THE VELOCITY OF LARGE-SCALE JETS IN A DECLINING DENSITY MEDIUM

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RESUMEN
Las clases de objetos compactos simétricos son morfológicamente similares a las fuentes en radio FR II. Para elucidar la relación física entre las dos, se propone aquí un modelo de evolución de chorros en Núcleos Activos de Galaxias (NAGs). Incluimos los efectos dinámicos de la caída en densidad del medio y el aumento en la sección transversal de la cabeza del chorro para determinar la velocidad de propagación del mismo. Con este modelo describimos la evolución de la velocidad, la presión, la densidad y el radio del punto caliente como función de la pendiente de decaimiento de la densidad ambiental. Nuestro modelo analítico puede explicar las observaciones de velocidad constante del punto caliente, y de la evolución de la presión y tamaño del mismo, \( P_{\text{HS}}(r) \propto r^{\alpha - 1.5}, \ r_{\text{HS}} \propto r^{\alpha = 0.75} \).

ABSTRACT
The classes of compact symmetric objects (CSOs) morphologically resemble the powerful large symmetric radio sources (FR II sources). In order to clarify the physical relation between CSOs and FR II sources, the evolution model of AGN jets in active galactic nuclei (AGNs) is proposed here. We include the dynamical effects of (i) the declining ambient mass density and (ii) the growth of head cross section area of the cocoon to determine the velocity of the jet propagation. Based on this model, we describe the evolution of the velocity \( (v_{\text{HS}}) \), pressure \( (P_{\text{HS}}) \), density \( (\rho_{\text{HS}}) \) and the radius \( (r_{\text{HS}}) \) of the hot spot as a function of slope index of ambient density and the growth rate of head parts of the cocoon. Moreover, our analytic model can well explain the observational trends of the constant hot spot velocity, the evolution of the hot spot pressure and the hot spot size \( (P_{\text{HS}}(r) \propto r^{\alpha - 1.5}, \ r_{\text{HS}} \propto r^{\alpha = 0.75}) \).

Key Words: GALAXIES: JETS — ISM: JETS AND OUTFLOWS — RADIO LINES: GALAXIES

1. INTRODUCTION
The evolution of powerful extragalactic radio sources is one of the primal issue in the study of AGNs. After the first discovery of the radio emission from the extragalactic radio galaxy of Cygnus A (Jennison & Das Gupta, 1954), a lot of extragalactic radio sources have been recognized by the following studies (e.g., Turland, 1975; Readhead, Cohen, & Blandford, 1978; Bridle & Perley, 1984). Stimulated by the observational progress, a number of hydrodynamic simulations of jet propagations have been performed to examine their physical state of the jet (e.g., Norman et al., 1982; Wilson & Scheuer, 1983; Smith et al., 1985; Clarke, Norman, & Burns, 1986; Lind et al., 1989; Clarke, Harris, & Carilli, 1997; Marti et al., 1997). These numerical studies now confirms that the jet is composed of “light” (i.e., lower mass) density plasma compared with that of the surrounding ambient medium. However the long term evolution of the cocoon it is not well known because of the limitation of the computational power.

Related to this, a new population of radio sources “compact symmetric objects (CSOs)” has been recently focused on. The CSOs was first identified by Philips & Mutel (1980, 1982). Soon after that, more complete sample was presented by Wilkinson et al. (1994) and Readhead et al. (1996a, b). Some CSOs were found to have a core between the two lobes or hot spots formed by the relativistic jet. They were called as CSOs because of their double-sided emission and their smaller size (linear size is smaller than 1 kpc). Concerning the origin of CSOs, two scenarios were initially proposed. One is so-called “frustrated jet scenario” in which the ambient medium is so dense that jet cannot break its way through, so sources are old and confined (van Breugel, Miley, & Heckman, 1984). The other is “youth radio source scenario” in which CSOs are the young progenitor of FR II radio galaxies (e.g., Philips & Mutel, 1982; Carvalho, 1985; Fanti et al., 1995; Begelman, 1996; Readhead et al., 1996a; O’Dea & Baum, 1997). Recent observations reveal that their speeds are better understood within the young source model since they estimate the age with \( 10^3 - 5 \) yr, which is much shorter...
DYNAMICAL EVOLUTION OF AGN JETS 193

than that of FR II sources with $10^{6-7}$ yr (e.g., Owa-
iank, Conway, & Polatidis 1998; Murgia et al. 1999;
Taylor et al. 2000). This indicates that CSOs are
the progenitor of FR II sources.

