

MOLECULAR CLOUD FRAGMENTATION DRIVEN BY GRAVITY, AMBIPOLAR DIFFUSION, AND NONLINEAR FLOWS: THREE-DIMENSIONAL SIMULATIONS

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RESUMEN

Empleamos la primera simulación completamente en tres dimensiones para estudiar el papel de los campos magnéticos y la fricción ion-neutro en la regulación de la fragmentación inducida gravitacionalmente en las nubes moleculares. Los núcleos en una nube inicialmente subcrítica evolucionan en una escala de tiempo de la difusión ambipolar, mientras que los núcleos en una nube inicialmente supercrítica se desarrollan en un tiempo dinámico. Encontramos que una medición instantánea de la relación entre la densidad (ρ) y la magnitud del campo magnético (B) en diferentes puntos espaciales de la nube, coinciden con el historial evolutivo de un núcleo individual. Cuando la densidad se vuelve grande, la relación tiende a ser $B \propto \rho^{0.5}$. También hemos demostrado que la formación de núcleos colapsantes en nubes subcríticas es acelerada por flujos supersónicos no lineales. Aunque el tiempo para la formación de un núcleo en nubes subcríticas se estima normalmente en unas cuantas veces 10^7 años, encontramos que se acorta aproximadamente varias veces 10^6 años por los flujos supersónicos. El resultado es consistente con simulaciones previas en dos dimensiones.

ABSTRACT

We employ the first fully three-dimensional simulation to study the role of magnetic fields and ion-neutral friction in regulating gravitationally driven fragmentation of molecular clouds. The cores in an initially subcritical cloud develop gradually over an ambipolar diffusion time while the cores in an initially supercritical cloud develop in a dynamical time. We found that a snapshot of the relation between density (ρ) and the strength of the magnetic field (B) at different spatial points of the cloud coincides with the evolutionary track of an individual core. When the density becomes large, both the relations tend to $B \propto \rho^{0.5}$. We also have demonstrated that the formation of collapsing cores in subcritical clouds is accelerated by the supersonic nonlinear flows. Although the time-scale of the core formation in subcritical clouds is normally estimated to be a few $\times 10^7$ years, we found that it is shortened to approximately several $\times 10^6$ years by the supersonic flows. The result is consistent with previous two-dimensional simulations.

Key Words: diffusion — ISM: clouds — ISM: magnetic fields — MHD — turbulence

1. INTRODUCTION

Magnetic fields are one of the important components in molecular clouds for the early stage of star formation. In particular, the relative strength of the magnetic field to that of the gravitational field is an important parameter. When the ratio of mass to magnetic flux is greater than a critical value, the cloud is fragmented by gravitational instability in a dynamical time. Such a cloud is called “supercritical.” On the other hand, when the mass-to-flux ratio is less than the critical value, a cloud obeying magnetic flux-freezing is gravitationally stable because the magnetic force can prevent the contraction of the

cloud (Nakano & Nakamura 1978). Such a cloud is called “subcritical.” However, subcritical clouds can also experience fragmentation through the magnetic diffusion induced by neutral-ion drift (ambipolar diffusion). Due to this effect, their fragmentation develops gradually over the ambipolar diffusion time, which can be ~ 10 times longer than the dynamical time for canonical values of cosmic-ray induced cloud ionization (Ciolek & Basu 2006).

Indebetouw & Zweibel (2000) and Basu & Ciolek (2004) carried out two-dimensional simulations of magnetized sheets in the thin-disk approximation, which are threaded by an initially perpendicular magnetic field. Starting with slightly subcritical or critical initial conditions, they followed the gravitational fragmentation of the cloud by inserting small perturbations into the cloud. They found that fragmentation regulated by the ambipolar diffusion oc-

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curred on a time scale intermediate between the dynamical time associated with supercritical collapse and the ambipolar diffusion time-scale associated with highly subcritical clouds.

