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QPOS AND RESONANCE IN ACCRETION DISKS

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RESUMEN

Hacemos una revisión de los argumentos que indican que la variabilidad en escalas de milisegundos en el flujo en rayos X observado en binarias de baja masa, se debe a oscilaciones en el disco de acreción. Las oscilaciones cuasi periódicas (OCPs) en el rango de kHz observadas en microcuasares parecen estar relacionadas con los mismos picos observados en sistemas con estrellas de neutrones. Es probable que ambos fenómenos sean el resultado de una resonancia no lineal en el disco de acreción, y que ésta sea posible sólo en el régimen gravitacional de campo fuerte. Una resonancia 3:2 entre los modos en cuestión corresponde con el cociente de frecuencias 3:2 observado en sistemas con agujeros negros, así como a la distribución más compleja de pares de OCPs en estrellas de neutrones.

ABSTRACT

We review arguments suggesting that millisecond variability detected in the X-ray flux of LMXBs has its origin in oscillation modes of the accretion disk. The twin high-frequency QPOs observed in microquasars seem to be related to the twin kHz QPOs in neutron-star systems, and both phenomena are likely the result of a non-linear resonance in the accretion disk that is possible only in strong-field gravity. A presumed 3:2 eigenfrequency ratio of the resonant modes corresponds to the 3:2 QPO frequency ratio clearly detected in black-hole systems, as well as to the more complex distribution of kHz QPO frequency pairs in neutron stars.

Key Words: **ACCRETION, ACCRETION DISKS — BLACK HOLE PHYSICS — HYDRODYNAMICS — STARS: NEUTRON**

1. THE EXCEPTIONAL TWIN PEAKS

Accretion is seldom a steady process and, historically, accretion powered X-ray sources have exhibited variability on all accessible time-scales. Ten years ago, the time-resolution (and telemetry rate) of satellite detectors improved to the point where there is now clear evidence of a cut-off frequency in the spectrum of X-ray variability of accreting neutron stars and black holes. Data taken with the Rossi XTE instrument have revealed that in the kilohertz range, with one puzzling exception, there is no variability other than that given by Poisson statistics (of incoming photons). The one exception is the celebrated pair of kHz QPOs (van der Klis et al. 1996; van der Klis 2000) that is present in the Fourier power density spectrum (PDS) of the light curves of several low mass X-ray binaries (LMXBs). In this contribution we will argue that the twin kHz QPO frequencies show a preference for a 3:2 ratio, and we will review the work suggesting that these highest frequency quasi-periodic oscillations (HF QPOs) originate in a non-linear resonance between two modes of oscillation of the accretion disk.

2. ACCRETION-FLOW ORIGIN OF KHZ QPOS

2.1. *Disk signatures*

In neutron stars, the kHz QPO frequencies drift in a matter of hours by up to several hundred hertz in any given source. This drift at once excludes vibrations of the neutron star as their origin, as these are computed and observed to have stable frequencies (McDermott et al. 1988; Watts & Strohmayer 2006). Non-axisymmetric distortions of the neutron star driven by a Chandrasekhar-Friedman-Schutz instability (Wagoner 1984) are excluded for the same reason.

A phenomenon very similar to the neutron star QPOs is observed in black holes, and white dwarfs in cataclysmic variables (CVs). This suggests very strongly that the QPOs originate in the accretion disk, the only structure common to the three types of sources: accreting black holes, white dwarfs and neutron stars.

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Remarkably, the maximum observed kHz QPO frequencies are within 10% of each other (about 1100 Hz) in several, very disparate neutron-star sources, so it seems unlikely that the frequency is connected to the more accidental attributes of the system, such as the magnetic field or mass-accretion rate (Zhang, Strohmayer, & Swank 1997). There is also a more direct argument in favor of an origin in the fluid dynamics of an accretion disk. Many physical processes do not display strong variability at the shortest time-scale, e.g., solar flares and avalanches have a $1/f$ spectrum (Bak, Tang, & Wiesenfield 1987; Lu & Hamilton 1991). In contrast, kHz QPOs are not accompanied by a background rumbling.

2.2. *Understanding the silence*

Why is it that the X-ray flux of LMXBs does not vary at arbitrarily high frequencies? As Bath (1973) pointed out, the natural time-scale of variability in a region of an accretion disk at a distance r from the central source corresponds to the Keplerian frequency

$$2\pi\nu(r) \equiv \omega(r) = (GM/r^3)^{1/2}. \qquad (1)$$

Here M is the mass of the central object, taken to be spherical in this formula, and G is Newton's constant. The maximum frequency of variability is now easily predicted, it occurs for the smallest possible value of r, i.e., close to the inner edge of the disk.