In order to explore the connection between CSOs and FR II sources, the self-similar model of their
dynamical evolution has been investigated by some
authors (e.g., Fanti et al. 1995; Begelman 1996;
Kaiser & Alexander 1997). From the hydrodynamical
point of view, both the effect of a declining ambient
cross section area of the cocoon are two key ingredients
that control the velocity of jet propagation as a whole
(see Figure 1). However, the self-similar model did not
solve the evolution of the head cross section area
of the cocoon, but assumed it in the self-similar way.
Therefore, we will construct the dynamical evolution
model of AGN jets, considering above two key
physics in the present work.

The rest of this proceeding is organized as follows.
In §2, we discuss “the interaction between a jet and an ambient medium” and “the growth of cross section area of the head part of cocoon”. By
combining them, we construct the dynamical evolution
model of the AGN jet in §2.5. In §3, we compare our result with previous works and we apply
our model to the actual observational trend of these
radio sources. The summary is given in §4.

2. DYNAMICAL CO-EVOLUTION OF JET AND ITS COCOON

We consider a pair of jets propagating into a de-
clining ambient medium of density $\rho_a$ (see Figure 1).
Assuming that the speed of the head part of jets $v_h$
is determined by balancing the momentum of jets
$L_j/v_j$ ($L_j$ and $v_j$ are the power and velocity of the
jet, respectively) in time against the ram pressure
force of the ambient medium $\rho_a v_h^2 A_h$, where $A_h$
is the cross sectional area of the head of the cocoon.
$A_h$ would be formed by the vortex occurred via the
shock (e.g., Smith et al. 1985) and/or the effect of
jittering drill (e.g., Williams & Gull 1985; Cox et al.

As mentioned in §1, the velocity of the head part
of jet (including the hot spot) is determined by the
following two key physics:

1. The slope index of the ambient mass density ($\alpha$)
   with $\rho_a(r) = \rho_{a,0}(r/r_0)^{-\alpha}$;

2. The growth rate of the head cross section area
   of the cocoon ($\beta$) with $A_h(r) = A_{h,0}(r/r_0)^{\beta}$;

where $r$, $\rho_0$ and $A_{h,0}$ are the distance from galactic
center, the ambient mass density and the cross sec-

![Fig. 1. Schematic picture of interaction of the ambient medium and the relativistic jet in the FR II radio galaxy. Most of the kinetic energy and mass of jet is deposited in the over-pressured cocoon with the cross section area of cocoon body ($A_c$). The sideways expansion speed of cocoon is $v_c$. The reverse shocked region is identified as a hot spot. The forward shock is identified as a bow shock. The area of the radio lobe at the position of hot spots ($A_h$) is larger than that of hot spots. The area of bow shock expands into the ambient medium with speed ($v_f$). The observable radio lobes are constituted near the ends of jets.](image)

2.1. Basic Assumptions

Our main assumptions are as follows:

1. We suppose that the speed of jet is relativistic
   on the large scale ($\sim 100$ kpc) and that the jet
   consists of the cold matter (the pressure $\simeq 0$);

2. The mass, energy and momentum of jet are con-
   served in time. Namely, we do not consider the
   effect of entrainment$^2$;

3. The magnetic fields are passive in large scale
   and we ignore their dynamical effect.

For (1), some authors suggested that the jet would
be relativistic (e.g., Celotti, Ghisellini, & Chiaberge

$^2$The effect of entrainment is that of the energy and mass losses caused via the complex and turbulent mixing layer that separates the jet from the surrounding medium.
2001) although the jet speed on large scales is still under debate. To explain the remarkable hot spots identified as a strong shock, the assumption of dynamically cold jet matter is reasonable. The assumption (2) is supported by numerical simulations for highly relativistic flows (e.g., Scheck et al. 2002). Our approach is a complementary one against the works considered the entrainment (De Young 1997). The assumption (3) is based on the results that a multi-frequency analysis of radio galaxies show the energy density of magnetic fields tend to be smaller than that of non-thermal electrons (e.g., Leahy & Giani 2001; Isobe et al. 2002).