Li & Nakamura (2004) and Nakamura & Li (2005) have studied fragmentation by inserting supersonic perturbations into the cloud. They performed two-dimensional simulations in the thin-disk approximation and showed that the timescale of cloud fragmentation is reduced by supersonic turbulence. Such a model can explain both relatively rapid star formation and the relatively low star formation efficiency in molecular clouds that is not well explained if star formation starts from a supercritical cloud.

In this paper, we study the three-dimensional extension of models such as those of Indebetouw & Zweibel (2000), Basu & Ciolek (2004), Li & Nakamura (2004), and Nakamura & Li (2005). The self-consistent calculation of the vertical structure of the cloud allows us to test the predictions of two-dimensional models as well as to make some new predictions. We model clouds that are either decidedly supercritical or subcritical and study the evolution after the introduction of small-amplitude or large-amplitude (supersonic) perturbations.

2. NUMERICAL METHODS

We solve the three-dimensional magnetohydrodynamic (MHD) equations including self-gravity and ambipolar diffusion, assuming that neutrals are much more numerous than ions. Instead of solving a detailed energy equation, we assume isothermality for each Lagrangian fluid particle (Kudoh & Basu 2003, 2006). For the neutral-ion collision time and associated quantities, we follow Basu & Mouschovias (1994). The basic equations are summarized in Kudoh et al. (2007).

As an initial condition, we assume hydrostatic equilibrium of a self-gravitating one-dimensional cloud along the z -direction in a Cartesian coordinate system (x, y, z) . Though nearly isothermal, a molecular cloud is usually surrounded by warm material, such as neutral hydrogen gas. We also assume that the initial magnetic field is uniform along the z -direction. The detail of the initial condition is described in Kudoh et al. (2007).

A set of fundamental units for this problem are c_{s0} , H_0 , and ρ_0 , where c_{s0} is the initial sound speed and ρ_0 is the initial density at $z = 0$, respectively, and $H_0 = c_{s0}/\sqrt{2\pi G\rho_0}$. These yield a time unit $t_0 = H_0/c_{s0}$. The initial magnetic field strength (B_0)

introduces a dimensionless free parameter

$$\beta_0 \equiv \frac{8\pi p_0}{B_0^2} = \frac{8\pi\rho_0 c_{s0}^2}{B_0^2} = 2 \frac{c_{s0}^2}{V_{A0}^2}, \quad (1)$$

the ratio of gas to magnetic pressure at $z = 0$. In the above relation, we have also used $V_{A0} \equiv B_0/\sqrt{4\pi\rho_0}$, the initial Alfvén speed at $z = 0$. In the sheet-like equilibrium cloud with a vertical magnetic field, β_0 is related to the mass-to-flux ratio for Spitzer's self-gravitating layer. The mass-to-flux ratio normalized to the critical value is

$$\mu_S \equiv 2\pi G^{1/2} \frac{\Sigma_S}{B_0}, \quad (2)$$

where $\Sigma_S = 2\rho_0 H_0$ is the column density of the Spitzer cloud. Therefore,

$$\beta_0 = \mu_S^2. \quad (3)$$

Although the initial cloud we used is not exactly the same as the Spitzer cloud, β_0 is a good indicator of whether or not the magnetic field can prevent gravitational instability. Dimensional values of all quantities can be found through a choice of ρ_0 and c_{s0} . For example, for $c_{s0} = 0.2 \text{ km s}^{-1}$ and $n_0 = \rho_0/m_n = 10^4 \text{ cm}^{-3}$, we get $H_0 = 0.05 \text{ pc}$, $t_0 = 2.5 \times 10^5 \text{ yr}$, and $B_0 = 40 \mu\text{G}$ if $\beta_0 = 0.25$.

The level of magnetic coupling in the partially ionized gas is characterized by numerical values of the ion number density n_i and neutral-ion collision timescale τ_{ni} . From equations (8) and (9) of Kudoh et al. (2007), and using standard values of parameters in that paper as well as the values of units used above, we find an initial midplane ionization fraction $x_{i,0} = n_{i,0}/n_0 = 9.5 \times 10^{-8}$ and a corresponding neutral-ion collision time $\tau_{ni,0} = 0.11 t_0$. The ionization fraction x_i and timescale τ_{ni} at other densities can be found from the initial midplane values given that they both scale $\propto \rho^{-1/2}$ (Elmegreen 1979).

