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MIGRATION OF A JUPITER IN A PROTOPLANETARY DISK

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RESUMEN

Estudiamos la formación de planetas en un modelo sencillo de acreción planetaria (Laskar 2000), que incluye además restricciones físicas en la acreción y un proto-Júpiter inmerso en el disco protoplanetario. Los efectos locales en tiempos cortos aumentan la migración del Júpiter y generan distribuciones más anchas de la excentricidad. Estos procesos de tres cuerpos podrían explicar las altas excentricidades observadas en los planetas exosolares.

ABSTRACT

We study the formation of planets in a simple model of planetary accretion (Laskar 2000), which includes additionally physical restrictions in the accretion processes and a proto-Jupiter immersed in the protoplanetary disk. Short-time local effects increase the migration of Jupiter and yield wider distributions for the eccentricity. Such three-body processes could explain the large eccentricities observed in exosolar planets.

Key Words: accretion — planets and satellites: formation — solar system: formation

Consider a planar system of N bodies (protoplanets) with masses $m_1, m_2, ..., m_N$ orbiting around a star of mass m_0 under the mutual gravitational interaction ($m_i \ll m_0$). In heliocentric coordinates the Hamiltonian of the system reads $H = H_0 + H_1$, where $H_0 = \sum_{i=1}^{N} \left[\mathbf{p_i}^2/(2m_i) - Gm_0 m_i/r_i\right]$ and $H_1 = \mathbf{p_0}^2/(2m_0) - G\sum_{1 \le i < j} m_i m_j/|\mathbf{r_i} - \mathbf{r_j}|$. Here, H_0 represents N independent Kepler problems and H_1 includes the indirect term and the mutual interactions among the protoplanets. With respect to H_0 the motion of m_i is Keplerian and is defined by the semi-major axis a_i and the eccentricity e_i of its elliptic orbit (the orbital plane is fixed for all protoplanets). H_1, viewed as a pertubation of H_0, induces the chaotic motion in the system.

Laskar (2000) introduced the following simplified model of planetary accretion, considering the secular limit of H, i.e., when the effects of mean-motion resonances are neglected (a_i remain constant). First, the chaotic motion induced by H_1 is modeled as a Brownian diffusion, a random walk in the space of eccentricities constrained by the conservation of the total angular momentum of the system. Secondly, accretion is implemented when the orbit of two protoplanets cross, considering the total inelastic collision that merges the particles, which conservs the mass and angular momentum. Consequently, the total angular momentum of the system $L = \sum_{i=1}^{N} m_i \sqrt{\mu a_i (1 - e_i^2)}$ is conserved, unless there are particles that escape away ($\mu = Gm_0$). The simplified model of Laskar consists of iterating the processes of accretion and chaotic diffusion until a stability condition is satisfied. The angular-momentum deficit (AMD) is defined by $\mathrm{AMD} = \sum_{i=1}^{N} m_i \sqrt{\mu a_i}(1 - \sqrt{1 - e_i^2})$ and is a measure of the lack of circularity of the protoplanetary orbits. Laskar (2000) shows that the AMD is reduced when particles merge. This led him to define AMD stability: a system is AMD-stable if its AMD does not allow planetary collisions (Laskar 2000).

In the statistical results presented below we fix the value of the total angular momentum of the system L for all realizations. Initially, the protoplanetary disk consists of a large number of equally massive protoplanets (with no gas), with semi-major axis homogeneously distributed over an interval (Laskar 2000). We implement Laskar's simplified accretion model until the AMD stability condition is fulfilled. We have additionally included a physical condition for the accretion processes: Accretion takes place if the relative velocity of the colliding bodies is smaller than the escape velocity $|\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1| \le v_{\mathrm{esc}}$ (Safronov 1972), where $v_{\mathrm{esc}} = \sqrt{6/r}$ (Ohtsuki 1993). Moreover, the random walk is implemented through angular momentum exchange between pairs of particles, which ensures the conservation of the total angular momentum of the system.