For ordinary stars, and for white dwarfs, the accretion disk can reach the stellar surface. It can also be terminated by strong magnetic fields, as is the case in Her X-1 and other X-ray pulsars. Thus, the accretion disk ends at the stellar radius, R_*, or the so called magnetic radius r_B (whose value is determined primarily by the stellar magnetic dipole and the accretion rate) and the maximum frequency expected on eq. (1) is $\nu_{\rm max} \sim \nu(r_{\rm in})$, with $r_{\rm in} \sim R_*$ or $r_{\rm in} \sim r_B$. The latter case seems to correspond rather well to the sub-hertz QPOs observed in strongly magnetized X-ray pulsars (Angelini, Stella, & Parmar 1989; Finger 2004), where the magnetic dipole is on the order of $10^{30}\,{\rm G}\cdot{\rm cm}^3$, and $r_B > 10^8\,{\rm cm} >> R_* \approx 10^6\,{\rm cm}$.

General relativity (GR) imposes a different upper limit to the orbital frequency that is relevant to black holes, as well as typical LMXB neutron stars. Stable circular orbits have radii larger than a certain minimum value, $r_{\rm ms}$, the radius of the innermost (marginally) stable circular orbit (ISCO). In a geometrically thin accretion disk, gradients of pressure are insufficient to provide radial support to the accreting fluid in nearly circular trajectories, the orbital frequencies are very close to Keplerian and the black-hole disk terminates close to the ISCO (Shakura & Sunyaev 1973; Muchotrzeb & Paczyński 1982). For many equations of state of neutron-star matter, the computed stellar radius is within the ISCO, $R_* < r_{\rm ms}$, and strong gravity may force also the accretion disk around the very weakly magnetized LMXB neutron stars to terminate close to the ISCO, above the stellar surface (Lipunov & Postnov 1984; Kluźniak & Wagoner 1985; Syunyaev & Shakura 1986). A clump orbiting a neutron star at the inner edge of the accretion disk would then modulate the X-ray flux at the frequency

$$\nu = (1 + 0.749j)(M_\odot/M) \times 2.198\,{\rm kHz}, \qquad (2)$$

the formula being valid through first order in the stellar angular momentum $J = jM^2(G/c)$ (Kluźniak, Michelson, & Wagoner 1990). A 1 kHz QPO frequency could then imply a mass of about 2 solar masses for a neutron star (Kaaret, Ford, & Chen 1997; Zhang, Strohmayer, & Swank 1997; Kluźniak 1998); alternatively, it could imply that the QPO frequency is a factor of ~ 2 lower than the highest orbital frequency attainable in a thin disk around a neutron star.

2.3. *The exceptional coherence of kHz QPOs*

The twin QPOs differ from lower frequency features in the PDS not only in that the kHz peaks occur in an otherwise noiseless region, but also in their exceptionally high coherence. Already in the discovery paper (van der Klis et al. 1996), the quality factor of the kHz QPO in Sco X-1 has been reported to reach values of up to about 10^2.

More recently, a study of the powerful kHz QPO in the atoll source 4U1608-52, revealed that the quality factor of one of the twin kHz QPOs remains persistently large ($Q > 150$) for thousands of seconds (Barret et al. 2005). A general conclusion follows immediately from the high value of Q: the QPO frequency is stable over intervals at least as long as few tenths of a second. This is a fairly long period when compared to the dynamical time-scale of a millisecond, or so, in the inner accretion disk. As Barret et al. (2005) point out, the fact (following from the measured Q value) that time and time again the QPO "rings" for several tens of cycles implies that the phenomenon of kHz QPOs cannot be explained by a simple model of orbiting clumps (such as e.g., in Kluźniak 1998; or Stella & Vietri 1999)—inhomogeneities in an accretion disk would be sheared out in a few turns into an axisymmetric configuration (Bath, Evans, & Papaloizou 1974).

In another strike against the clump model—and, indeed, any model in which the QPO is a local phenomenon—eq. (1) allows noise up to the highest frequency $\nu(r_{\rm in})$, but this is not what is observed. In the kHz range, only a pair of characteristic, yet variable frequencies is present.