In this equilibrium sheet-like gas layer, we input a linear or nonlinear (supersonic) perturbation to v_x and v_y at each grid point. Independent realizations of v_x and v_y are generated. The rms value of the initial velocity perturbation in physical space, v_a , is about $0.1c_{s0}$ for linear perturbations, and $3c_{s0}$ for nonlinear perturbations, so that $v_a \simeq V_{A0}$ for $\beta_0 = 0.25$ as well.

The method of solution and boundary conditions are described by Kudoh et al. (2007) (see also Kudoh, Matsumoto, & Shibata 1999; Ogata et al. 2004). The computational region is $|x|, |y| \leq 8\pi H_0$ and $0 \leq z \leq 4H_0$, with a number of grid points for each direction $(N_x, N_y, N_z) = (64, 64, 40)$.

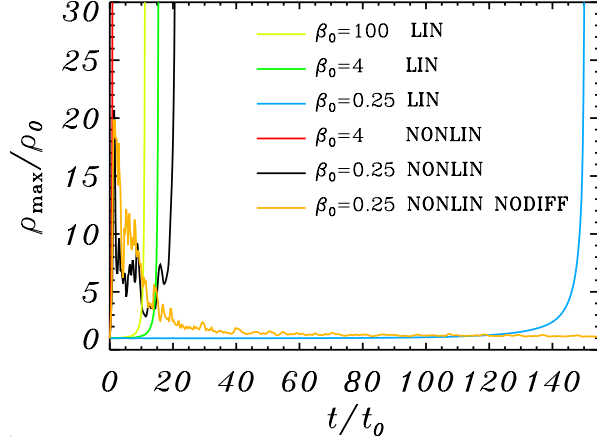


Fig. 1. Time evolution of maximum densities at $z = 0$. The blue line ($\beta_0 = 0.25$) and the green line ($\beta_0 = 4$) show the evolution for models with a linear initial perturbation. The black line ($\beta_0 = 0.25$) and the red line ($\beta_0 = 4$) shows the evolution for an initially nonlinear supersonic perturbation. The orange line shows the evolution for an initially nonlinear supersonic perturbation and $\beta_0 = 0.25$, but without ambipolar diffusion.

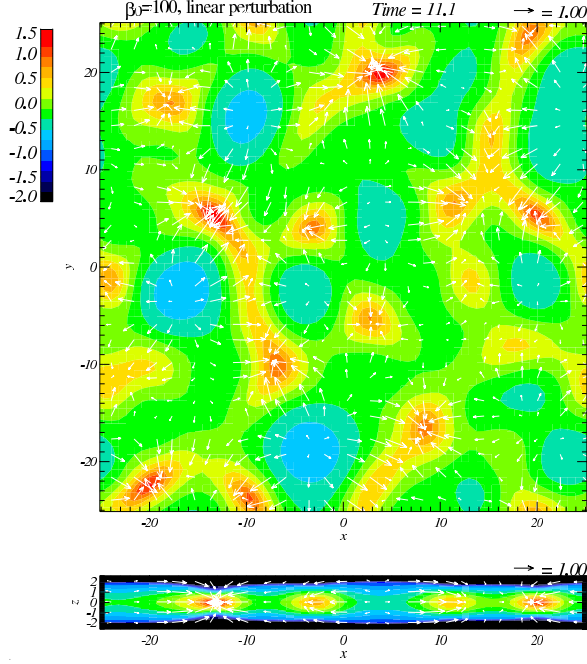


Fig. 2. The logarithmic density contours for $\beta_0 = 100$ at $t = 11.1 t_0$. Arrows show velocity vectors on each cross section. The upper panel shows the cross section at $z = 0$, and the lower panel shows the cross section at $y = 5.1 H_0$.

3. RESULTS

We summarize and expand upon results presented by Kudoh et al. (2007) and Kudoh & Basu (2008).

