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PRIMORDIAL MAGNETIC FIELDS WITH THE WMAP AND PLANCK SATELLITE EXPERIMENTS

J. Aumont[1] and J. F. Macías-Pérez[2]

RESUMEN

Presentamos un estudio de la rotación de Faraday de las anisotropías en temperatura y polarización del Fondo Cósmico Microondas (CMB) causada por un campo magnético primordial (PMF) a gran escala presente durante el desacoplamiento entre fotones y electrones cuando los primeros átomos fueron formados. Suponemos un campo magnético de baja amplitud, típicamente del orden del nG hoy en día, y un desacoplamiento instantáneo entre fotones y electrones. Bajo esas hipótesis y asumiendo perturbaciones tensoriales despreciables, las ecuaciones de evolución de la polarización del CMB dan lugar a la generación de modos B de polarización a partir de los modos E asociados a las perturbaciones escalares. Nuestro estudio permite la simulación de mapas de CMB que incluyen el efecto del PMF para un amplio espectro de geometrías del mismo. A partir de estas simulaciones extraemos los espectros de potencias en polarización del CMB y los comparamos con cálculos analíticos del mismo efecto. Observamos que nuestra aproximación reproduce el orden de magnitud del espectro de potencias del CMB $C_\ell^{\rm BB}$. Finalmente, obtenemos un límite superior en la intensidad actual del PMF, $B_0 \lesssim 27$ nG (95% CL), comparando nuestros modelos con las observaciones de WMAP.

ABSTRACT

We present a study of Faraday rotation of the Cosmic Microwave Background (CMB) polarization anisotropies by a large scale primordial magnetic field (PMF) present at the time of decoupling. We assume weak intensity for the PMF, of the order of a nG nowadays, and tight coupling between photons, electrons and baryons around recombination. Under these hypotheses and considering no tensor perturbations, the equations of evolution of the CMB polarization lead to the direct generation of B polarization modes from the E modes generated by scalar perturbations. We are able to produce simulated CMB maps including this effect for a wide range of geometries of the PMF. From these simulations we extract the modified polarization angular power spectra and the results are compared to an analytic approach showing that our hypothesis leads to the same order of magnitude signature on the CMB $C_\ell^{\rm BB}$ spectrum. Finally, we obtain an upper limit on the today intensity of the PMF, $B_0 \lesssim 27$ nG (95% CL), by comparing our models to the $C_\ell^{\rm TE}$ spectrum measured by WMAP.

Key Words: cosmic microwave background — cosmology: observations — magnectic fields

1. INTRODUCTION

The study of the Cosmic Microwave Background radiation (CMB) enters a new era. Since the precise measurements of the CMB temperature anisotropies by the WMAP satellite experiment (Hinshaw et al. 2007), the challenge is now to extract information from the polarization anisotropies, several orders of magnitude fainter than temperature anisotropies. Among the wealth of information contained in the CMB polarization, one is of particular interest for cosmologists and particle physicists, the B modes which might have been created from tensor perturbations in the form of primordial gravitational waves generated at the Inflation epoch. The level of the BB angular power spectrum, $C_\ell^{\rm BB}$, is directly related to the tensor to scalar ratio, r. This is defined as the ratio of the amplitudes of the temperature anisotropies for the quadrupole generated from tensor and scalar perturbations and it is directly related to the energy scale of Inflation (Zaldarriaga 2004). Thus, measuring the CMB $C_\ell^{\rm BB}$ spectrum is a thrilling challenge since it is a direct tracer of the Inflation epoch. Currently, only upper limits have been set, $r < 0.30$ (95% CL) (Spergel et al. 2007).

Independently, large-scale primordial magnetic fields (hereafter PMF), with present amplitudes of order 1 nG, have been considered in cosmology to explain the Mpc-scale magnetic fields observed in galaxies and galaxy clusters. Dynamo amplification of a *seed* field which can be of weak amplitude (e.g. Parker 1971) or adiabatic compression of large

[1]Centre d'Étude Spatiale des Rayonnements, 9 avenue du Colonel Roche, BP 44346, 31028 Toulouse Cedex 4, France (aumont@cesr.fr).