2.4. *Stability of QPO frequencies*

If it may seem surprising that an ocean of fluid in turbulent motion modulates its X-ray emissions at definite frequencies, it is positively astounding that in some systems the frequencies do not change in time. The $\sim 10^2$ Hz QPOs in microquasars display remarkably constant frequencies. These transient systems are bright X-ray sources only during outbursts. Homan et al. (2005) report that after several years of quiescence, the microquasar GRO J1655-40 has been observed in 2005 to display QPO modulations of its X-ray flux at the same two frequencies, 300 Hz and 450 Hz, that have been previously detected in its 1996 outburst.

Presumably, the accretion disk has completely reformed itself in between the two outbursts, and yet this black hole system retains a memory of the QPO frequencies. The only known answer to this puzzling behavior is that the frequencies are determined by the space-time metric of the black hole, and not by the transient properties of the accretion disk (such as the thermodynamic variables in the fluid, the magnetic field, etc.).

2.5. *Disko-seismology*

The observed stability of HF QPOs in black holes corresponds to predictions of "disko-seismology", the linear theory of oscillations of standard thin accretion disks in the Kerr metric. Normal-mode analysis confirms that, typically, disk oscillations that are most likely to give rise to an appreciable modulation of X-rays occur at sub-orbital frequencies (Wagoner 1999; Kato 2001). Some of these modes are trapped in a region of the accretion disk close to the radius in which the radial epicyclic frequency goes through a maximum, and their frequency is less than that maximum (Okazaki, Kato, & Fukue 1987), others are related to the vertical epicyclic frequency (Perez et al. 1997). The frequency of specific g-modes, and $c-$modes is essentially a function of the space-time metric alone. Therefore, if the twin HF QPOs in black holes are identified with these modes, their frequencies should depend only on M and j, and should not vary with time (Wagoner, Silbergleit, & Ortega-Rodríguez 2001).

Disko-seismology rests on the assumption that a smooth background solution exists in the accretion disk, whose small perturbations may then be worked out. In fact, it is not obvious that the turbulent accretion flow allows a mathematical description of the accretion disk in terms of smoothly varying variables. Indeed, the variability on all time-scales observed with the older generation of detectors seemed to argue against such a description. Simulations of accretion also suggest that smooth spatial variation of fluid variables is possible only in some time-averaged sense. However, the kHz desert in the PDS derived from RXTE observations can be interpreted as reflecting a lack of chaotic variation in the inner disk region of neutron stars. Some states of black-hole systems may also be characterized by a very quiet disk, as evidenced by the very low r.m.s. variation of the X-ray flux (McClintock & Remillard 2006).

2.6. *The importance of* $1/M$ *scaling*

The QPO phenomenology is very complex, and some non-relativistic models (e.g., Titarchuk & Wood 2002) are in impressive agreement with a number of phenomenological correlations, including those that extend from neutron stars in LMXBs to white dwarfs in CVs (Mauche 2002; Warner, Woudt, & Pretorius 2003). Are strong-field effects of Einstein's gravity to be ignored in QPO theory?

The stellar radius predicted by neutron star models (Arnett & Bowers 1977; Cook, Shapiro, & Teukolsky 1994) is sufficiently close to that of the ISCO ($R_* \approx r_{\rm ms}$), and the actual frequency of kHz QPOs probably differs sufficiently from the orbital frequency at the inner edge of the disk, that it would be very hard to determine from the value alone of the QPO frequency whether or not the neutron-star accretion disk is terminated by GR effects. However, there is an important difference between predictions of those QPO models where the frequency is directly related to an effect of general relativity, as in disko-seismology, and those where the relationship to GR is absent or incidental.

In general-relativistic models, the frequency scales inversely with the mass of the compact object, as in eq. (2). The obvious fact that the ≤ 450 Hz QPO frequencies observed in black holes are lower than the kHz QPO frequencies observed in neutron stars, is clearly consistent with this $1/M$ scaling (Figure 1). Crucially, McClintock & Remillard (2006) report a $1/M$ scaling for the three twin HF QPOs occurring in black holes with a measured value of M. This scaling is a very strong argument in favor of a general-relativistic origin of the kHz QPO phenomenon.