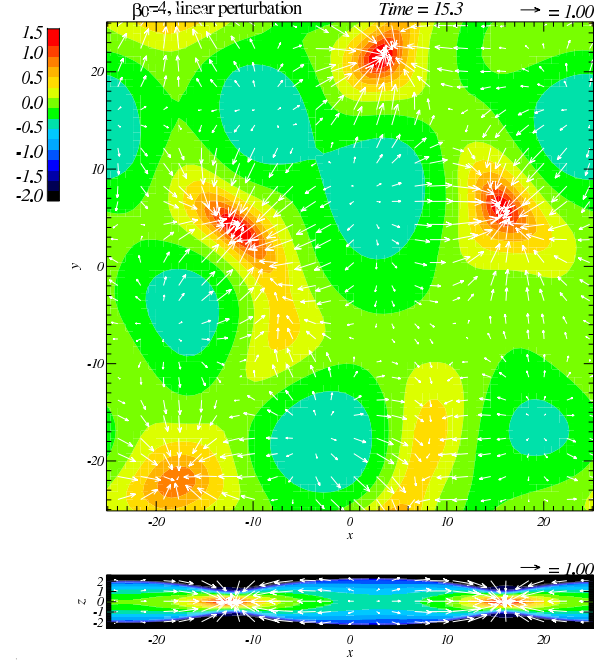


Fig. 3. Logarithmic density contours at $t = 15.3 t_0$ for $\beta_0 = 4$ and the linear perturbation case. Arrows show velocity vectors on each cross section. The top panel shows the cross section at $z = 0$, and the bottom panel shows the cross section at $y = 5.1 H_0$.

3.1. Linear perturbation studies

Figure 1 shows the time evolution of the maximum density $\rho_{\max}$ at $z = 0$. The blue line ($\beta_0 = 0.25$) and the green line ($\beta_0 = 4$) show the evolution for models with a linear initial perturbation of $v_a \simeq 0.1c_{s0}$. When β_0 is 100 or 4, the magnetic field is not strong enough to suppress the self-gravitational instability of the cloud. In these cases, the density evolves rapidly, over the sound-crossing time of the most unstable wavelength ($\sim 4\pi H_0$). However, when $\beta_0 = 0.25$, the cloud is self-gravitationally stable unless neutral-ion slip is present. Therefore, the density evolves gradually over the diffusion time of the magnetic field. According to the two-dimensional linear analysis by Ciolek & Basu (2006), the evolutionary time scale of a significantly subcritical cloud is about ten times longer than the dynamical time, for a standard ionization fraction, as used here. Our numerical result is consistent with their analysis.

Figures 2, 3, and 4 show the logarithmic density contours for linear perturbation cases of $\beta_0 = 100$ at $t = 11.1 t_0$, $\beta_0 = 4$ at $t = 15.3 t_0$, and $\beta_0 = 0.25$ at $t = 150 t_0$, and respectively. Each upper panel shows the cross section at $z = 0$, and the lower panel shows the cross section at $y = 5.1 H_0$ for $\beta_0 = 100$, $y = 5.1 H_0$ for $\beta_0 = 4$, and $y = 4.3 H_0$ for $\beta_0 = 0.25$

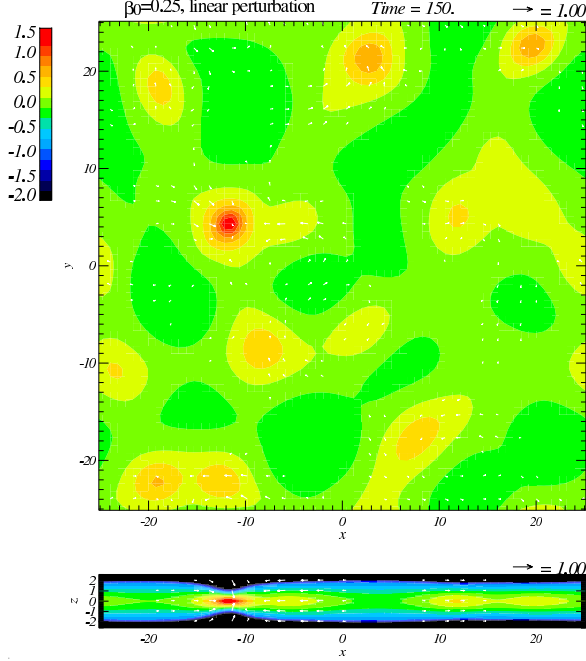


Fig. 4. Logarithmic density contours at $t = 150 t_0$ for $\beta_0 = 0.25$ and the linear perturbation case. Arrows show velocity vectors on each cross section. Upper panel shows the cross section at $z = 0$, and the bottom panel shows the cross section at $y = 4.3 H_0$.