[2]Laboratoire de Physique Subatomique et de Cosmologie, 53 avenue des Martyrs, 38026 Grenoble Cedex, France (macias@lpsc.in2p3.fr).

scale magnetic field during structure formation (e.g. Howard & Kulsrud 1997) are two mechanisms that may explain galactic fields of amplitude between 1 and 100 μG. Many arguments have been proposed to explain the origin of PMFs, some authors consider them as natural initial conditions of our Universe, others propose that they might have been generated during Inflation or electro-weak transition (e.g. Giovannini 2008) but none of these argument significantly came to the fore. For a review on this topic, see Widrow (2002).

Such a field might have been present at the time of decoupling, when the Universe was totally ionized and when the CMB radiation formed. The magnetic field energy momentum tensor would modify the space-time geometry and therefore affect the propagation of photons, leading to additional observable imprints on the CMB anisotropies. Therefore, the study of the CMB anisotropies can be used to constrain the PMF amplitude and geometry. Upper limits on the amplitude of the PMF have been set, for example to $B_0 < 6.8 \cdot 10^{-9} \cdot (\Omega_m h^2)^{1/2}$ G (95% C.L.) (Barrow et al. 1997) and to $B_0 < 7.7$ nG (68% C.L.) (Yamazaki et al. 2006).

Furthermore, if a PMF was present at decoupling, Faraday rotation of the CMB photons polarization vectors would occur. Under this rotation, the Stokes parameters Q_r and U_r in the radial frame associated with scalar perturbations, which are spin-2 quantities, are mixed and B mode polarization is formed from the scalar perturbations E mode polarization (Zaldarriaga 2001). Thus, the presence of a large-scale magnetic field at the time of decoupling will lead to a non-zero cosmological $C_\ell^{\rm BB}$ CMB angular power spectrum even in the case of no tensor perturbation. Studying this effect is then of particular interest for cosmology as it can help to improve our understanding on the physics of structure formation down to galactic scales. In addition, it can be also considered as a contaminant to the detection of tensor perturbations via their induced CMB $C_\ell^{\rm BB}$ angular power spectrum.

Previous works have tried to quantify Faraday rotation on CMB polarization at the last scattering surface deriving the equations of evolution of the Stokes parameters in the presence of a PMF. They typically extract in the case of simplified PMF geometries analytical expressions of the resulting CMB angular power spectra after line-of-sight integration (e.g. Scóccola et al. 2004). In this work we are more interested in producing maps of the Faraday rotated CMB polarization for complex geometries of the PMF. For this purpose we make simple assumptions on the equations of evolution of the CMB photons at the time of decoupling: weak Faraday rotation and tight coupling. Under these hypotheses, we are able to construct Q and U maps of the Faraday rotated CMB sky at different observation frequencies. This work is of great interest to study the possibility of detection of PMFs with the Planck satellite mission and has been developed within the Planck consortium.

This paper is organized as follows. In § 2, we first present the expression of the Faraday rotation in the case of the presence of a PMF around time of decoupling and in the equations of evolution of the Stokes parameters, deriving our hypothesis. Under these assumptions, in § 3, we present the modeling of its effect on CMB polarized angular power spectra and we compare our approach to the analytic one from Scóccola et al. (2004), in § 4 we simulate this effect on the $C_\ell^{\rm BB}$angular power spectra and finally, in § 5, we extract an upper limit on the nowadays intensity of the PMF by comparing our model to the WMAP $C_\ell^{\rm TE}$ data.

2. EVOLUTION OF CMB ANISOTROPIES IN THE PRESENCE OF A PMF

We can consider our Universe before decoupling as a plasma formed of ions, electrons and photons. In this plasma, we will assume the presence of a PMF. An electromagnetic wave propagating in an ionized medium in the presence of a magnetic field undergoes a Faraday rotation of its polarization vector. This rotation has the following angular velocity (Kosowsky & Loeb 1996):

$$\frac{d\varphi}{dt} = \frac{e^3 x_e n_e}{2\pi m_e^2 \nu^2} (\mathbf{B} \cdot \hat{\mathbf{q}}) \,, \quad (1)$$

where $\hat{\mathbf{q}}$ is an unit vector in the propagation direction of the wave, $\mathbf{B}$ is the PMF vector, ν is the frequency, x_e the ionization fraction of the medium, e and m_e the charge and the mass of an electron and n_e the electron density (throughout the paper we use natural units with $\hbar = c = G = 1$).