3. IMPLICATIONS OF A VARIABLE FREQUENCY

Perhaps the most interesting clue to the origin of the kHz QPO frequencies in neutron-star sources is provided by their variability. On the one hand, it provides an argument for accretion-flow origin of QPOs (Section 2.1). On the other hand, the eigenfrequencies of the most promising modes of a standard thin accretion disk do not vary strongly with the luminosity of the disk (Wagoner 1999; Kato 2001), in fact their allowed variation is much less than the observed frequency excursions of the kHz QPOs in neutron stars. A possible resolution of this conflict can be found in the emerging theory of oscillations of accretion torii, whose eigenfrequencies depend on the shape and position of the torus (Kluźniak & Abramowicz 2002; Zanotti, Rezzolla, & Font 2003; Rubio-Herrera & Lee 2005; Abramowicz et al. 2006b). Another possible solution, described in detail in the companion contribution (Abramowicz et al. 2007), relies on the fact that the actual frequencies of oscillation in a non-linear system need not equal the eigenfrequency.

It has been pointed out that the presence of two variable characteristic frequencies is a hallmark of non-linear resonance, that a resonance should result in a frequency ratio close to $m : n$ (with m and n small integers), and that it should be occurring between two modes of the accretion disk—with the corollary that *two* high frequency QPOs should be present also in black hole systems—(Kluźniak & Abramowicz 2001a,b). The success of this last prediction, evident in the discovery of the 450 Hz QPO in the microquasar GRO J1655-40 (Strohmayer 2001), where only one 300 Hz QPO had been previously known (Remillard et al. 1999), led us to a more detailed study of non-linear resonances in accretion disks, as well as a re-examination of some of the neutron star QPO data.

Resonances in accretion disks in the context of QPOs were also discussed by Psaltis & Norman (2000), Titarchuk (2002), and Kato (2004a,b,2005).

4. A MODEL FOR QPOS IN A 3:2 RESONANT RATIO

As pointed out by Abramowicz & Kluźniak (2001), the frequencies 450 Hz and 300 Hz reported for GRO J1655-40 (Strohmayer 2001), are in a 3:2 ratio, in agreement with the prediction of the non-linear resonance model, and this should allow a determination of the dimensionless angular momentum, j (or the Kerr parameter) of the black hole, once the two frequencies are correctly identified. Because the two frequencies are assumed to be in resonance, only one of them is independent, and the mass has to be determined by other means, e.g., from optical studies of the binary. In contrast, a determination of *both* j and M is possible in (linear) disko-seismology when the two QPO frequencies are identified with a g-mode and a c-mode (Wagoner et al. 2001)—however, the observed frequency ratio of 1.5 would then have to be considered accidental.

Today, we know that all (three or four) black hole sources exhibiting twin high-frequency QPOs exhibit a 3:2 frequency ratio (McClintock & Remillard 2006), and the resonance model for these *black-hole* QPOs has been widely accepted. In this section we discuss the simplest model of twin QPOs in a 3:2 frequency ratio.

4.1. *Parametric resonance*

In general, non-linear resonances allow various rational ratios of frequencies, e.g., 1:2, 1:3, etc. What is special about 3:2? The simplest solution to the puzzle of pervasive 3:2 ratios is given by the theory of parametric oscillators (e.g. Landau & Lifschitz 1974). When the eigenfrequency of a harmonic oscillator varies periodically:

$$\omega(t) = (1 + h \cos \omega_1 t) \cdot \omega_0, \qquad (3)$$

a resonance can occur, in which the frequency of oscillation of the oscillator, ω_2, is a half-integer multiple of the frequency of variation, ω_1, of the oscillator parameter:

$$\omega_2/\omega_1 = n/2 \qquad (4).$$

The resonant frequency of oscillation of the parametric oscillator is close to the eigenfrequency of the unperturbed oscillator, $\omega_2 \approx \omega_0$, the range of allowed frequency variation in the resonance depending on the amplitude h, and on damping. Now, **if** at the same time $\omega_0 >> \omega_1$, as is the case for the two epicyclic modes discussed in the next section, **then** necessarily $n = 3, 4, 5...$, with the first possibility,

$$\omega_2/\omega_1 = 3/2 \qquad (5),$$

giving the strongest resonance (Kluźniak & Abramowicz 2002; Abramowicz et al. 2003b).

4.2. *Epicyclic modes*

The resonance under discussion may occur between any two modes, whose eigenfrequencies are close to a 3:2 ratio (how close, depends on the amplitude of the lower-frequency oscillator). In disko-seismology these could be g-modes with azimuthal number $m = 2$ and $m = 3$ (Wagoner 2006). Here, we consider two simplest axisymmetric oscillation modes of a torus.