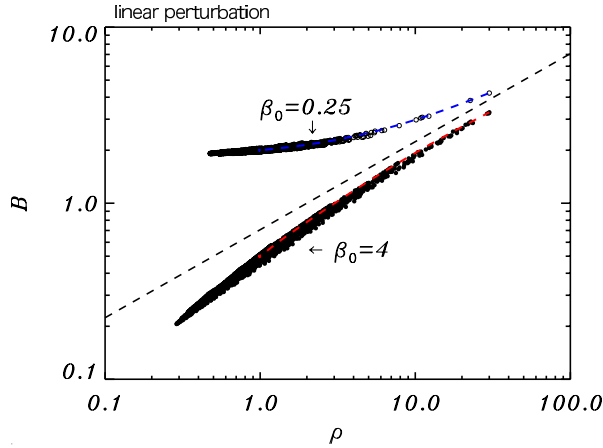


Fig. 5. Open circles show the magnetic field strength as a function of density along $z = 0$ at $t = 150 t_0$ for the model with $\beta_0 = 0.25$. The strength of the magnetic field is normalized by $\sqrt{8\pi\rho_0 c_{s0}^2}$. Filled circles are the same for $\beta_0 = 4$, at $t = 15.3 t_0$. The blue line shows the evolutionary track of the point at which the density achieves its maximum value for the model with $\beta_0 = 0.25$. The red line is the same for $\beta_0 = 4$.

respectively. The values of y for the lower panels are chosen so that the vertical cut passes through

at least one dense core. (In these numerical simulations, we use the term “core” to refer to the region where the density is greater than the mean background density by about a factor of 3.) The size of cores for $\beta_0 = 4$ is bigger than that for $\beta_0 = 100$. The size becomes smaller again when the magnetic field is stronger than critical ($\beta_0 = 0.25$). This result is consistent with the two-dimensional linear analysis of Ciolek & Basu (2006), who found a hybrid mode for critical or mildly supercritical clouds in which the combined effect of field-line dragging and magnetic restoring forces enforce a larger than usual fragmentation scale. Arrows show velocity vectors on each cross section. Maximum velocities become supersonic for $\beta_0 = 4$ and $\beta_0 = 100$, but remain subsonic for $\beta_0 = 0.25$. This is also consistent with the two-dimensional numerical simulations of Basu & Ciolek (2004).

Figure 5 shows the relation between density and magnetic field on the plane $z = 0$. The strength of the magnetic field is normalized by $\sqrt{8\pi\rho_0 c_{s0}^2}$. Open circles show the magnetic field strength as a function of density along $z = 0$ at $t = 150 t_0$ for the model with $\beta_0 = 0.25$ (see Figure 4). Filled circles are the same for $\beta_0 = 4$, at $t = 15.3 t_0$ (see Figure 3). The blue line shows the evolutionary track of the point at which the density achieves its maximum value for the model with $\beta_0 = 0.25$. The red line is the same but for $\beta_0 = 4$. This figure shows that *the snapshot of the relation between density and magnetic field at different spatial points in the midplane of the cloud overlaps with the evolutionary track of an individual core*. The dashed line shows $B \propto \rho^{0.5}$. When the density becomes large, each relation approximately tends to $B \propto \rho^{0.5}$. In the case of $\beta_0 = 0.25$, the relation shows that core initially evolves to greater density without increasing the magnetic field strength. This is caused by the slip of neutral gas through the magnetic field during the subcritical phase of evolution.

