy and angular momentum loss rates are smaller than in phase (3). However, because \( R_\gamma < R_L \), the spindown rate is larger than what would be inferred from an application of the force-free limit. The results of B06 indicate that this is true despite the fact that \( \sigma \) is larger than and, increasingly with time as the neutrino luminosity decreases, much larger than unity. Eventually, \( L_\nu \) and \( \dot{M} \) decrease sufficiently that \( R_\gamma \approx R_L \), and phase (5) begins. A simple and very uncertain extrapolation of the results of B06 suggest that this transition occurs at a very high \( \sigma \) (perhaps \( \sim 10^6 \)). In this epoch, the spindown luminosity is given by the recent force-free calculations described in §2.4.

There is a last phase (or set of phases) beyond the scope of the present work, which follow after the Kelvin-Helmholtz cooling epoch, as the MHD approximation in the magnetosphere of any young neutron star or magnetar begins to break down, the flow becomes charge-separated, and particles are accelerated electromagnetically and to high Lorentz factors directly off of the neutron star surface.

3.2. Energetics, Relativity, & Variability

As first emphasized by Usov (1992), the energy budget and luminosity of millisecond magnetar spindown is in the range needed to explain cosmological long-duration GRBs (eqs. 3 & 13; see also Thompson 1994; Wheeler et al. 2000; similar models also by Katz 1997; Kluzniak & Ruderman 1998). The recent work of Thompson et al. (2004), Metzger et al. (2006), and B06, which we have reviewed here, have explored a more complete picture of proto-magnetar spindown, from the initial non-relativistic wind stage through the Kelvin-Helmholtz cooling epoch.

In order to power a GRB, the outflow must simultaneously achieve high spindown luminosity and high Lorentz factor (\( \gamma_{\infty} \gtrsim 100 \); e.g., Lithwick & Sari 2001). The asymptotic Lorentz factor depends on the magnetization of the flow, measured by \( \sigma \) (eq. 11), which is, in turn, set by the time dependence of \( \dot{M} \) (eq. 12), itself determined by \( L_\nu(t) \). Thus, the transition to a relativistic flow (\( \sigma > 1 \)) is governed by the Kelvin-Helmholtz cooling timescale \( \tau_{KH} \sim 10 - 100\) s. If we take the average duration of a long GRB to be \( \tau_{GRB} \sim 30\) s, the constraints that (1) the total energy ejected must be of order \( E_{\GRB} \sim 10^{51.5} \) ergs and (2) that \( \gamma_{\infty} \sim 100 \) imply that there must be efficient conversion of electromagnetic energy to kinetic energy in the outflow beyond \( R_L \). The reason follows from noting that \( \sigma \propto \dot{M}^{-1} \) and that \( \dot{E} \) is a decreasing function of \( \dot{M} \) before the force-free limit is reached, after which \( \dot{E} \) is independent of \( \dot{M} \). These facts limit the space of possible \( \sigma \) obtainable at a given \( \dot{E} \) to the range of several hundred, depending on \( B \) and \( \Omega \) at the proto-magnetar surface. Therefore, in order to have the Lorentz factors required for GRBs, \( \gamma_{\infty} \sim \sigma(R_L) \). This is in contradiction to expectations from the force-free monopole, which gives \( \gamma_{\infty} \sim \sigma(R_L)^{1/3} \) and is potentially accomplished by efficient magnetic dissipation in the out-flowing wind at radii larger than or comparable to \( R_L \) and/or the fast magnetosonic point (Drenkhahn & Spruit 2002; Lyutikov & Blandford 2003).

We expect variability in both \( \dot{E} \) and \( \sigma \) due to rapid changes in mass loading. In an average sense, this may cause the wind to alternate rapidly between \( \sigma < 1 \) and \( \sigma > 1 \). Proto-magnetars, like all neutron stars, are expected to be fully convective during the Kelvin-Helmholtz cooling epoch, as the MHD approximation in the magnetosphere of any young neutron star or magnetar begins to break down, the flow becomes charge-separated, and particles are accelerated electromagnetically and to high Lorentz factors directly off of the neutron star surface.

3.3. Emergence & Geometry

Evidence for collimation abounds in afterglow observations of GRBs. On the other hand, there is much theoretical work supporting the conclusion that it is difficult to collimate Poynting-flux dominated flows (e.g., Begelman & Li 1994; Lyubarsky & Eichler 2001). If the relativistic proto-magnetar wind can be collimated, we expect its emergence from the progenitor to resemble models of collapsar jets escaping their Type-Ibc hosts (e.g., Aloy et al. 2000; Zhang, Woosley, & MacFadyen 2003). But, can relativistic proto-magnetar winds be collimated?