The rotating fluid torus is assumed to be initially in hydrostatic equilibrium, with its symmetry axis coinciding with the black hole axis, and its symmetry plane in the equatorial plane of the black hole. Let us consider the torus to be sufficiently thin that we can neglect the variation of the vertical epicyclic frequency across the extent of the torus. If the torus is displaced vertically (i.e., parallel to its axis), it can be shown that the restoring force causes a rigid-like motion of the torus, causing it to oscillate harmonically along its axis at the local vertical epicyclic frequency (Kluźniak & Abramowicz 2002; Abramowicz et al. 2006b). The same authors have shown that a radial displacement of the torus leads to harmonic variation in the radial distance of the torus to the black hole, the eigenfrequency of this second axisymmetric mode coincides with the local radial epicyclic frequency.

In other words, in the limit of a slender torus, the frequencies of its oscillation in these two axisymmetric modes are exactly equal to the two epicyclic frequencies (vertical, ω_z, and radial ω_r) of a test particle in a circular orbit in the equatorial plane, at that radius where the pressure of the unperturbed torus is maximal. Now, it is generally true for black holes that

$$\omega_z > \omega_r .$$

This is precisely the condition required for the 3:2 parametric resonance to arise naturally (Section 4.1).

Each of these modes, when considered separately, satisfies a harmonic eq.:

$$\ddot{z} + \omega_z^2 z = 0, \tag{6}$$

$$\ddot{\delta r} + \omega_r^2 \delta r = 0, \tag{7}$$

z and δr being the vertical and radial displacements from the equilibrium position of the torus. When the two motions are considered together, eq. (6) becomes the Mathieu eq. (Kluźniak & Abramowicz 2002; see Kluźniak 2005a for details), which is the equation describing parametric oscillators, and admits resonance solutions corresponding to eq. (5). For a numerical model of the resonance see Abramowicz et al. (2003b). For a comparison with the black-hole HF QPOs of frequencies predicted in this and other resonance models as a function of angular momentum j, see Török et al. (2005).

In this model, which holds the two epicyclic modes responsible for the observed twin HF QPOs in black holes, the assumption is that the properties of the torus change with the accretion rate. Only in certain states of the accreting system is the position of the torus favorable for a resonance, when $\omega_z/\omega_r \approx 3/2$ holds. This simple model certainly explains the observed ratio of frequencies. The stability of the HF QPOs is also explained in the limit of slender torii, where the eigenfrequencies are unique functions of the position of the torus. A somewhat different prediction for the stability is obtained when the slender-torus restriction is relaxed. The two modes discussed here persist for thick torii, but the eigenfrequencies depart from those of a slender torus placed in the same location (Zanotti, Rezzolla, & Font 2003; Rubio-Herrera & Lee 2005; Abramowicz et al. 2006b). The observed frequencies, while still in a 3:2 ratio, should now vary somewhat with the thickness of the torus.

4.3. *Modulation of X-rays*

To be detected as QPOs, the eigenmodes of an accretion disk or torus must, of course, modulate the emitted X-rays. The radial epicyclic mode described in the previous section is associated with volume changes of the torus and thus can modulate the emission directly. However, the vertical mode would be undetectable in Newtonian physics, as it simply corresponds to an axial displacement with no change in the internal thermodynamic variables of the torus, and no change of aspect of the torus as observed at infinity. The situation is quite different in relativity. Ray-tracing calculations of the oscillating torus in GR reveal that, in a range of favorable inclinations, Doppler boosting and light bending (gravitational lensing at the source) lead to a substantial modulation of the X-ray flux detected by a distant observer (Bursa et al. 2004; Schnittman & Rezzolla 2006).

It is interesting to note that in this model the highest frequency QPOs are a manifestation of Einstein's strong-field gravity in three essential ways. First, the two epicyclic oscillation modes have distinct frequencies only because their degeneracy, which is present in Newtonian gravity, is removed in GR. Second, strong field is required so that the ratio of the two epicyclic frequencies may be as high as 3:2, see eqs. (4), (5). Third, the vertical oscillation mode modulates the light curve because of light bending. We may look upon the QPOs as an extreme application of two of the classic test of GR, the precession of the perihelion of Mercury being related to the first point, deflection of star-light by the Sun to the third point.

