

## AN INTRODUCTION TO ASTROMETRY AND CELESTIAL MECHANICS: A PROPOSED SYLLABUS

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### RESUMEN

Si hemos de aprovechar la astrometría con precisión de micro segundos de arco que nos ofrecerán las nuevas técnicas de observación, debemos reformular nuestros estudios sobre sistemas de referencia y ecuaciones de movimiento en el contexto de la relatividad especial y general. También debemos desarrollar métodos más avanzados para el análisis estadístico de los datos y para la calibración de los instrumentos. En consecuencia, tenemos que revisar drásticamente los planes de estudio para adecuarlos a las necesidades de los estudiantes del siglo 21. Para ello, hemos desarrollado el programa de estudios para un curso introductorio de un semestre en Astrometría y Mecánica Celeste. El curso proporciona una introducción moderna y amplia a casi todos los temas de nuestro campo, y forma una base de conocimientos a partir de la cual el estudiante podrá elegir áreas para el estudio individual o para seguir cursos avanzados en centros especializados.

### ABSTRACT

If we are to take advantage of the Micro-arcsecond astrometry to be offered by new observational techniques, we need to reformulate our study of reference frames, systems and the equations of motion in the context of special and general relativity. Methods also need to be developed to statistically analyze our data and calibrate our instruments to levels beyond current standards. As a consequence, our curricula must be drastically revised to meet the needs of students in the 21st Century. With the above considerations in mind, we developed a syllabus for an introductory one-semester course in Astrometry and Celestial Mechanics. This course gives broad introductions to most topics in our fields and a base of knowledge from which a student can elect areas for self-study or attendance at centers where advanced courses, workshops or internships are available.

*Key Words:* astrometry — celestial mechanics

### 1. INTRODUCTION

Astrometry and Celestial Mechanics have entered a new era with the advent of Micro-arcsecond positions, parallaxes and proper motions. Cutting-edge science topics will be addressed that were far beyond our grasp only a few years ago. It will be possible to determine definitive distances to Cepheid variables, the center of our Galaxy, the Magellanic Clouds and other Local Group members. We will measure the orbital parameters of dwarf galaxies that are merging with the Milky Way, define the kinematics, dynamics and structure of our Galaxy and search for evidence of the Dark Matter that makes up most of the mass in the universe. Stellar masses will be determined routinely to 1% accuracy and we will be able to make full orbit solutions and mass determinations for extrasolar planetary systems. If we are to take advantage of micro-arcsecond astrometry, we

need to reformulate our study of reference frames, systems and the equations of motion in the context of special and general relativity. Methods also need to be developed to statistically analyze our data and calibrate our instruments to levels beyond current standards. As a consequence, our curricula must be drastically revised to meet the needs of students in the 21st Century. With the above considerations in mind, a syllabus has been developed for an introductory one-semester course in Astrometry and Celestial Mechanics (van Altena & Stavinschi 2007). This course gives broad introductions to most topics in our fields and a base of knowledge from which a student can elect areas for self-study or attendance at centers where advanced courses, workshops or internships are available.

### 2. THE SYLLABUS

As mentioned above, the syllabus is designed for a one-semester introductory course that will hopefully motivate students to continue their study of Astrometry. The syllabus is divided into five sections.

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### 2.1. *Opportunities and challenges for Astrometry in the 21st Century (3 hours)*

§2.1 provides the impetus to study Astrometry by reviewing the opportunities and challenges for micro-arcsecond positions, parallaxes and proper motions that will be determined by new space astrometry missions as well as ground-based telescopes that will yield milli-arcsecond data for enormous numbers of objects. Current hot topics include the cosmological distance scale, the dynamics of dwarf-galaxy remnants that are merging with the Galaxy, the continuing search for dark matter, understanding the Disk, Thick Disk and Halo kinematics and dynamics, measuring the masses and orbits of extra-solar planets and understanding their origin and determining the masses of binary stars to 1% so that critical constraints can be put on stellar structure and evolution models.

### 2.2. *Relativistic Foundations of Astrometry and Celestial Mechanics (9 hours)*

§2.2 begins the course with introductions to the relativistic foundations of astrometry and celestial mechanics (3 hours). The basics of special and general relativity are introduced followed by the post-Newtonian approximation scheme, relativistic astronomical reference systems and frames, coordinate-dependent and measurable quantities, relativistic astronomical time scales and their realizations, relativistic data reduction and modeling, the relativistic equations of motion of a test body around a spherically symmetric body, and the N-body problem. The next subsection introduces the celestial mechanics of N-body systems (4 hours). Subjects dealt with here include the Sun and solar oblateness, the major planets and planetary rings, the minor planets and the asteroid belts, the role of analytic techniques and numerical methods, pulsar timing, laser and radio ranging, high-resolution radial velocities and analyzing binary and multiple systems from micro-arcsecond positions and high-resolution radial velocities. The third subsection deals with stellar coordinate systems and positions (2 hours), where we introduce the terminology in current use as well as flow charts and formulae for the transformation from ICRS to the observed places of the stars.