There are a number of possible answers. The first potential answer may be that collimation simply cannot be accounted for in the millisecond proto-magnetar model for GRBs. The second reply is that relativistic Poynting-flux dominated winds actually can be efficiently collimated (see Vlahakis & Königl 2003; Vlahakis 2004). A third possibility is that a small disk forms outside the rapidly rotating magnetar and that this aids collimation, or that the proto-magnetar is so distorted by centrifugal forces that it is disk-like at early times (e.g., see Figs. 7 & 8 of...
Dessart et al. 2006b). This option might provide a picture which connects logically with the collapsar model of GRBs (Macfadyen & Woosley 1999). A fourth option, as suggested by Thompson (2005), is to appeal to a somewhat different geometry. Pulsars drive energetically dominant high Lorentz factor equatorial outflows (Komissarov & Lyubarsky 2004; Spitkovsky & Arons 2004). The models of B06 show that when $\sigma > 1$, $\gamma(\theta)$ is peaked in the equatorial plane. In addition, the presence of the current sheet in this region may facilitate the magnetic dissipation required for efficient conversion of magnetic energy to kinetic energy. In analogy with pulsar winds, perhaps it is possible that the geometry of many GRBs is “sheet”- or “fan”-like rather than jet-like so that the solid angle subtended by the GRB is $\sim \theta$ instead of $\sim \theta^2$. Modeling shows that the expected sheet-break (the analog of the jet-break; e.g., Rhoads 1999; Sari, Piran & Halpern 1999) is not steep enough to explain all achromatic breaks in GRB afterglow lightcurves (T. Thompson, unpublished; Granot 2005). In addition, for the same observed isotropic equivalent energy and break time, a sheet-like geometry increases the true energy of the burst with respect to a uniform jet by a factor $\propto \theta^{-1}$. Nevertheless, this geometry remains an interesting alternative for a subset of bursts with shallower break profiles (see, e.g., Fig. 2 of Panaitescu 2005).

There is another interesting possibility that may bear on the question of collimation in millisecond proto-magnetar winds. The relativistic Poynting-flux dominated wind of phases (4) and (5) comes after the non-relativistic mass-loaded $\sigma < 1$ wind of phase (3) (see §3.1). This means that the envelope structure that the relativistic wind encounters has been “pre-processed” by the preceding non-relativistic flow. The results of B06 (in analogy with models of non-relativistic stellar winds; e.g., Smith 1998) show that the energetic flux is strongly directed along the rotation axis by hoop stress when $\sigma < 1$. Thus, if the total energy extracted from the proto-magnetar in phase (3) is on the order of the supernova explosion energy, then there will be a relatively “hollow,” asymmetric, and elongated channel that the subsequent relativistic flow emerges into. Pressure forces and wind material bounding this channel may force the less-energetic and relativistic flow into a jet-like structure.

3.4. Remnants & Nucleosynthesis

For fiducial proto-magnetar parameters, the timescale for extraction of an amount of rotational energy comparable to $10^{51}$ ergs is small on the timescale for the supernova shockwave to traverse a Type-II supernova progenitor and comparable to that for a Type-Ibc progenitor. Much of this energy is extracted during phase (3) ($\sigma < 1$), when the energetic flux is strongly directed along the axis of rotation because of hoop stress (as in B06). The action of this outflow may cause an asymmetry in the supernova remnant (see also Wheeler et al. 2000). B06 show that the zenith angle at which the energetic flux is maximized is an increasing function of $\sigma$ so that for $\sigma > 1$, the energetic loss is primarily equatorial. Depending on the timing of the start of phase (3) with respect to the position and energy of the preceding supernova shockwave, the action of the energetic wind could modify the nucleosynthesis in the remnant in an asymmetric way.

As a bit of speculation and for illustrative purposes, we note that the Cass A supernova remnant has a strong jet/counter-jet morphology with a distinctive nucleosynthetic enrichment signature (Hwang et al. 2001, 2004; Willingale et al. 2002; Fesen et al. 2006) and that it has been at least discussed in the context of GRB remnants (Laming et al. 2006). Indeed, many core-collapse supernova events exhibit asymmetry in their spectropolarimetry (e.g., Wang et al. 2001). However, for large-scale asymmetries representing total energy comparable to the asymptotic supernova remnant energetics (as in Cass A), we require surface magnetic fields of $\gtrsim 10^{14}$ G and rotation rates at birth in the millisecond range. Interestingly, Chakrabarty et al. (2001) suggest that the X-ray point source in the Cass A remnant is an Anomalous X-ray Pulsar, a magnetar, and so perhaps this object satisfied our requirements at birth. Evidence also exists for infrared light echoes from the X-ray point source, which are interpreted as burst-like events, potentially analogous to the giant flares seen from magnetars (Krause et al. 2005; as in, e.g., SGR 1806-20, Palmer et al. 2005).

Because $\gtrsim 10^{51}$ ergs can be extracted on a timescale shorter than or comparable to the timescale for the supernova shockwave to traverse the progenitor, we expect that the wind may significantly affect the nucleosynthetic yield and its angular distribution. If significant energy can be extracted and communicated to the surrounding envelope of expanding supernova shocked gas rapidly, the $^{56}$Ni yield of proto-magnetars may be enhanced. In this way it may be possible to generate hyper-energetic or 1998bw-like supernovae (Thompson et al. 2004; Woosley & Heger 2003). The inferred energetics and $^{56}$Ni yield of SN2003dh and SN1998bw
put strong constraints on any GRB mechanism. In the collapsar model a disk wind is thought to generate the $^{56}\text{Ni}$ required to power the SN lightcurve (Macfadyen & Woosley 1999; Pruet et al. 2004). In the millisecond proto-magnetar model, the energetic wind shocks the material already processed by the supernova shock, perhaps generating the large inferred $^{56}\text{Ni}$ yields. Such a mechanism relies on timing. We are currently investigating this scenario more fully.

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