COCOON DYNAMICS IN AGN JETS AS A PROBE OF THEIR POWER AND AGE

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RESUMEN

Hacemos un análisis de las restricciones que se pueden poner sobre la potencia cinética ($L_j$) y la edad ($t_{age}$) de chorros relativistas en fuentes radio FR II. Calculamos la expansión dinámica del chorro en un medio intra-cúmulo y obtenemos una solución analítica en términos de $L_j$ y $t_{age}$. Posteriormente se comparan las estimaciones de $L_j$ y $t_{age}$ con los valores obtenidos a partir de observaciones.

ABSTRACT

We examine the constraints imposed on the total kinetic power ($L_j$) and the age ($t_{age}$) of relativistic jets in FR II radio sources. We solve the dynamical expansion of its cocoon embedded in the intra-cluster medium (ICM) and obtain the analytic solution of its physical quantities in terms of $L_j$ and $t_{age}$. The estimate of $L_j$ and $t_{age}$ is done by the comparison of the model and the observed shape of the cocoon. The analysis is focused on Cygnus A.

Key Words: GALAXIES: JETS — HYDRODYNAMICS — ISM: JETS AND OUTFLOWS — QUASARS: GENERAL

1. INTRODUCTION

After the detections of inverse Compton component in X-ray from FR II radio sources, our knowledge of energetics, especially on the kinetic power of non-thermal electrons, is progressed in recent years (e.g., Leahy & Gizani 2001; Harris & Krawczynski 2002; Hardcastle at al. 2002; Kataoka et al. 2003; Croston et al. 2005). When we explore further physical conditions in the jets, we encounter a big difficulty to constrain on the fraction such as thermal electrons and/or protons co-existing with non-thermal electrons because it is hard to observe these component (e.g., Celotti et al. 1998; Sikora & Madejski 2000). This problem prevented us from estimating the total mass and energy flux ejected from a central engine. To conquer this, a simple procedure is proposed in the study of jets in FR II sources in Kino & Takahara (2004, hereafter KT04). In the pressure and mass density of jet, the contributions from the invisible components are also involved. Hence the rest mass density and energy density estimated from the shock dynamics definitely prove the quantities of total plasma. Also for radio bubbles in cluster cores, the similar dynamical approach has been adopted to constrain on their physical state (Fabian et al. 2002; Dunn & Fabian 2004). Here we will explore the cocoon dynamics in FR II radio sources as a robust tool to know the total kinetic power $L_j$ and source age $t_{age}$.

2. DYNAMICS OF COCOON

We deal with the cocoon dynamics in FR II radio source. The adopted basic equations are almost the same as those in Begelman & Cioffi (1989, hereafter BC89). The main differences between BC89 and the present work are (i) we explicitly solve the physical quantities as a function of $L_j$ and $t_{age}$, (ii) we take account of the effect of radial profile of the mass density of ICM, and, as a result, (iii) the growth of the cocoon head $A_h$ is consistently solved from the basic equations.

2.1. Basic Assumptions

Our main assumptions are as follows; (1) We limit our attention to a jet relativistic speed ($v_j = c$), (2) The jet supplies a constant $L_j$ in time, (3) We focus on the over-pressured cocoon phase, and (4) We assume that the magnetic fields are passive and ignore their dynamical effect. For (1), although it is still under debate, some jets are suggested to be relativistic (e.g., Tavecchio et al. 2000; Celotti, Ghisellini, & Chiaberge 2001). Since little is known about the evolution of $L_j$, the assumption (2) is adopted as a first-step working hypothesis. The assumption of (3) is automatically guaranteed by the sideways expansion of the cocoon (e.g., Cioffi & Blondin 1992). The assumption of (4) is based on the results that a multi-frequency analysis of radio galaxies show that the energy density of magnetic field tend to be smaller than that of non-thermal electrons (e.g., Leahy & Giani 2001; Isobe et al. 2002).
The declining mass density of the cocoon, the radius of the cocoon body, the specific heat ratio of the plasma inside the cocoon, the velocity of cocoon sideways expansion, the pressure of the cocoon, the cross sectional area of the head part of the cocoon (or the cross sectional area of cocoon body), respectively (see Figure 1). Equation of motion along the jet axis, and sideways expansion, and energy conservation in the cocoon are given by

\[
\frac{L_j}{v_j} = \rho_a(r_h) v_h^2(t) A_h(t),
\]

\[
P_c(t) = \rho_a(r_v) v_c(t)^2,
\]