Following Kosowsky & Loeb (1996) the redshift dependence for the magnetic field intensity is $||\mathbf{B}|| \equiv B \propto z^2$ and for the wave frequency $\nu \propto z$. If we assume an optical depth of the order of unity at decoupling and thin last scattering surface ($\Delta z_D / z_D \ll 1$), we can also substitute $\int x_e n_e dt$ by σ_T^{-1}, where σ_T is the Thomson total cross section. Thus, the rms rotation angle can be estimated by integrating in time equation (1), noticing that B/ν^2 is time-independent and by averaging over all the orientations of $\mathbf{B}$

(Kosowsky & Loeb 1996)

$$\begin{aligned} \langle \varphi^2 \rangle^{\frac{1}{2}} &\approx \frac{B_0}{2\sqrt{2} m_e^2 \sigma_T \nu_0^2} \\ &= 1.6^\circ \left(\frac{B_0}{10^{-9}\ \mathrm{G}} \right) \left(\frac{30\ \mathrm{GHz}}{\nu_0} \right)^2 , \quad (2) \end{aligned}$$

where B_0 is the current value of the PMF intensity and ν_0 the observation frequency. The rms of the rotation angle is inversely proportional to the square of the frequency of observation and have a distinct spectral signature that may allow to separate this effect from the primordial fluctuations of the CMB. For a field having a current intensity of 1 nG and observing at 30 GHz, the rms rotation angle would be of the order of 1°.

Faraday rotation can also be considered directly in the equations of evolution of the CMB polarization, the so-called Boltzmann equations. These equations are a set of radiative transfer equations for the Fourier modes of the radiation Stokes parameter brightness $\Delta_I(\mathbf{k}, \hat{\mathbf{q}}, \eta)$, $\Delta_Q(\mathbf{k}, \hat{\mathbf{q}}, \eta)$, $\Delta_U(\mathbf{k}, \hat{\mathbf{q}}, \eta)$ and $\Delta_V(\mathbf{k}, \hat{\mathbf{q}}, \eta)$ where $\mathbf{k}$ is the Fourier mode wavevector and η is the conformal time. We present these equations in the case of scalar perturbations in the radial reference frame defined by them and assuming a PMF (Kosowsky & Loeb 1996; Scannapieco & Ferreira 1997)

$$\begin{aligned} \left[\frac{\partial}{\partial \eta} + \frac{1}{a} ik\mu + \dot{\tau} \right] \Delta_{Q_r} &= -\frac{\dot{\tau}}{2}(1 - P_2(\mu)) \cdot S_{IQ} \\ &\quad + 2\omega_{\mathbf{B}} \Delta_{U_r} , \\ \left[\frac{\partial}{\partial \eta} + \frac{1}{a} ik\mu + \dot{\tau} \right] \Delta_{U_r} &= -2\omega_{\mathbf{B}} \Delta_{Q_r} , \quad (3) \end{aligned}$$

where $\mu \equiv \cos(\hat{\mathbf{k}} \cdot \hat{\mathbf{q}})$ is the cosine between a given Fourier mode and the propagation direction, $k \equiv ||\mathbf{k}||$, $\dot{\tau} \equiv x_e n_e \sigma_T a / a_0$ is the differential optical depth, S_{IQ} is a source term depending on the monopole and the quadrupole in I and Q_r and $\omega_{\mathbf{B}} \equiv d\varphi/d\eta = (a/a_0)(d\varphi/dt)$ is the conformal angular velocity of the Faraday rotation defined in equation (1).

If no PMF is present at the time of decoupling the terms in $\omega_{\mathbf{B}}$ are null (Kosowsky 1996). Thus, no U_r polarization can arise from scalar perturbations only, since the equation of evolution of Δ_{U_r} does not present a source term. As the Stokes parameters in the radial reference frame Q_r and U_r are strictly and respectively related to the E and B CMB polarization modes, it means that scalar perturbations produce only E modes. However, if a PMF is present at the last scattering surface, U_r is induced by Q_r and then B modes can also arise from scalar perturbations.

In order to quantify this effect on the CMB angular power spectra an analytical integration of the above evolution equations was performed by Scóccola et al. (2004) assuming a PMF oriented in the direction of the Fourier-mode wave vectors in equation 3. In this paper we are more interested in accounting for complex magnetic field geometries and in producing CMB Faraday rotated maps. We therefore adopt a different and simpler approach to solve the evolution equations based on the following assumptions

⋆ *Weak-field*: As discussed above the observed Galactic magnetic fields, of the order of 1 μG, require seed fields of the oder of 1 nG today. We can therefore assume that the intensity of the PMF is of the same order, i.e. weak. Following equation (2), the rms Faraday rotation angle would be for such a PMF of the order of 1 degree. Then, the conformal angular velocity would be $\omega_{\mathbf{B}} \ll 1$.