### 2.3. *Observing through the Atmosphere (7 hours)*

§2.3 introduces the deleterious effects of observing through the atmosphere and methods developed to compensate and even take advantage of those effects, followed by introductions to selected topics in optics and detectors. The first subsection (2.5 hours)

introduces models of the atmosphere and turbulence and refraction through a turbulent atmosphere as a function of wavelength, including the special case of radio astronomy. We then review the limits on positional precision imposed by the atmosphere and the agreement with theoretical interpretations, followed by methods that have been developed to compensate for the atmosphere such as tip-tilt and adaptive optics, phase referencing, image reconstruction, speckle imaging, observing in the infrared and from space satellites. The next subsection (4 hours total) introduce the theory and practicalities of optical and radio interferometry from the ground and in space with reference to existing and planned systems.

### 2.4. *From detected photons to the Celestial Sphere (14 hours)*

§2.4 develops the techniques for analyzing the images formed by our telescopes and the relations necessary to project complex focal plane geometries onto the celestial sphere.

§2.4.1 reviews geometrical optics in the context of astrometry (3 hours). Special emphasis is given to the beam shifts, image position errors and aberrations introduced by the principal optics as well as the secondary optics such as filters and CCD windows and the misalignment of the optics.

§2.4.2 (2 hours) introduces the complexities of our modern imaging detectors including CCD and CMOS detectors and their use in both the tracking and drift-scan modes. Topics include understanding the detectors in some detail, calibrating for systematic effects, determining photometric parameters and astrometric image centers and an introduction to statistical techniques useful for CCD analyses.

While photographic plate measurement is a subject that is rapidly losing relevance, there are still many old photographic plates that remain unmeasured and could provide valuable first-epoch positions. We have therefore included (1 hour) a discussion of the various measuring machines still in use and the procedures used in their calibration as well as the methods used to derive photometric and astrometric parameters of the measured images.

Since much of astrometry depends on the use of statistical analyses procedures to derive the desired astronomical information, Subsection 2.4.4 (1.5 hours) deals with an introduction to error analysis, the Malmquist bias and the Lutz-Kelker corrections as applied to trigonometric parallaxes and the calibration of the luminosities and masses of stars, as well as Monte-Carlo modeling, statistical and secular parallaxes.

§2.4.5 (0.5 hour) introduces the topic of image deconvolution as a technique useful for recovering high-resolution images from ones degraded by aberrations, such as the HST initial images, as well as for improving the resolution and magnitude limit of ground-based imaging.

§2.4.6 (4 hours) derives the techniques for modeling the focal plane of telescopes and mapping it onto the celestial sphere. Special attention is given to searching for systematic errors, dealing with arrays of CCD's found in most of the modern large telescopes and drift-scan systems and developing astrometric calibration regions over a large range of magnitudes.

The final §2.4.7 (2 hours) reviews the derivation of the final repository of the astrometric positions that we have derived above, i.e. catalogs. A wide range of astrometric-quality catalogs are reviewed including the Fundamental ICRF radio catalog, the optical reference systems related the Hipparcos and looking forward to Gaia. Extension of the optical reference system to fainter magnitudes is examined in relation to the determination of image positions of very faint objects observed with large telescopes using the UCAC, NPM and SPM catalogs. The value of non-astrometric catalogs, e.g. 2MASS and DENIS, and Schmidt-based catalogs, e.g. USNO A&B GSC and Cosmos, and the variations of the Virtual Observatory are examined.

### 2.5. *Applications of Astrometry to Astronomical Topics (9 hours)*

The concluding §2.5 gives some applications of astrometry to a variety of astronomical topics of current interest that we hope will stimulate students to further explore our exciting field. The specific topics included in this section should reflect the ongoing research interests of the institute where this course is taught so that the students can apply the astrometric techniques learned to research relevant to their home base. Examples include applying astrometry to studies of the spatial and kinematical structure of the Galaxy and of the merging dwarf galaxies; calibrating the luminosities and masses of stars using Hipparcos, Gaia and SIM parallaxes; measuring the mass of the Black Hole at the center of our Galaxy; using speckle and long-baseline interferometry to determine the orbits of binary stars; determining the membership and internal kinematics of star clusters from proper motions; finding extra-solar planets and determining their orbital parameters and masses; using lunar occultations and astrometry to derive the orbits of solar system bodies; and using astrometry to help constrain critical cosmological parameters.

## 3. CONCLUSIONS

This syllabus for a one-semester introductory course in Astrometry and Celestial Mechanics is designed to introduce students to a broad range of topics in astrometry and to provide them with the foundation to continue with more specialized courses in the field, when available, or to continue with self study when no other courses exist. A variety of methods for continuing study are available including attending international workshops on specialized topics or participating in international summer research programs at institutes specializing in astrometry and celestial mechanics. The syllabus was developed with a 40-hour one-semester course in mind, but it can be easily scaled to other standards, such as a 20-hour course, by reducing the depth in the subjects or selectively omitting some subject areas according the needs of the individual institute where the course is taught. The instructor will have to consider the specific audience for the course, e.g. Masters or Ph.D. students, since the topic stress for Masters students might vary according to the training needs of the students. In some cases a lecture-style course may be appropriate, while in other cases lecture plus internships at observatories and institutes and workshops may be appropriate. A critical component of this course is to present strong science introductions to each topic to help motivate the students. Finally, we have assumed that, since astrometry and celestial mechanics are rather specialized fields, the course would be taught in a curriculum that includes a variety of courses in astronomy and astrophysics to which astrometry could be applied, such as the formation, structure and evolution of stars, exo-planetary systems and galaxies.

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