\[
\frac{P_c(t) V_c(t)}{\gamma - 1} \approx 2 L_j t,
\]

where \(r_h(t) = \int_{t_{min}}^{t} v_h(t') dt', \) \(r_v(t) = \int_{t_{min}}^{t} v_c(t') dt', \) and \(\gamma_c\) are the length from the center of the galaxy to the head of the cocoon, the radius of the cocoon body, the volume of the cocoon, the start time of source evolution and specific heat ratio of the plasma inside the cocoon, respectively. The declining mass density of ICM \(\rho_a\) is given by \(\rho_a(r) = \tilde{\rho}_a(r/r_0)^{-\alpha}\) where \(r_0\) and \(\tilde{\rho}_a\) are reference position and the ICM mass density at \(r_0\), respectively. We set \(r_0\) as \(r_h(t_{age})\) in throughout this paper. Most of the kinetic energy is deposited in the cocoon, which is initially suggested by Scheuer (1974) and recent studies of hot spots also shows the radiative efficiency is very small (e.g., KT04). Following to Cioffi and Blondin (1992), we add the factor of \(1/(\gamma_c - 1)\) in Eq. (3) to express the amount of the deposited internal energy. In other words, this corresponds to the neglect of the PdV work because of its smallness.

The numbers of quantities are 4, while those of basic Eqs. are 3. Hence, we set \(A_c(t) \propto t^X\) where \(X\) as a free parameter. We can constrain on the value of \(X\) from observations. A specific case is shown in §3. Hence, we obtain \(v_h, v_v, P_c,\) and \(A_h\) by using a free parameter \(X\).

As a subsidiary equation, the area of the cocoon body is given by

\[
A_c(t) = \pi \left( \int_{t_{min}}^{t} v_c(t') dt' \right)^2.
\]

It is clear that the change of the unknown from \(A_c\) to \(A_h\) does not change the result. However note that when we choose \(A_h\) as an unknown instead, we cannot obtain the solution for \(\alpha = 2\).

2.3. Analytic Solution

We assume that physical quantities have a power law dependence in time and the coefficient of each physical quantity is barred. Each quantity has the form of \(A = \tilde{A} (t/t_{age})^Y\) where \(Y\) is an arbitrary index. The time evolution of \(v_c\) is

\[
v_c(t) = \tilde{v}_c \left( \frac{t}{t_{age}} \right)^{0.5X - 1} = \frac{\tilde{A}_c}{t_{age}} C_{vc} \left( \frac{t}{t_{age}} \right)^{0.5X - 1},
\]

for a given \(A_c\). With this, the analytic form of cocoon quantities in decreasing ICM density is obtained as follows;

\[
P_c(t) = \tilde{\rho}_a \tilde{v}_c^2 C_{pc} \left( \frac{\tilde{v}_c}{v_0} \right)^{-\alpha} \left( \frac{t}{t_{age}} \right)^X (1-\alpha/2)^{-2},
\]

\[
v_h(L_j, t) = \frac{L_j}{\tilde{\rho}_a v_c} A_c \left( \frac{v_h}{v_0} \right)^\alpha \left( \frac{t}{t_{age}} \right)^X (-2 + 0.5\alpha)^{+2},
\]

\[
A_h(L_j, t) = \frac{L_j C_{ah}}{\tilde{\rho}_a \tilde{v}_c} \left( \frac{v_h}{v_0} \right)^\alpha \left( \frac{t}{t_{age}} \right)^X (\alpha - 2 + 0.5\alpha + 3\alpha - 4),
\]

where \(C_{ah} = (\gamma - 1)[3 - (1 - 0.5\alpha)X]X(0.5X)^{-\alpha}\), \(C_{pc} = 0.5X/\pi^{1/2}\), \(C_{vc} = (0.5X)^\alpha\), and \(C_{ah} = [X(-2 + 0.5\alpha) + 3]^\alpha\), respectively. \(v_0 \equiv r_h/t_{age}\) corresponds to the head speed for constant velocity in time.
Here we use the conditions of $0.5X > 0$ and $X(-2 + 0.5\alpha) + 3 > 0$ which make the contribution at $t_{\min}$ in the integrations of $r_h$ and $r_c$ small enough. The case we focus on in §3 is that $X = 12/7$ and $\alpha = 1.5$, which clearly satisfies these conditions.