3.2. Nonlinear perturbation studies

In Figure 1, the black line ($\beta_0 = 0.25$) and the red line ($\beta_0 = 4$) show the evolution for models with a nonlinear initial perturbation of $v_a \simeq 3c_{s0}$. This figure shows that the timescale of collapsing core formation for the nonlinear perturbation case is much shorter than that for the linear perturbation case, when β_0 is the same. Even when the initial cloud is subcritical ($\beta_0 = 0.25$), the core formation occurs on almost the same timescale as that of the supercritical ($\beta_0 = 4$) linear perturbation case. The orange line ($\beta_0 = 0.25$) shows the model with a nonlinear initial

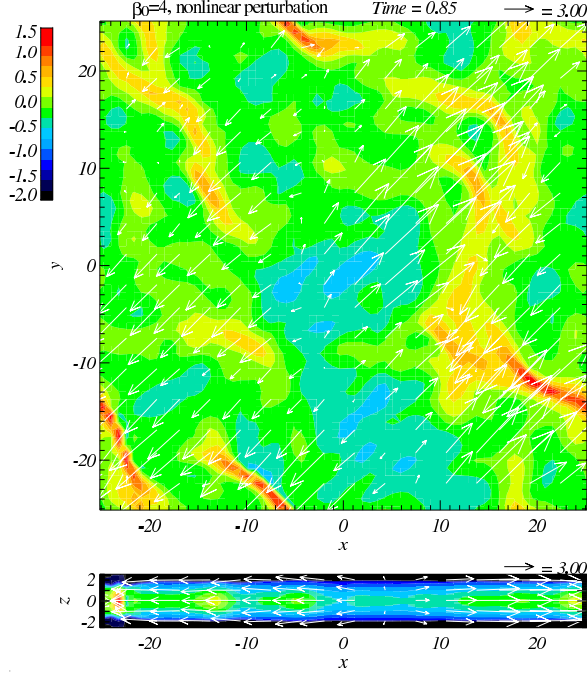


Fig. 6. Logarithmic density image at $t = 0.85 t_0$ for $\beta_0 = 4$ and the nonlinear perturbation case. Arrows show velocity vectors on each cross section. The top panel shows the cross section at $z = 0$, and the bottom panel shows the $x - z$ cross section at $y = -17.7 H_0$.

perturbation of $v_a \simeq 3c_{s0}$, but without ambipolar diffusion. It clearly shows that the collapsing core formation does not happen without ambipolar diffusion, when the cloud is subcritical.

Figures 6 and 7 show an image of the logarithmic density for nonlinear perturbation cases of $\beta_0 = 4$ at $t = 0.85 t_0$ and $\beta_0 = 0.25$ at $t = 20.5 t_0$. The top panel shows the cross section at $z = 0$, and the bottom panel shows the cross section at $y = -17.7 H_0$ for $\beta_0 = 4$ and $y = -5.9 H_0$ for $\beta_0 = 0.25$. The value of y for the bottom panel is chosen so that the vertical cut passes through the maximum density point. When the initial cloud is supercritical ($\beta_0 = 4$) and the perturbation is supersonic, the collapsing core formation happens quickly, at $t \simeq 0.85 t_0$, from the initial flow. This may be *too* rapid to agree with observations. In the case of the initial subcritical cloud ($\beta_0 = 0.25$), a collapsing core is located in the vicinity of $x = -20 H_0, y = -5 H_0$. The size of the core is similar to that created by linear initial perturbations (see Figure 4), although the shape is notably less circular.

Figure 8 shows the time evolution of the maximum value of density at $z = 0$ ($\rho_{\max}$) and β at the location of maximum density ($\beta_{\rho_{\max}}$) for the case