2.2. Jet propagation

In the case of a cold relativistic jet, the mass \(J\), energy \(L\) and momentum \(Q\) flux are given as follows (Blandford & Rees 1974):

\[
J(r) = \Gamma_j(r)A_j(r)p_j(r)c, \quad (2)
\]

\[
L(r) = \Gamma_j^2(r)A_j(r)p_j(r)c^3, \quad (3)
\]

\[
Q(r) = \Gamma_j^3(r)A_j(r)p_j(r)c^2, \quad (4)
\]

where \(\Gamma_j\), \(p_j\), and \(A_j\) are the Lorentz factor, the mass density, and the cross section, respectively. Based on these conditions, we found the following relations for a cold relativistic jet;

\[
\Gamma_j(r) = \text{constant}, \quad (5)
\]

\[
p_j(r)A_j(r)c = \text{constant}. \quad (6)
\]

Interestingly, the Lorentz factor \(\Gamma_j(r)\) does not depend on the distance from the galactic center.

2.3. Interaction between jets and ambient -local 1D shock junction-

In order to treat the interaction between jets and ambient medium in a declining ambient mass density with slope index \(\alpha\), we review the 1D shock jump conditions briefly (see Kino & Takahara 2004 for details). As shown in Figure 1, we use terminology of region \(i\) (i=1, 2, 3, and 4) with the number marking the four regions in the head part of AGN jets as follows: (1) the un-shocked ambient medium, (2) the shocked ambient medium, (3) the shocked jet which is identified with hot spots, and (4) the un-shocked jet. In general, we solve for \(3+3=6\) downstream quantities when \(3+3=6\) upstream quantities are given.

As for FR II sources, it is able to select 6 givens in a different way. The properties of ambient medium are known from X-ray observations to give \(P_1(=P_a), \rho_1(=\rho_a), \text{ and } v_1 = 0\). Here, \(P_a\) is the ambient pressure. The hot spot advance speed \(v_{HS}\) is inferred from the synchrotron aging method (e.g., Carilli et al. 1991). From the assumption of a cold jet, \(P_a = 0\). Here we adopt the \(\Gamma_4(=\Gamma_1)\) as a given parameter. Thus, by using the plausible shock conditions, we can obtain \(\rho_2, P_2 = P_3 = P_{HS}, \rho_3 = \rho_{HS}, \rho_4 = \rho_1, v_{FS} \text{ and } v_{HS}\), where \(v_{FS} \text{ and } v_{HS}\) are the velocity of the forward shock front (FS) and that of the reverse one (RS).

The forward shocked region quantities are given by the well-known shock jump conditions in non-relativistic limit (Landau & Lifshitz 1959),

\[
v_{HS} = \frac{3}{4}v_{FS}(1 - 1/M_1^2), \quad (7)
\]

\[
\rho_2 = \frac{4}{1 + 3(1/M_1^2)}\rho_3, \quad (8)
\]

\[
P_{HS} = \left(\frac{3 - (3(1/M_1^2)/5)}{4}\right)\rho_3v_{FS}^2, \quad (9)
\]

where \(M_1 = v_{FS}/\sqrt{(5P_a/3\rho_3)}\) is the Mach number of the upstream of FS. In this work, we treat the forward shock as a non-relativistic one because the speed of hotspots is estimated to be in the range 0.01c to 0.1c (e.g., Liu, Pooley, & Riley 1992), and adopt the adiabatic index of the downstream as 5/3.