First, let us consider the evolution of cocoon. The growth of both $A_h$ and $A_c$ must be positive. As for the cocoon expansion speeds, three different behaviors are theoretically predicted such as (I) accelerated-head ($X(-2 + 0.5\alpha) + 2 > 0$), (II) constant-head ($X(-2 + 0.5\alpha) + 2 = 0$), and (III) decelerated-head ($X(-2 + 0.5\alpha) + 2 < 0$). The case of (I), (II), and (III) correspond to $X < 1$, $X = 1$, and $X > 1$ for $\alpha = 0$, while in the case of $\alpha = 2$ (I), (II), and (III) correspond to $X < 2$, $X = 2$, and $X > 2$, respectively. Related to this, it is useful to express the aspect ratio of the cocoon $\frac{R}{h_e} = R$ as a function of time. This is written as

$$R(t) = \frac{X(-2 + 0.5\alpha) + 3}{0.5X} \frac{v_h(t)}{v_c(t)} \left(\frac{t}{t_{\text{age}}}\right)^{X(2.5 - 0.5\alpha) - 3}.$$  \hspace{1cm} (8)

It is worth to note that the solution describes not only the self-similar evolution (e.g., Kaiser & Alexander 1997; Bicknell, Dopita, & O’Dea 1997) but also the non self-similar one. Although it is not concerned with in this paper, the evolution of $R(t)$ may probe the evolution of radio galaxies such as “compact symmetric objects (CSO)” which are thought to be the progenitors of the FR II source (e.g., Fanti et al. 1996; Readhead et al. 1996a). Note that, observationally, the large deviation from $R \sim O(1)$ does not seem to be unnatural for actual radio sources (e.g., Readhead et al. 1996b).

Next, let us consider the $\alpha$ and $X$ dependences on the quantities based on Eqs. (1), (2) and (3). Here we fix the physical quantities at $t = t_{\text{age}}$. With regard to the $\alpha$ dependence in fixed $X$, larger $\alpha$ leads to larger $\rho_\alpha$, slower $v_h$, same $v_c$, larger $P_c$, during $t < t_{\text{age}}$. We can understand it as follows. Larger $\alpha$ leads to stronger the deceleration effect on the head speed $v_h$ due to larger $\rho_\alpha$. Larger $\alpha$ also predict larger $P_c$ in order to keep the same velocity of sideways expansion $v_c$.

Next we consider the $X$ dependence. In fixed $\alpha$, larger $X$ lead to faster $v_h$, slower $v_c$, smaller $P_c$, and smaller $A_h$ during $t < t_{\text{age}}$. We can explain it as follows. Larger $X$ leads to slower sideways expansion $v_c$ and smaller $A_c$ by definition. From the equation of motion to the sideways, it is clear that smaller $P_c$ is required. To satisfy the energy equation at the same time, the faster velocity of $v_h$ is needed. This is realized by smaller area head of thecocoon $A_h$.

Finally, we compare our solution with the previous works. The solution for flat ICM density by BC89 model corresponds to the case of $X = 1$ and $\alpha = 0$ and the quantities are written as $v_h \propto t^{-1/2}$, $v_c \propto \text{const.}$, $P_c \propto t^{-1}$, $A_h \propto \text{const.}$ The comparison with the analytic model and numerical studies, is also important issue. By Cioffi & Blondin (1992), the issues of “head cross section growth” and “decreasing head velocity” is assessed by hydrodynamic simulation and their result shows $A_h \propto t^{0.4}$. As a good example, our solution of $X \simeq 1.1$ describes the studies Nath (1995) corresponds to case of flat solution above $v_h \propto t^{-0.2}$, $v_c \propto t^{-0.45}$, $P_c \propto t^{-0.9}$, and $A_h \propto t^{0.4}$. Concerning $A_c$, since most of numerical studies have mainly focused on the propagation of cylindrical geometry jets (e.g., Marti et al. 1997; Clarke, Harris, & Carilli 1997), it remains as an important future work on the time evolution of $A_c$.

3. TOTAL KINETIC POWER AND SOURCE AGE OF CYGNUS A

Here we explore $L_j$ and $t_{\text{age}}$ by matching the observed cross-sectional areas and lengths of a cocoon (i.e., $r_h$, $r_c$, $A_h$, and $A_c$). In this paper, we focus on the archetypal radio galaxy Cygnus A (Carilli & Barthel 1996a; Carilli & Harris 1996b for reviews).