⋆ *Tight coupling*: We place ourselves in the *tight-coupling* regime, assuming a strong coupling between electrons and photons around the time of decoupling (Peebles & Yu 1970). In this regime, we can assume that decoupling is fast enough to be considered as instantaneous ($\Delta t_D / t_D \ll 1$).

If we consider only scalar perturbations the above hypotheses lead to the generation of a small amount of U_r polarization from Q_r polarization with no feedback. In other words, B modes are generated from scalar E modes via Faraday rotation.

Tensor perturbations are subdominant with respect to scalar perturbations ($r < 0.3$ at 95% CL; Spergel et al. 2007). We can therefore consider that the generation of Q_r polarization from the tensor associated U_r polarization is a second order effect if the above assumptions hold. Then, to first order we can neglect any feedback between tensor generated and scalar generated polarization. In other words the tensor E and B polarization modes can just be added to the scalar ones.

To summarize, these assumptions allow us to neglect the $-2\omega_{\mathbf{B}} \Delta_{U_r}$ term in the equation of evolution of Q_r. At the same time, the presence of a PMF at the time of decoupling will generate in the case of scalar perturbations U_r polarization through the $+2\omega_{\mathbf{B}} \Delta_{Q_r}$ term. We neglect in here the contribution to U_r polarization from Faraday rotation of the tensor Q_r polarization. Thus, Faraday rotation would generate *primordial* C_ℓ^{BB} spectrum from the C_ℓ^{EE} one. Under these hypotheses, we may consider, as a first approximation that the time integration of equations (3) assuming a PMF is equivalent to the direct rotation of the CMB photon polarization vector. Notice that hereafter we do not consider any polarization component generated after decoupling as for example by global reionisation of the Universe when the first stars form (Page et al. 2007).

3. MODELIZATION OF THE PMF FARADAY ROTATED CMB POLARIZATION

3.1. *Method*

To model Faraday rotation of the CMB polarization by a PMF, we simulate the CMB sky for the Stokes parameters I, Q and U, perform a rotation of the polarization vector and extract the modified polarized angular power spectra as follows

⋆ We generate a model of temperature and polarization angular power spectra using the CAMB software (Lewis et al. 2000), for a set of cosmological parameters corresponding to the ΛCDM model consistent with the WMAP data (Spergel et al. 2007), with no tensor perturbations ($r = 0$) and excluding late reionization. As discussed above under our hypotheses the ionization history of the Universe would present a sharp peak at decoupling and the CMB photons would have not interacted again in their journey from the last scattering surface to us. Nevertheless, the optical depth for global reionisation at the time of the formation of the first stars is one order of magnitude fainter than at decoupling ($\tau_r = 0.089 \pm 0.03$ (Page et al. 2007)) leading to an effect ten times smaller on the Faraday rotation of the CMB polarization vector. To sidestep this problem, when considering angular power spectra, we focus only on multipoles ℓ above 20, where the effect of reionization is subdominant in the polarization case. However, this issue remains open and only a resolution of the evolution equations for the Stokes parameters of the CMB, in the case of an Universe presenting a reionization epoch, would undercome this ambiguity. In this model, we also consider that B modes produced by lensing of the E modes are perfectly removed.

⋆ We generate full sky maps in $\mu K_{\rm CMB}$ for the Stokes parameters I, Q and U from these angular power spectra using the HEALPIX package software (Górski et al. 1999), at a given HEALPIX resolution.

⋆ We apply the Faraday rotation of the CMB polarization vector through the Stokes parameters algebra using equation (2), given the current value of the PMF intensity B_0 and the observational frequency ν_0. We have supposed two types of PMF. A *random* field, oriented randomly with a normal distribution and a PMF always oriented parallel to the line-of-sight. The second one has no physical motivation but maximizes the effect and lead to upper limits. We then obtain I, Q' and U' CMB maps Faraday rotated by the PMF.

⋆ Finally, we compute the angular power spectra of the I, Q' and U' maps using again the HEALPIX package software.