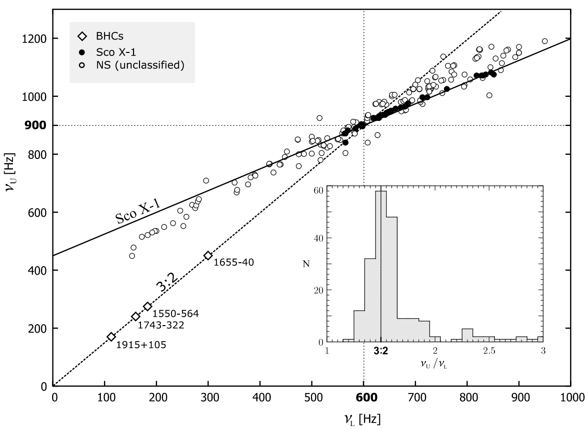

Fig. 1. The frequency-frequency relation for twin HF QPOs in four micro-quasars and several neutron-star sources, compiled from the literature. For references see Abramowicz et al. 2003a (neutron-stars), and Remillard et al. 2006 (black holes). The four black hole sources have QPO frequency ratio accurately equal to 3:2 (the red line). The green line goes through the Sco X-1 points. The inset shows a histogram of the frequency ratio for the neutron-star points.

5. PERTURBATIONS BY THE CENTRAL SPINNING STAR

Throughout this contribution we have argued that the twin kHz QPOs in neutron stars and the twin HF QPOs in black holes are the same phenomenon: a non-linear resonance between two modes of an accretion disk in the strong-field regime of general relativity. However, there is a qualitative difference between the two systems. Neutron stars have a surface capable of stopping infalling matter and of supporting a magnetic field. The first property allows a modulated mass-accretion rate to be translated into modulated X-ray emission in the boundary layer, perhaps this is the origin of the larger amplitude of HF QPOs in neutron stars (Kluźniak & Abramowicz 2004a; Horák 2005b). The second property allows the accretion disk to be perturbed at the spin frequency of the neutron star. It is not known at present, whether the differences in the phenomenology of kHz QPOs and HF QPOs in black-hole and neutron-star systems are related to this possibility. However, in one system at least, there seems to be clear evidence of a non-linear resonance excited in the accretion disk by the spinning neutron star.

Wijnands et al. (2003) have reported twin HF QPOs in the transient LMXB accreting pulsar, and noted that the difference in the observed frequencies (694 ± 4 Hz and 499 ± 4 Hz), is consistent with being equal to one-half of the known spin frequency of the neutron star (401 Hz). Kluźniak et al. (2004b), pointed out that the presence of subharmonic frequencies is a generic feature of non-linear resonances and gave a model for the observed QPOs consistent with the epicyclic oscillations model of black-hole twin QPOs of Section 4, above. Numerical simulations of a periodically disturbed torus confirm that the strongest resonant response occurs when the frequency difference of the two axisymmetric oscillation modes of the torus is equal to one-half the forcing frequency, precisely as observed in the accreting pulsar SAX J1808.4-3658 (Lee, Abramowicz, & Kluźniak 2004). For recent MHD shearing-box simulations see Brandenburg (2005).

The accretion disk may also be resonantly excited by the spinning white dwarf in CVs (Kluźniak et al. 2005b). However, GR effects are negligible for those systems, and only one variable frequency is present.

6. THE FREQUENCIES OF TWIN KHZ QPOS

To better illustrate how the HF QPOs may be related by a 3:2 frequency ratio (red line), we exhibit the frequency-frequency plot (Figure 1) of HF QPO data, compiled by us from the literature. The four black hole twin HF QPOs have stable frequencies which are clearly in a 3:2 frequency ratio. The role played by the 3:2 frequency ratio in neutron stars is more subtle. A histogram of the frequency ratio (inset), reveals that pairs of frequencies accumulate close to a 3:2 ratio (Abramowicz et al. 2003a).

The kHz QPO frequencies for individual neutron star sources vary (in time), but the two frequencies appear to be linearly correlated. e.g., the QPO frequencies of Sco X-1 seem to be well described by the eq. $\nu_{\rm upp} - \nu_2 = (3/4)(\nu_{\rm down} - \nu_1)$, with $\nu_1 = 600\,$Hz, and $\nu_2 = 900\,$Hz. It is not clear whether the integer ratio in this slope of 3/4 has any significance. For other neutron stars the slope of the correlation line is different, but all the lines seem to pass through (or very near) the same point (ν_1, ν_2).