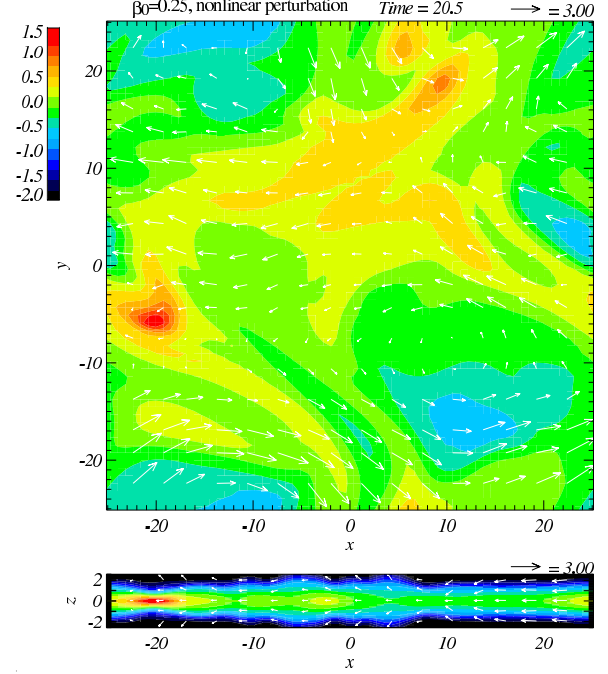


Fig. 7. Logarithmic density image at $t = 20.5 t_0$ for $\beta_0 = 0.25$ and the nonlinear perturbation case. Arrows show velocity vectors on each cross section. The top panel shows the cross section at $z = 0$, and the bottom panel shows the $x - z$ cross section at $y = -5.9 H_0$.

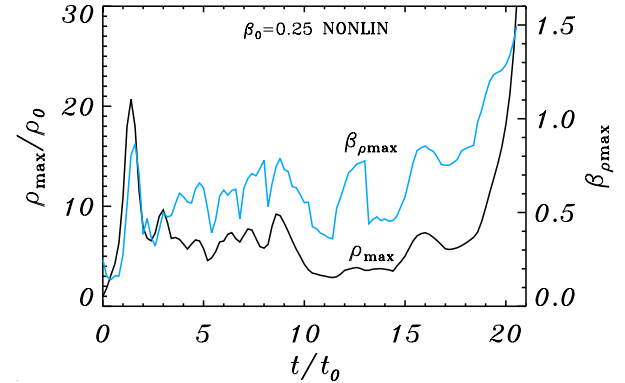


Fig. 8. Evolution of the maximum density (black line) at $z = 0$ of the simulation box for the model with $\beta_0 = 0.25$ and nonlinear initial perturbation, and evolution of β at the location of maximum density (blue line).

of $\beta_0 = 0.25$ with nonlinear perturbation. At first, $\beta_{\rho_{\max}}$ increases rapidly up to ~ 0.9 due to rapid ambipolar diffusion in the highly compressed regions caused by the initial supersonic perturbation. However, there is enough stored magnetic energy in the compressed region that it rebounds and starts oscillations, with $\beta_{\rho_{\max}}$ around 0.7 and increasing grad-

ually. Eventually, $\beta_{\rho_{\max}}$ becomes > 1 and the dense region collapses to form a core. This figure implies that β is a good indicator to see whether a subregion of the cloud is supercritical or not. The evolution of $\rho_{\max}$ confirms that there is an initial compression followed by a rebound to a lower density (still greater than the initial background value) and subsequent oscillations until a runaway collapse starts when continuing ambipolar diffusion has created a region with $\beta > 1$.

4. SUMMARY AND DISCUSSION

We have studied fragmentation of a sheet-like self-gravitating cloud by three-dimensional MHD simulations. The main results are as follows.

- We confirmed that in the case of an initially subcritical cloud ($\beta_0 = 0.25$), cores developed gradually over an ambipolar diffusion time when a small perturbation is input, while the cores in an initially supercritical cloud ($\beta_0 = 4$ or $\beta_0 = 100$) developed in a dynamical time.

- In the $B - \rho$ plane, the snapshot of the relation between magnetic field strength (B) and density (ρ) at different spatial points of the cloud overlaps with the evolutionary track of an individual core. When the density becomes large, each relation approximately tends to $B \propto \rho^{0.5}$.

- The supersonic nonlinear flows significantly reduce the timescale of collapsing core formation in subcritical clouds. It is of order several $\times 10^6$ years for typical parameters, or ~ 10 times less than found in the linear initial perturbation studies.