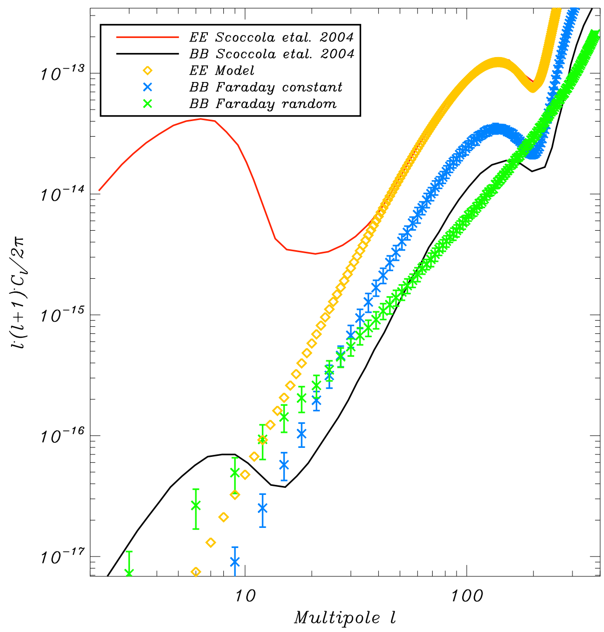

Fig. 1. $C_\ell^{\rm EE}$ and $C_\ell^{\rm BB}$ angular power spectra in arbitrary units, at 30 GHz, in the case of a PMF field having a nowadays intensity of 10 nG. *Lines* are the $C_\ell^{\rm EE}$ (*red*) and $C_\ell^{\rm BB}$ (*black*) spectra as computed analyticaly by (Scóccola et al. 2004). *Orange diamonds* represent the $C_\ell^{\rm EE}$ model we have used for our simulations, without reionization and adjusted to the value of (Scóccola et al. 2004) at $\ell = 100$. *Blue crosses* represent the $C_\ell^{\rm BB}$ spectrum modeled by our approach in the case of a *constant* PMF. *Green crosses* represent the $C_\ell^{\rm BB}$ spectrum modeled by our approach in the case of a *random* PMF. In both cases, error bars are the dispersion upon 100 realizations of the sky.

As an example we show in Figure 1 the CMB $C_\ell^{\rm BB}$ power spectrum, blue crosses, obtained by Faraday rotation of the CMB polarization from scalar perturbations only. For this model we assume a PMF oriented parallel to the line-of-sight and with an intensity of 10 nG and observations at 30 GHz. Error bars for this power spectrum are also traced representing cosmic variance as computed from 100 independent simulations. We also plot in orange diamonds the input $C_\ell^{\rm EE}$ power spectrum.

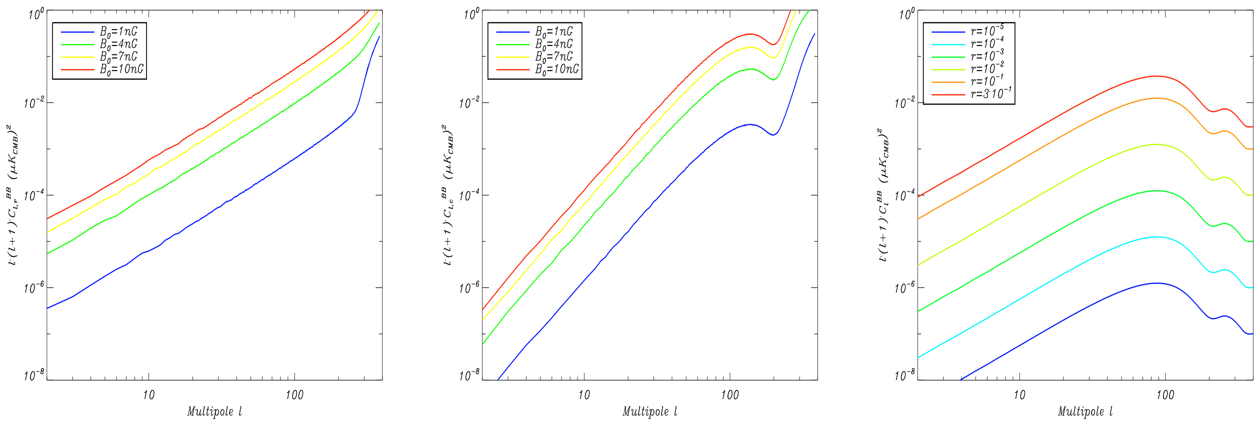

Fig. 2. $C_\ell^{\rm BB}$ angular power spectrum in the case of the ΛCDM concordance model of Spergel et al. (2007), with no reionization and no lensing, in $\mu{\rm K}^2_{\rm CMB}$. *Left*: $r = 0$ and a *random* PMF present (see text for details) at last scattering surface, for different values of the today intensity of the PMF. *Center*: $r = 0$ and a PMF oriented parallel to the line-of-sight (see text for details) present at last scattering surface, for different values of the today intensity of the PMF. *Right*: for different values of r and no PMF.