The masses are known for three of the four black holes in the plot, and the frequencies for these three are proportional to $1/M$ (McClintock & Remillard 2006). The frequencies in the least massive black hole, GRO J1655-40, are (300 Hz, 450 Hz), for a mass of about $6M_\odot$. Note that the point where the neutron star frequencies are in a 3:2 ratio is at (600 Hz, 900 Hz). That the frequencies are *higher* for the neutron stars than for the black holes is entirely consistent with the inverse mass scaling expected in GR. The same is true of the spread in frequencies: the black holes have a wide range of masses (about 6 to 15 solar mass), while the masses of neutron stars in LMXBs probably differ by at most 50% (they are widely expected to be between $1.4M_{\odot}$ and about $2M_{\odot}$). It is very reassuring to see that the cloud of neutron-star frequencies crosses the 3:2 line in a narrow zone. Finally, the frequencies and X-ray spectra suggest that these black holes spin very rapidly (Abramowicz & Kluźniak 2001; Török et al. 2005; Shafee et al. 2006), with the disk eigenfrequencies elevated accordingly, and this may explain why there is only a factor 2 difference between (300 Hz, 450 Hz) and (600 Hz, 900 Hz), while the masses of the black hole in GRO J1655-40 and the neutron stars probably differ by the larger factor of 3 to 4.

We only very briefly mention that the observed facts that 1) the twin kHz QPO frequencies are linearly correlated for individual neutron stars, and 2) the slope of the correlation is substantially different from 1.5, are entirely consistent with the theory of non-linear resonance of two oscillators, whose eigenfrequencies are in a 3:2 ratio (Abramowicz et al. 2003a; Rebusco 2004; Horák 2004,2005a). In fact, within the context of this theory, the eigenfrequency ratio can be recovered directly from the data, and the result is consistent with 3:2 (the recovered eigenfrequency ratio being 1.47 ± 0.18, Abramowicz et al. 2006a).

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REFERENCES

Abramowicz, M. A., & Kluźniak, W. 2001, A&A, 374, L19
Abramowicz, M. A., Bulik, T., Bursa, M., & Kluźniak, W. 2003a, A&A, 404, L21
Abramowicz, M. A., Karas, V., Kluźniak, W., Lee, W. H., & Rebusco, P. 2003b, PASJ, 55, 467
Abramowicz, M. A., Barret, D., Bursa, M., Horák, J., Kluźniak, W., Olive, J.-F., Rebusco, P., & Török G. 2006a, submitted
Abramowicz, M. A., Blaes, O. M., Horák, J., Kluźniak, W., & Rebusco, P. 2006b, CQGra, 23, 1689
Abramowicz, M. A., Kluźniak, W., Bursa, M., Horák, J., Rebusco, P., & Török G. 2007, RevMexAA (SC), 27, 9
Angelini, L., Stella, L., & Parmar, A. N. 1989, ApJ, 346, 906
Arnett, W. D., & Bowers, R. L. 1977, ApJS, 33, 415
Bak, P., Tang, C., & Wiesenfield, K. 1987, Phys. Rev. Lett., 59, 381
Barret, D., Kluźniak, W., Olive, J. F., Paltani, S., & Skinner, G. K. 2005, MNRAS, 357, 1288
Bath, G. T. 1973, Nature, 246, 84
Bath, G. T., Evans, W. D., & Papaloizou, J. 1974, MNRAS, 167, P7
Brandenburg, A. 2005, Astr. Nach., 326, 787
Bursa, M., Abramowicz, M. A., Karas, V., & Kluźniak, W. 2004, ApJ, 617, L45
Cook, G. B., Shapiro, S. L., & Teukolsky, S. A. 1994, ApJ, 424, 823
Finger, M. H., 2004, in AIP Conf. Proc. 714, X-Ray Timing 2003: Rossi and Beyond, ed. Ph. Kaaret, F. K. Lamb, & J. H. Swank (Melville: American Institute of Physics), 342
Homan, J., Miller, J. M., Wijnands, R., & Lewin, W. H. G. 2005, BAAS, 37, 1331
Horák, J. 2004, in Processes in the Vicinity of Black Holes and Neutron Stars, 4, ed. S. Hledík, & Z. Stuchlík (Opava: Silesian University), 91
Horák, J. 2005a, Astr. Nach., 326, 824
_________. 2005b, Astr. Nach., 326, 845
Kaaret, Ph., Ford, E. C., & Chen, K. 1997, ApJ, 480, L27
Kato, S. 2001, PASJ, 53, 1
_________. 2004a, PASJ, 56, L25
_________. 2004b, PASJ, 56, 905
_________. 2005, PASJ, 57, 699
Kluźniak, W. 1998, ApJ, 509, L37
_________. 2005a, Astr. Nach., 326, 820
_________. 2001a, astro-ph/0105057
_________. 2001b, Acta Phys. Polonica B, 32, 3605
_________. 2002, astro-ph/0203314
_________. 2004a, in Proc. 22nd Texas Symposium on Relativistic Astrophysics at Stanford, ed. P. Chen, et al. (eConf041213), 2314
Kluźniak, W., Abramowicz, M. A., Kato, S., Lee, W. H., & Stergioulas, N. 2004b, ApJ, 603, L89
Kluźniak, W., Lasota, J.-P., Abramowicz, M. A., & Warner, B. 2005b, A&A, 440, L25
Kluźniak, W., Michelson, P., & Wagoner, R. V. 1990, ApJ, 358, 538
Kluźniak, W., & Wagoner, R. V. 1985, ApJ, 297, 548
Landau, L. D., & Lifshitz E. M. 1974, Course of Theoretical Physics: Mechanics (2nd. ed.; Oxford: Pergamon Press)
Lee, W. H., Abramowicz, M. A., & Kluźniak, W. 2004, ApJ, 603, L93
Lipunov, V. M., & Postnov, K. A., 1984, Ap&SS, 106, 103
Lu, E., & Hamilton, R. 1991, ApJ, 380, L89
Mauche, C. W. 2002, ApJ, 580, 423
McClintock, J. E., & Remillard, R. A. 2006, in Compact Stellar X-Ray Sources, ed. W. H. G. Lewin & M. van