From eqs. (7) and (9), we express \(P_{HS}(r)\) by two observable quantities \(v_{HS}(r)\) and \(\rho_4(r)\) as follows;

\[
P_{HS}(r) = \frac{15}{4}\left(\frac{(1 - (1/M_1^2))^2}{5 - (1/M_1^2)}\right)\rho_4(r)v_{HS}(r)^2. \quad (10)
\]

On the other hand, in the reverse shocked region, by using the strong limit jump condition (Blandford & McKee 1976) and eq. (10), we have the radial dependence of the mass density in the hot spot and the jet as follows;

\[
\rho_{HS}(r) = \frac{P_{HS}(r)}{(\gamma_3 - 1)(\Gamma_1 - 1)c^2} \quad (11)
\]

\[
\rho_3(r) = \frac{P_{HS}(r)}{\gamma_3\Gamma_j + 1)(\Gamma_j - 1)c^2}, \quad (12)
\]

where \(\gamma_3 = 4/3\). From these equations, we find that the radial dependences of \(\rho_{HS}(r)\) and \(\rho_j(r)\) are the same as \(P_{HS}(r)\).
2.4. Cocoon dynamics

The formation of the cocoon was initially proposed by Schenker (1974). The cocoon is filled with the relativistic medium escaping through the hot spots (see Figure 1). Begelman & Ciolfi (1989) described both propagation of the head part of jets and the transverse growth of the cocoon itself. Then, they expressed the evolution of radio sources with the self-similar solutions.

Recently, Kino & Kawakatu (2005, hereafter KK05) dealt with the cocoon dynamics in FR II radio sources without using the self-similar model (see KK05 for the differences from self-similar model). By solving the equation of motion along the jet axis, sideways expansion, and energy conservation in the cocoon, they obtained the $v_{c}(r)$, $v_{h}(r)$, $P_{c}(r)$, and $A_{h}(r)$ as a function of the distance from the galactic center (for details see KK05). Here, $v_{c}$, $v_{h}$, $P_{c}$ and $A_{h}$ are the velocity of cocoon sideways expansion, that of the advance velocity of the cocoon head part, the pressure of cocoon, and the cross section of the head part of cocoon, respectively. Note that in order to convert $t$-dependence into $r$-dependence we use the equation $r = \int_{0}^{t}v_{c}(t')dt'$ and $r_{c} = \int_{0}^{t}v_{h}(t')dt'$ where $r_{c}$ is the length of the cocoon. By using the analytic solutions of KK05, these quantities are given by

\begin{align}
    v_{c}(r) & \propto \left( \frac{r}{r_{0}} \right)^{\frac{X(1-\alpha/2)-2}{X(-2+0.5\alpha)+3}}, \\
    P_{c}(r) & \propto \left( \frac{r}{r_{0}} \right)^{\frac{X(-2+0.5\alpha)+3}{X(-2+0.5\alpha)+3}}, \\
    v_{h}(r) & \propto \left( \frac{r}{r_{0}} \right)^{\frac{X(-2-0.5\alpha)+3\alpha-4}{X(-2+0.5\alpha)+3}}, \\
    A_{h}(r) & \propto \left( \frac{r}{r_{0}} \right)^{\frac{X(-2-0.5\alpha)+3\alpha-4}{X(-2+0.5\alpha)+3}},
\end{align}

where $X$ is the power law index of the effective cross section of the cocoon body $A_{c} \propto t^{X}$.

2.5. Connection between jet and its cocoon

The growth rate of the head part of jets $\beta$ can be obtained explicitly as the function of $\alpha$ and $X$ (eqs. (1), (15)and (16)) as follow:

$$\beta = \frac{X(\alpha-2)(-2+0.5\alpha)+3\alpha-4}{X(-2+0.5\alpha)+3}. \quad (17)$$

From this, we can connect the dynamical evolution between a jet and a cocoon. With combining the 1D shock jump conditions, we get the evolution of $v_{HS}$, $P_{HS}$, $\rho_{HS}$, and $\rho_{j}$ as follows:

\begin{align}
    v_{HS}(r) & = v_{HS,0} \left( \frac{r}{r_{0}} \right)^{\frac{X(2-0.5\alpha)}{X(-2+0.5\alpha)+3}}, \\
    P_{HS}(r) & = P_{HS,0} \left( \frac{r}{r_{0}} \right)^{\frac{X(2-0.5\alpha)(\alpha-2)+3\alpha-4}{X(-2+0.5\alpha)+3}}, \\
    \rho_{HS}(r) & = \rho_{HS,0} \left( \frac{r}{r_{0}} \right)^{\frac{X(2-0.5\alpha)(\alpha-2)+3\alpha-4}{X(-2+0.5\alpha)+3}}, \\
    \rho_{j}(r) & = \rho_{j,0} \left( \frac{r}{r_{0}} \right)^{\frac{X(2-0.5\alpha)(\alpha-2)+3\alpha-4}{X(-2+0.5\alpha)+3}},
\end{align}

where $v_{HS,0}$, $P_{HS,0}$, $\rho_{HS,0}$, and $\rho_{j,0}$ are the physical quantities at $r_{0}$, respectively. On the velocity of hot spots ($v_{HS}$), larger $\alpha$ leads to the stronger acceleration effect due to the lighter ambient medium, while larger $X$ leads to the stronger deceleration effect because of the stronger interaction with the ambient medium.

It is useful to define the aspect ratio of the cocoon, $R(r) = r_{c}/r$. Using this, the evolution of the aspect ratio of cocoon is written by

$$R(r) = R_{0} \left( \frac{r}{r_{0}} \right)^{\frac{X(2-0.5\alpha)-3}{X(-2+0.5\alpha)+3}}, \quad (22)$$

where $R_{0}$ is the aspect ratio at $r_{0}$.

The jet thrust is defined by $F \equiv A_{j}(r)P_{HS}(r)$. From eq. (6) and 1D shock junctions, we found $F = \rho_{a}(r)v_{HS}^{2}(r)A_{j}(r) = \text{constant}$. On the other hand, the equation of motion along the jet axis shows $\rho_{a}(r)v_{HS}^{2}(r)A_{j}(r) = \text{constant}$ (see §2). Hence, the radial dependence of $A_{j}(r)$ is the same as that of $A_{h}(r)$. Then, the evolution of the cross section of hot spots $A_{j}(r)$ is obtained as

$$A_{j}(r) = A_{j,0} \left( \frac{r}{r_{0}} \right)^{\frac{X(2-0.5\alpha)-3}{X(-2+0.5\alpha)+3}}. \quad (23)$$

Since the cross section of the hot spot is expressed by $A_{j}(r) = \pi r_{HS}^{2}(r)$, the evolution of hot spot radius is estimated as

$$r_{HS}(r) = r_{HS,0} \left( \frac{r}{r_{0}} \right)^{\frac{X(2-0.5\alpha)-3\alpha-4}{2X(-2+0.5\alpha)+6}}. \quad (24)$$

In summary, we solve the all physical quantities as functions of two key physical quantities, namely $\alpha$ (the slope index of the ambient matter density) and $X$ (the growth rate of cross section of cocoon body) without using the self-similar models.
3. DISCUSSIONS

3.1. Comparison with previous works

In order to compare with previous works, it is worth to describe our solutions in the case of the constant aspect ratio in time ($r/r_c = $ constant). In this case, $X(2.5 - 0.5\alpha) - 3 = 0$ is required from eq. (22) and thus we can eliminate the parameter $X$ from eqs. (13)-(23). As a result, we obtain the following results: $v_c(r) \propto v_{HS}(r) \propto r^{(\alpha-2)/3}$, $P_c(r) \propto P_{HS}(r) \propto r^{-(\alpha+4)/3}$, $\rho_{HS}(r) \propto r^{-(\alpha+4)/3}$, $r_{HS}(r) \propto r^{-(\alpha+4)/6}$, $\rho_j(r) \propto r^{-(\alpha+4)/3}$.