3.2. *Comparison to the analytic approach*

We have compared the results from our modeling technique to the analytical resolution of the Boltzmann equations (equation 3) by Scóccola et al. (2004). For the latter, the authors use the *total angular momentum* formalism, developed by Hu & White (1997) to integrate these equations along the line-of-sight. They have also assumed that the PMF is oriented in the direction of each of the the Fourier-mode wave vectors, $\mathbf{k}$, in equations (3).

As above we assume observations at 30 GHz and a current intensity for the PMF of 10 nG. We have adapted our inputs so that the amplitude of our $C_\ell^{\rm EE}$ power spectrum mimic the one from Scóccola et al. (2004) at multipole $\ell = 100$. Notice that as our model does not include reionization the $C_\ell^{\rm EE}$ power spectra differ at low multipoles ($\ell < 40$).

The Scóccola et al. (2004) $C_\ell^{\rm EE}$ and $C_\ell^{\rm BB}$ CMB angular power spectra are represented in figure 1 in red and black respectively. We observe that the $C_\ell^{\rm BB}$ angular power spectrum as computed using our technique can be qualitatively compared to the one from Scóccola et al. (2004). However our $C_\ell^{\rm BB}$ power spectrum is in average a factor of two larger in the case of the *constant* field. This probably comes from the difference in the choice of orientation of the PMF. Indeed, if we consider a PMF with random orientation, the $C_\ell^{\rm BB}$ spectrum, green crosses in the figure, is featureless and tilted towards the low multipole. This shows that a full comparison of the results is not possible unless the same orientation of the PMF is considered (for the kind of PMF orientation proposed by Scóccola et al. (2004) this is out of the scope of this paper). However, our simulations and the Scóccola et al. (2004) results prove that for ordered PMFs the resulting $C_\ell^{\rm BB}$ preserves the structure in the input $C_\ell^{\rm EE}$ spectrum while for random ones the structure is wiped out.

4. SIMULATIONS

Using the above scheme, we can produce both Faraday rotated I, Q and U maps and power spectra of the CMB for any given PMF intensity, orientation model and observation frequency.

From left to right, Figure 2 presents the CMB $C_\ell^{\rm BB}$ power spectrum from scalar perturbations Faraday rotated by both randomly and orientated parallel to the line-of-sigth PMFs as well as the CMB $C_\ell^{\rm BB}$ power spectrum from tensor perturbations, respectively. The PMF Faraday rotated $C_\ell^{\rm BB}$ spectra are computed from maps at the HEALPIX resolution $N_{\rm side} = 128$ and for an observation frequency of 30 GHz, where the effect is maximal for Planck. We consider different values of the current intensity of the PMF, ranging from 1 to 10 nG. In the tensor perturbation case we consider values of r ranging from 10^{-5} to $3 \cdot 10^{-1}$.

First, we can see that in the *randomly* oriented PMF case, the $C_{\ell,r}^{BB}$ spectra have white-noise like shape with total power proportional to the current PMF intensity (e. g. at $\ell = 100$, $\ell \cdot (\ell+1) \cdot C_{\ell,r}^{BB}/2\pi = 6.0 \cdot 10^{-4}$ $\mu{\rm K}^2_{\rm CMB}$ for $B_0 = 1$ nG and $\ell \cdot (\ell+1) \cdot C_{\ell,r}^{BB}/2\pi = 5.2 \cdot 10^{-2}$ $\mu{\rm K}^2_{\rm CMB}$ for $B_0 = 10$ nG). In the case of a PMF oriented parallel to the line-of-sight, the $C_{\ell,c}^{BB}$ spectra inherit the shape of the $C_\ell^{\rm EE}$ spectra with total power also proportional to the current intensity of the PMF (e.g. at $\ell = 100$, $\ell \cdot (\ell+1) \cdot C_{\ell,c}^{BB}/2\pi = 2.4 \cdot 10^{-3}$ $\mu\mathrm{K}^2_{\mathrm{CMB}}$ for $B_0 = 1$ nG and $\ell \cdot (\ell + 1) \cdot C_{\ell,c}^{BB}/2\pi = 0.21$ $\mu\mathrm{K}^2_{\mathrm{CMB}}$ for $B_0 = 10$ nG). Nevertheless we can notice that the the maximum of the power in this case is not significantly larger than in the *randomly* oriented case.