With this model we address the migration of a giant protoplanet of mass m_J (proto-Jupiter) embedded in the disk of protoplanets. The initial disk contains $N = 10000$ protoplanets distributed

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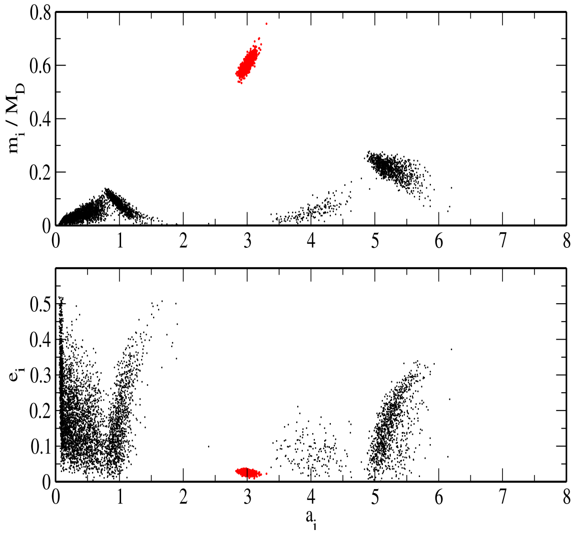

Fig. 1. Final mass and eccentricity in terms of the semi-major axis for the totality of planets formed for 1000 realizations of the simple accretion model. The red dots (grey) correspond to Jupiters, while the black ones are the other formed planets.

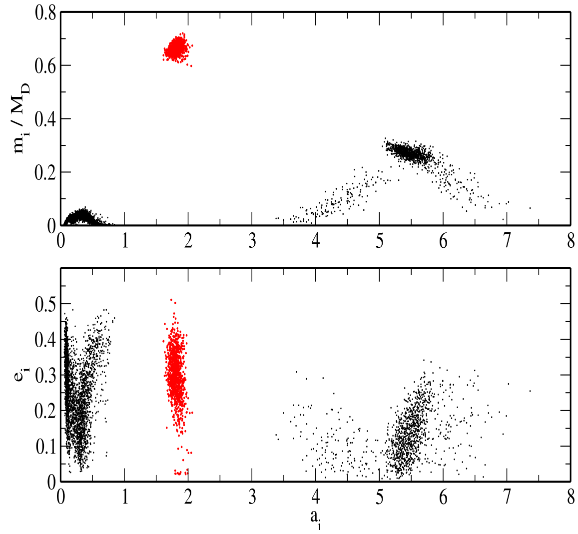

Fig. 2. Same as Figure 1, but the model includes effects from the 1:1 mean-motion resonance.

randomly in 10AU, and random eccentricities in $[0, 0.99]$. The initial conditions for the proto-Jupiter are: $a_J^0 = 5\text{AU}, e_J^0 = 0.05, m_J^0 = 0.1 M_D$. The units are defined by $m_0 = 1$, $\mu = G m_0 = 4\pi^2$, and we fix $M_D = m_J + \sum_i m_i = 1.35 \times 10^{-3}$ and $L = 3.479 \times 10^{-2}$, which are the values for the Solar System.

In Figure 1 we show all the planets formed by 1000 realizations of planetary systems once the AMD stability criterion is fulfilled. Each planetary system has between 6 and 11 planets. The results indicate that the mass of Jupiter is in the range $(0.53, 0.76) M_D$, migrating until $a_J \sim 3$AU, and essentially keeping its initial eccentricity. Note that the mass distribution of the inner planets, between the star and Jupiter, is similar to the observed for the inner planets of our Solar System. Moreover, the outer planets formed are typically more massive than the inner ones.

The presence of a proto-Jupiter among the protoplanets modifies in short time-scales their Keplerian trajectories. We therefore consider a model for the effect of the 1:1 mean-motion resonance. We define the width of the resonance as $2R_H$, with R_H the Hill radius of the proto-Jupiter. After each accretion step, all protoplanets within the resonance change their semi-major axis by $a_i \to a_i + \delta a$, with $\delta a = R_H - |a_J - a_i|$, which simulates a kind of wake. Correspondingly, a_J and e_J are changed to conserve the energy and angular momentum of the system. In Figure 2 we show the results. With respect to the former case, the Jupiters accrete approximately to the same final mass. However, they migrate reaching $a_J \sim 2$ AU and the values of their final eccentricities display a broader distribution, $e_J \in (0.02, 0.51)$. Hence, the presence of three-body like effects could account for the large eccentricities observed in exosolar planets (Armitage 2007).

Summarizing, we have studied the statistics of planet formation and migration in a simple planetary accretion model which includes physical constrains for the accretion processes, considering a protoplanetary disk and a proto-Jupiter. Including a simple implementation for effects related to the 1:1 mean-motion resonance, we obtained an enhanced migration for Jupiter, and a wider distribution for their eccentricities. Such three-body effects could explain the large eccentricities observed in exosolar systems (Armitage 2007). Interestingly, the distribution of mass for the inner planets is comparable with that observed in the inner Solar System.

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