First, we estimate the typical values of physical quantities. Concerning the mass density profile of ICM, we adopt $\alpha = 1.5$ based on Reynolds & Fabian (1996) and Smith et al. (2002). We examine the case of $X = 12/7$ as a fiducial one which predict the constant $R$ in time. Other observed quantities $\rho_a = 0.5 \times 10^{-2} m_p$ g cm$^{-3}$, and $r_h = 60$ kpc, based on Arnaud et al. (1984), Carilli et al. (1998), and Smith et al. (2002). Here we assume $\gamma_c = 4/3$.

Next, to clarify the allowed range of $t_{\text{age}}$ and $L_j$, we impose the following conditions; the condition which should be satisfied is that (I) $R(t_{\text{age}}) \sim 0.5 - 0.7$, from the Chandra image (Wilson et al. 2000), (II) cocoon pressure is over-pressured $P_c > P_a = 8 \times 10^{-11}$ dyn cm$^{-2}$ (Arnaud et al. 1984; Smith et al. 2002), (III) the area size of $A_h$ lies in the range of $30$ kpc$^2 < A_h < 150$ kpc$^2$. The minimum value corresponds to the one adopted in BC89. From the radio image of Perley, Dreher, & Cowan (1984), we employ the maximum value as $A_h = 150$ kpc$^2$, which corresponds to the cross sectional area of the radio lobe at the location of the hot spot. In Figure 2, we show the resultant source age and total kinetic power of the jet. The region of the source age larger than $\sim 30$ Myr is not allowed by the condition (II). Larger (smaller) $A_h$ predict larger (smaller) $L_j$ and younger $t_{\text{age}}$. Obtained values are $3 \times 10^{45}$ erg s$^{-1} < L_j < 3 \times 10^{47}$ erg s$^{-1}$ and $6$ Myr $< t_{\text{age}} < 30$ Myr.

Let us consider how the uncertainties of $L_j$ and $t_{\text{age}}$ are determined. From Eqs. (7) and (8), $A_h \propto$
L_j^2 \tau_{\text{age}} \sim R \propto L_j^{-1/(\alpha-4)} \tau_{\text{age}}^{-3/(\alpha-4)} \) are obtained. Since A_h and R have uncertainties with the factors of 5 and 1.4 respectively, the allowed region is mainly controlled by A_h. In the present work, in view of the comparison of BC89, we took the minimum value as A_h = 30kpc^2. However, from the hydrodynamical point of view, it seems natural to suppose that A_h to be the close value to the cross sectional area of the radio lobe at the distance of head, i.e., A_h = 150 kpc^2.

### 3.1. Comparison with previous works

The resultant age well agree with the independent result of synchrotron age model by Carilli et al. (1991), which claims that 6 Myr < \tau_{\text{age}} < 30 Myr. The velocity of the hot spot \beta_{\text{hs}} \sim 0.06 corresponds to the source age of 6 Myr, while \beta_{\text{hs}} \sim 0.01 corresponds to the source age of 30 Myr.

As a complementary result, L_j estimated in KT04 in the range of 6 Myr < \tau_{\text{age}} < 30 Myr based on the result of Carilli et al. (1991) is shown in Figure 2 (the solid line). KT04 estimate the total kinetic power of the relativistic jet as L_j = \rho_j c^2 R / \gamma_j \beta_j^2, where A_j = \pi R_{\text{hs}}^2, \gamma_j, \beta_j c, \rho_j, and R_{\text{hs}} = 2 kpc (Wilson et al. 2000) are the cross-sectional area of the jet the Lorentz factor, the velocity, and mass density of the jet, and the hot spot radius, respectively. It should be stressed that the cocoon model can predict both L_j and \tau_{\text{age}} at the same time, while the synchrotron aging model (Carilli et al. 1991) and the 1D jet model (KT04) only determine \tau_{\text{age}} or L_j. Although these three models are independent, they show a reasonable agreement on the values of L_j and \tau_{\text{age}} with each other in the case of Cygnus A.

Rawlings & Saunders (1991, hereafter RS91) is the pioneering work on the correlation between the L_j and the luminosity of the narrow line regions, which lies in close to the central engine. Then the comparison with the present work and RS91 is intriguing issue. For the sources where synchrotron spectral aging are available, it is simply by \eta = \frac{E_{\text{en}}}{E_{\text{age}}} and \eta = 0.5 where Q, \eta, and E_{\text{eq}} are the kinetic power of jet, a parameter expressing the fraction of the work done on the ICM, and the equipartition energy with the electrons and the magnetic field field make an equal contribution to the total energy density (e.g., Miley 1980), respectively. We focus on the sample sources where \tau_{\text{age}} is independently obtained by synchrotron aging method. One difference between the present work and RS91 is that we solved the equations of motion and energy equation (3 equations in total), while the RS91 only employ energy equation. Because of this, we can eliminate the free parameter \eta. More important and essential difference is that Q in RS91 is not a total kinetic power but it is just a equipartition power. We emphasize the advantage of our work of dealing with the total kinetic power whilst RS91 only handles the equipartition power of extragalactic jets.