Finally, we can compare the C_ℓ^{BB} spectra produced by PMF Faraday rotation to the C_ℓ^{BB} spectra produced by tensor perturbations. We observe that at 30 GHz and for PMF today intensities of the order of a few nG, the total power around $\ell = 100$ is comparable to that coming from tensor perturbations with r around 0.1.

We do not present in here how Faraday rotation by a PMF modifies the C_ℓ^{EE} and C_ℓ^{TE} power spectra as the effect is not as obvious as for C_ℓ^{BB}. However, as the CMB C_ℓ^{TE} power spectrum has been precisely measured at large angular scales by WMAP, we use it in the following section to set upper limits on the current intensity of the PMF.

5. COMPARISON TO THE WMAP DATA

The Faraday rotation of the CMB polarization by a PMF is of cosmological interest. On the one hand, it could be considered a contaminant to the detection of primordial B polarization modes produced by tensor perturbations. On the other hand, it provides a unique way to detect a PMF which may be considered as an additional cosmological parameter.

The WMAP satellite all-sky observations (Page et al. 2007) provide the most precise measurements of the CMB polarization angular power spectra at large angular scales up-to-date. In particular, WMAP has accurately measured the C_ℓ^{TE} power spectrum in a wide range of multipoles from $\ell = 2$ to $\ell = 400$. Currently, the C_ℓ^{EE} and C_ℓ^{BB} power spectra are dominated by instrumental noise. The WMAP C_ℓ^{TE} power spectrum is plotted in Figure 3 for each multipole ℓ as black crosses with their associated error bars.

To constrain the intensity of the PMF using the WMAP data, we have simulated, using the technique described in this paper, PMF Faraday rotated CMB maps. We have considered different values of the PMF amplitude ranging from 1 to 120 nG with a 2 nG step. We have then computed the C_ℓ^{TE} angular power spectrum by averaging 100 independent simulations. The simulations have been performed at the two observation frequency bands, Q and V (41 and 61 GHz), used by the WMAP team to estimate the C_ℓ^{TE} angular power spectrum (Page et al. 2007). To be able to compare to the WMAP data, we have averaged the two simulated frequency-bands C_ℓ^{TE} power spectra and binned them in ℓ in

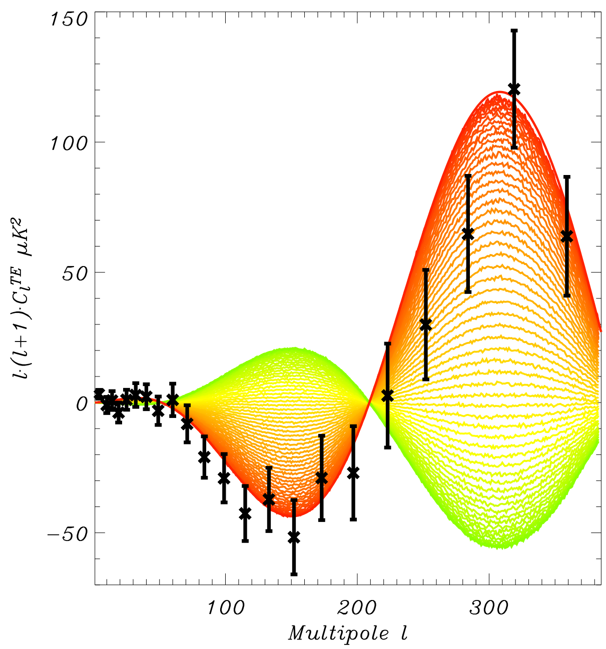

Fig. 3. C_ℓ^{TE} spectrum as measured by WMAP (*black*, Page et al. 2007), in $\mu\mathrm{K}^2_{\mathrm{CMB}}$. C_ℓ^{TE} spectra computed by our method, in the case of a *constant* PMF are also displayed for several current values of the PMF ranging between 1 nG (*red*) and 120 nG (*green*).

the same way that the WMAP data. The simulated C_ℓ^{TE} spectra for the different values of the PMF are overplotted in Figure 3 as colored solid lines.