The dynamical evolution of radio sources with self-similar model were suggested by some authors (e.g., Fanti et al. 1995; Begelman 1996; Kaiser & Alexander 1997). In this work, we focus on the model of Begelman (1996; other comparison will be in Kawakatu, & Kino 2005, in preparation). In his model, the different assumptions from our model are (i) the hot spot pressure is proportional to the cocoon one ($P_{HS}(r) \propto P_c(r)$), and (ii) the constant aspect ratio of the cocoon in time. The latter results in a constant ratio of the hot spot radius to the distance from the nuclei ($r_{HS}(r) \propto r$). Note that our model solves the evolution of a hot spot pressure and a cocoon shape without assumptions. As a result, he found the following self-similar solutions: $v_c(r) \propto v_{HS}(r) \propto r^{(\alpha-2)/3}$, $P_c(r) \propto P_{HS}(r) \propto r^{-(\alpha+4)/3}$, $\rho_{HS}(r) \propto r^{-(\alpha+4)/3}$, $r_{HS}(r) \propto r^{-(\alpha+4)/6}$, $\rho_j(r) \propto r^{-(\alpha+4)/3}$. These results coincide with ours when $\alpha = 2$. In the case of $\alpha \neq 2$, his model does not satisfy with the constant jet thrust in time because of $F(r) = \pi r^2_{HS}(r) P_{HS}(r) \propto r^{(2-\alpha)/3}$. On the other hand, his model should fill with the condition of the constant jet thrust since a loss-free jet was supposed. According to our results (eqs. (14) and (19)), the assumption (i) is not valid in the case of $\alpha \neq 2$. Therefore, an important difference between Begelman’s model and our model is that we describe the evolution of physical properties in hot spots ($P_{HS}$ and $r_{HS}$) consistently with keeping the conservation law for the jet.

3.2. Evolution of radio sources

Recent observations have suggested that the advance speed of the hot spot is roughly constant for the different scale radio sources (Readhead et al. 1996b, Carilli et al. 1991; Conway 2002). In this section, we discuss the case of $v_{HS} = $ constant. From eq. (18), $2 - X(2 - 0.5\alpha) = 0$ is required in the case of constant hot spot velocity. By eliminating the parameter $X$, the following results are obtained: $v_c(r) \propto r^{-(\alpha-2)/(\alpha-4)}$, $P_c(r) \propto r^{1/(\alpha-4)}$, $P_{HS}(r) \propto r^{-\alpha}$, $\rho_{HS}(r) \propto r^{-\alpha}$, $r_{HS}(r) \propto r^{\alpha/2}$, $\rho_j(r) \propto r^{-\alpha}$, and $R(r) \propto r^{-(\alpha-2)/(\alpha-4)}$.

As for the ambient density profile, we adopt a single power law model, $\rho_a(r) \propto r^{-1.5}$ based on Reynolds & Fabian (1996) and Smith et al. (2002). In this case, the dynamical evolutions of jets are as follows: $v_{HS}(r) = $ constant, $P_{HS}(r) \propto r^{-1.5}$, $r_{HS}(r) \propto r^{-0.75}$, and $R(r) \propto r^{-0.2}$. We should keep in mind the above results depend on the slope index $\alpha$ ($0 < \alpha < 2.0$).

Observationally, Perucho & Marti (2002) found that $P_{HS}(r) \propto r^{-1.7\pm0.4}$ and $r_{HS}(r) \propto r^{1.0\pm0.3}$ for CSOs. Readhead et al. (1996a) also mentioned the evolution of hot spot pressure with $P_{HS}(r) \propto r^{-1.3\pm0.13}$ for CSOs and FR II sources. Our model reproduces this observational data well in the case of constant $v_{HS}$. In our model, moreover, the weak evolution of cocoon shape is predicted ($R(r) \propto r^{-0.2}$).

4. SUMMARY

In order to reveal the evolution of radio sources, we propose a new dynamical co-evolution model of jet and its cocoon. As key components, we take into account the effects of (i) the mass density gradient of the ambient medium, and (ii) the growth rate of head cross section area of the cocoon. Based on this model, we express the evolution of physical properties in hot spots as a function of slope index of ambient density ($\alpha$) and the growth rate of the cocoon body ($X$). An important difference from the self-similar model of Begelman (1996) is that we solve the evolution of hot spots consistently satisfying the conservation law for the mass, energy and momentum flux of the jet. Moreover, our analytic model can explain the observational trends of $v_{HS}(r) \approx constant$, $P_{HS}(r) \propto r^{-2.5}$, $r_{HS}(r) \propto r^{\pm0.75}$.

As a ongoing project, we are comparing with the observational data quantitatively (Kawakatu & Kino 2005, in preparation).

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