### 3.2. Efficiency of accretion power to kinetic power

In Tadhunter et al. (2003), the mass of the supermassive black hole (SMBH) of Cygnus A is reported as 2.5 \times 10^9 M_\odot which leads to the Eddington luminosity as L_{\text{Edd}} = 3 \times 10^{47} \text{ erg s}^{-1}. From this it follows that L_j / L_{\text{Edd}} \sim 0.01 - 1. On the basis of the observational evidence for the accretion-flow origin for the jet in AGNs (e.g., Marscher et al. 2002), it is clear that the value of L_j / L_{\text{Edd}} directly shows the required minimum rate of the mass accretion onto the SMBH normalized by the corresponding Eddington mass accretion rate.

Merloni, Heinz, & Di Matteo (2003; see also Maccarone, Gallo, & Fender 2003) examine the disk-jet connection by studying the correlation between the radio (L_R) and the X-ray (L_X) luminosity and the black hole mass. With the large samples of the stellar mass black holes and SMBHs, they claim these sources have the “fundamental plane” in the three dimensional space of these quantities. On the way, the quantity of L_X / L_{\text{Edd}} is brought up to probe the
activity of the central engine. For Cygnus A, Young et al. (2002) reports \( L_X \simeq 3.7 \times 10^{44} \text{erg s}^{-1} \) which leads to \( L_X / L_{\text{Edd}} \sim 10^{-3} \). Whereas we recognize the usability of the quantity of \( L_X / L_{\text{Edd}} \) to probe the engine activity, we emphasize that the \( L_X \) largely depends on the accretion flow model and the radiation efficiency at X-ray band. On the other hand, the quantity of \( L_j / L_{\text{Edd}} \) addressed in the present work does not have the model dependence and directly shows us the engine activity.

We again emphasize that as we see above, the value of \( L_j / L_{\text{Edd}} \) for Cygnus A in the present work is obtained with fewer assumptions. Moreover it make us a fairly robust probe of the central engine activity of AGN jets. The application of our method to a large sample of other AGN jets surely bring about new and important knowledges. This is actually our ongoing project (Itoh et al. in preparation).

4. CONCLUSION

Our main conclusions in the present work are as follows:

(i) A new method to estimate the total kinetic power of the jet \( L_j \) and source age \( t_{\text{age}} \) in powerful FR II radio sources is proposed. For that, the study of cocoon dynamics by BC89 is revisited and physical quantities associated with the cocoon are analytically solved as functions of \( L_j \) and \( t_{\text{age}} \). The comparison of the analytic solution with the observed cocoon shape lead to the \( L_j \) and \( t_{\text{age}} \) in general.

(ii) The analysis is focused on Cygnus A, with the conditions of \( 0.5 \leq R \leq 0.7 \) and \( 30 \text{ kpc}^2 < A_\text{h} < 150 \text{ kpc}^2 \). The estimated age 3 Myr < \( t_{\text{age}} \) < 30 Myr shows a good agreement of the independently estimate age by synchrotron aging model by Carilli et al. (1991). The estimated total kinetic power \( 0.2 \times 10^{46} \text{erg s}^{-1} < L_j < 1 \times 10^{48} \text{erg s}^{-1} \) has also a reasonable agreement with the independent 1D jet model of KT04 with the aid of 6 Myr < \( t_{\text{age}} < 30 \text{ Myr} \).

(iii) For Cygnus A, we find that the total kinetic power lies in the range of \( L_j / L_{\text{Edd}} \sim 0.01 - 1 \), while the X-ray luminosity \( L_X \simeq 3.7 \times 10^{44} \text{erg s}^{-1} \) (Young et al. 2002) satisfies \( L_X / L_{\text{Edd}} \sim 10^{-3} \). The result of \( L_j / L_{\text{Edd}} \sim 0.01 - 1 \) indicate the lower limit of the mass accretion rate, which gives the crucial hint for resolving the jet formation problem.

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