Finally, we have obtained the best-fit model to the data by χ^2 minimisation. As we have not taken into account late reionization in our model the fit has been performed on the multipole bins in the range $15 < \ell < 300$. From this fitting procedure, we can set an upper limit on the PMF intensity of $B_0 < 27$ nG (95% CL) and $B_0 < 15$ nG (68% CL). The complete likelihood function is displayed in Figure 4.

6. SUMMARY AND CONCLUSIONS

We have developed a simple Monte Carlo technique to estimate the Faraday rotation of the CMB polarization induced by a PMF present at the time of decoupling and avoiding the full resolution of the photon evolution equations. This technique is based on two main hypotheses: tight coupling and weak PMF. The latter is justified by observations if we consider the PMF as the seed of current galactic magnetic fields of the order of 1 μG and therefore with amplitudes of the order of few nG. Under these two hypotheses and in the case of scalar perturbations, Faraday rotation by a PMF transforms a fraction of the CMB E modes into B modes. The Fara-

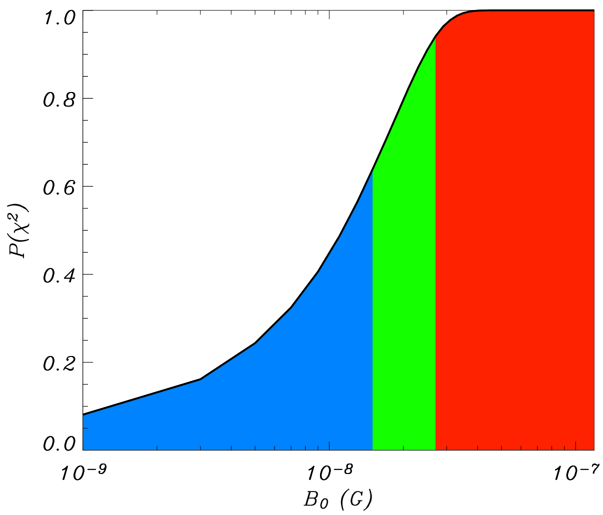

Fig. 4. Cummulative $P(\chi^2)$ of the fit of the $C_\ell^{\rm TE}$ spectrum as measured by WMAP (Page et al. 2007) for several current values of the PMF ranging between 1 nG and 120 nG.

day rotation effect depends both on the PMF intensity and geometry, and on the frequency of observation with a ν^{-2} dependence.

We have compared the results obtained using our technique to the analytical resolution of the Boltzmann equations proposed by Scóccola et al. (2004). For this we concentrated in the $C_\ell^{\rm BB}$ power spectrum although the other polarized power spectra are also modified. A direct comparison is difficult as the orientation of the PMF plays a major role both in the shape and amplitude of the $C_\ell^{\rm BB}$ power spectrum. However, our results qualitatively agree with the analytical ones. Furthermore, we observe that ordered PMF preserve the structure of the primordial $C_\ell^{\rm EE}$ power spectrum while random ones wipe the structure out.

We have observed that both for random and ordered PMF the amplitude of the resulting $C_\ell^{\rm BB}$ power spectrum around $\ell = 100$, for observations at 30 GHz and for reasonable values of the PMF intensity of few nG, is comparable to the that expected from primordial tensor perturbations assuming a tensor to scalar ratio $r \sim 0.1$. We then conclude that Faraday rotation of the CMB photons by a PMF can be observed in future CMB polarization experiments like Planck and is of cosmological interest. Indeed, it would provide a way to measure the PMF amplitude and orientation. Furthermore, it could be a contaminant to the detection of primordial gravitational waves from the analysis of the $C_\ell^{\rm BB}$ CMB power spectrum. As Faraday rotation presents a clear dependence with observation frequency in ν^{-2}, component separation methods in polarization (e.g Aumont & Macías-Pérez 2007) could be used to separate both effects.

Finally, we have inferred an upper limit on the PMF intensity, $B_0 \lesssim 27$ nG (95 % CL), by comparing our simulations of the modified $C_\ell^{\rm TE}$ spectra to the WMAP $C_\ell^{\rm TE}$ data. To date, upper limits on the PMF intensity have been set only by a few works (e.g. Barrow et al. 1997; Yamazaki et al. 2006) and ours is the first one arising from the CMB polarization study.

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