To see how accelerated ambipolar diffusion can occur, we consider the magnetic induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \nabla \times \left[\frac{\tau_{\text{ni}}}{c\rho} (\mathbf{j} \times \mathbf{B}) \times \mathbf{B} \right], \quad (4)$$

where $\mathbf{v}$ is the velocity, $\mathbf{B}$ is the magnetic field, and $\mathbf{j} = (c/4\pi)\nabla \times \mathbf{B}$ is the electric current density. Our assumption of ionization balance ($n_i \propto n^{1/2}$) can be used to estimate the diffusion time $\tau_d \propto \rho^{3/2} L^2 / B^2$, where L is the gradient length scale introduced by the initial turbulent compression and B is the magnetic field strength. Because the compression by the nonlinear flow is nearly one-dimensional, the magnetic field scales roughly as $B \propto L^{-1}$ within the flux freezing approximation. If the compression is rapid enough that vertical hydrostatic equilibrium cannot be established (unlike in previous calculations using the thin-disk approximation), then $\rho \propto L^{-1}$ as well (i.e., one-dimensional contraction without vertical settling), and $\tau_d \propto L^{5/2}$. This means that diffusion can occur quickly (and lead to a rapidly rising

value of β) if the turbulent compression creates small values of L . If diffusion is so effective during the first turbulent compression that a dense region becomes magnetically supercritical, then it will evolve directly into collapse. Alternately, the stored magnetic energy of the compressed (and still subcritical) region may lead to a reexpansion of the dense region. The timescale for this, in the flux-freezing limit, is the Alfvén time $\tau_A \propto L\rho^{1/2}/B$, which scales $\propto L^{3/2}$ for the above conditions. Thus, τ_d decreases more rapidly than τ_A , and sufficiently small turbulent-generated values of L may lead to enough magnetic diffusion that collapse occurs before any reexpansion can occur. See Elmegreen (2007) for some similar discussion along these lines. Ultimately, whether or not reexpansion of the first compression can occur depends on the strength of the turbulent compression, mass-to-flux ratio of the initial cloud, and neutral-ion collision time.

If reexpansion of the initial compression does occur, as in the standard model presented in this paper, then there is enough time for the vertical structure to settle back to near-hydrostatic equilibrium, in which case $B \propto \rho^{1/2}$. Since the compressed and reexpanded region executes oscillations about a new mean density, it is convenient to analyze the scalings in terms of the density ρ . The diffusion time now scales as $\tau_d \propto \rho^{-1/2}$. This yields a scaling of τ_d that is the traditionally used one (and is satisfied by design in the thin-disk approximation). However, the diffusion occurs more rapidly than it would in the initial state due to the elevated value of ρ in the compressed but oscillating region (see Figure 8).

REFERENCES

- Basu, S., & Mouschovias, T. Ch. 1994, ApJ, 432, 720
 Basu, S., & Ciolek, G. E. 2004, ApJ, 607, L39
 Ciolek, G. E., & Basu, S. 2006, ApJ, 652, 442
 Elmegreen, B. G. 1979, ApJ, 232, 729
 ———. 2007, ApJ, 668, 1064
 Indebetouw, R. M., & Zweibel, E. G. 2000, ApJ, 532, 361
 Kudoh, T., Matsumoto, R., & Shibata, K. 1999, Computational Fluid Dynamics Journal, 8, 56
 Kudoh, T., & Basu, S. 2003, ApJ, 595, 842
 ———. 2006, ApJ, 642, 270
 ———. 2008, ApJ, 679, 97
 Kudoh, T., Basu, S., Ogata, Y., & Yabe, T. 2007, MNRAS, 380, 499
 Li, Z.-Y., & Nakamura, F. 2004, ApJ, 609, L83
 Nakano, T., & Nakamura, T. 1978, PASJ, 30, 671
 Nakamura, F., & Li, Z.-Y. 2005, ApJ, 631, 411
 Ogata, Y., Yabe, T., Shibata, K., & Kudoh, T. 2004, International Journal of Computational Methods, 1, 201