der Klis (Cambridge: Cambridge Univ. Press)
McDermott, P. N., et al. 1988, ApJ, 325, 725
Muchotrzeb, B., & Paczyński, B. 1982, Acta Astron., 32, 1
Okazaki, A. T., Kato, S., & Fukue, J. 1987, PASJ, 39, 457
Perez, C. A., Silbergleit, A. S., Wagoner, R. V., & Lehr, D. E. 1997, ApJ, 476, 589
Psaltis, D., & Norman C. 2000 (astro-ph/0001391)
Rebusco, P. 2004, PASJ, 56, 533
Remillard, R. A., McClintock, J. E., Orosz, J. A., & Levine, A. M. 2006, ApJ, 637, 1002
Remillard, R. A., Morgan, E. H., McClintock, J. E., Bailyn, C. D., & Orosz, J. A. 1999, ApJ, 522, 397
Rubio-Herrera, E., & Lee, W. H. 2005, MNRAS, 357, L31
Shafee, R., et al. 2006, ApJ, 636, L113
Schnittman, J. D., & Rezzolla, L. 2006, ApJ, 637, L113
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
Stella, L., & Vietri, M. 1999, Phys. Rev. Lett., 82, 17
Strohmayer, T. 2001, ApJ, 552, L49
Syunyaev, R. A., & Shakura, N. I. 1986, Soviet Astron. Lett., 12, 117
Török, G., Abramowicz, M. A., Kluźniak, W., & Stuchlík, Z. 2005, A&A, 436, 1
Titarchuk, L. 2002, ApJ, 578, L71
Titarchuk, L., & Wood, K. 2002, ApJ, 577, L23
van der Klis, M. 2000, ARA&A, 38, 717
van der Klis, M., Swank, J. H., Zhang, W., Jahoda, K., Morgan, E. H., Lewin, W. H. G., Vaughan, B., & van Paradijs, J. 1996, ApJ, 469, L1
Wagoner, R. V. 1984, ApJ, 278, 345
Wagoner, R. V. 1999, Phys. Rev., 311, 259
Wagoner, R. V. 2006, private comm.
Wagoner, R. V., Silbergleit, A. S., & Ortega-Rodríguez, M. 2001, ApJ, 559, L25
Warner, B., Woudt, P., & Pretorius, M. L. 2003, MNRAS, 344, 1193
Watts, A. L., & Strohmayer, T. E. 2006, ApJ, 637, L117
Wijnands, R., et al. 2003, Nature, 424, 44
Zanotti, O., Rezzolla, L., & Font, J. A. 2003, MNRAS, 341, 832
Zhang, W., Strohmayer, T. E., & Swank, J. H. 1997, ApJ, 482, L167

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