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EFFECTS OF PRIMORDIAL MAGNETIC FIELDS ON THE ELECTROWEAK BARYOGENESIS PROCESS

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RESUMEN

Uno de los problemas más sobresalientes en cosmología es la asimetría existente entre materia y antimateria, es decir, el problema de la bariogénesis. El planteamiento más natural para la solución de este problema es considerar que esta asimetría es el resultado de un proceso dinámico en la evolución del universo temprano. Éste tendría que satisfacer las tres condiciones de Sakharov: (1) violación de número bariónico, (2) rompimiento de las simetrías C y CP y (3) condiciones fuera de equilibrio. El modelo estándar electrodébil satisface las tres condiciones, aunque sólo parcialmente, de manera que se muestra insuficiente para resolver el problema. Sin embargo, la idea de introducir campos magnéticos durante la evolución de la transición de fase electrodébil revive la posibilidad de desarrollar el proceso de bariogénesis en el modelo estándar. Los campos magnéticos parecen invadir todo el universo y, aunque su origen no está actualmente bien establecido, no se puede descartar su presencia en el universo temprano. Presentamos aquí los posibles efectos de campos magnéticos sobre el proceso de bariogénesis, en particular sobre la segunda y la tercera condiciones de Sakharov.

ABSTRACT

One of the outstanding problems in cosmology is the explanation of the asymmetry between matter and antimatter: the baryogenesis problem. The most natural approach to this problem is to consider that this asymmetry results from a dynamical process during the evolution of the early universe. A dynamical scenario for baryogenesis must satisfy the three Sakharov conditions: (1) baryon number violation, (2) breaking of the C and CP symmetries and (3) out of equilibrium conditions. Each of these conditions is satisfied in the electroweak standard model, although only partially, making the model insufficient to solve the problem. However, the idea to include magnetic fields during the development of the electroweak phase transition revives the possibility to embed baryogenesis in the standard model. Magnetic fields pervade the entire universe and although their origin is presently not well established, their presence in the early universe can certainly not be ruled out. We present here the possible effect of magnetic fields on the baryogenesis process, in particular, on the second and third Sakharov conditions.

Key Words: cosmology: theory — early universe — elementary particles — magnetic fields

1. INTRODUCTION

1.1. *Baryon asymmetry*

From the point of view of elementary particle physics, there is a symmetry between particles and antiparticles which suggests that their abundance should be the same. On the other hand, in the cosmological approach, in the hot early epoch of the universe evolution, particles and antiparticles are expected to be in thermal equilibrium with radiation, leading to a universe with a small, equal, amount of both species. However, the observed universe is made almost entirely of matter, with no traces of present or primordial antimatter (see e.g. Kolb & Turner 1990; Stecker 2002).

1.2. *Electroweak Baryogenesis*

Sakharov (1967) proposed a dynamical generation of the observed asymmetry, in which the universe, in an initial symmetric state, evolves to an asymmetric one. He established the three necessary conditions for this process:

- Baryonic number violation
- C and CP symmetries violation
- Out-of-equilibrium conditions

The standard model of weak interactions fulfills these three conditions during the electroweak phase transition (EWPT), although too weakly (Gavela et al. 1994; Kajantie et al. 1996).

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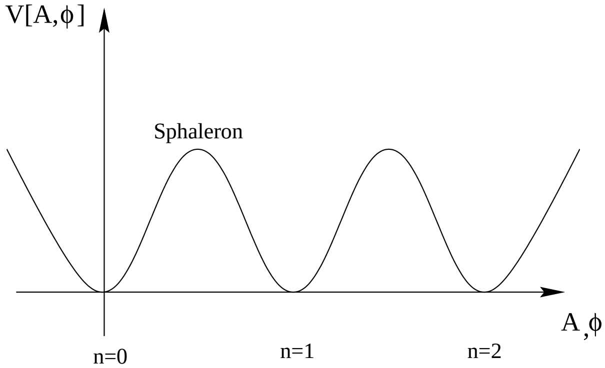

Fig. 1. The potential energy of the $SU(2)$ gauge field as a function of the winding number n (Chern–Simons number). The minima correspond to configurations with zero gauge field energy but different baryon number. A and ϕ are, respectively, the gauge and Higgs bosons of the theory.

• The transition between different topological vacua of the non-abelian $SU(2)$ theory (Klinkhamer & Manton 1984), depicted schematically in Figure 1, generates baryonic number. For this potential, there is an static and unstable solution, called *sphaleron*, of the field equations of the electroweak model, corresponding to the top of the energy barrier (the name is based on the classical Greek adjective meaning "ready to fall"). Transitions between vacua –that correspond to different *winding numbers* n– are associated with the violation of baryon (B) and lepton (L) numbers, with leptons and baryons produced with the same rate (B − L conservation) (t'Hooft 1976a,b).

• The violation of symmetries C and CP then gives a direction to the baryon number generation. In the standard model, C violation comes from the existence of vector and axial currents and CP violation in the CKM mass matrix, although they are too tiny to be at the origin of the present baryon asymmetry (Gavela et al. 1994).

• From CPT invariance, particles and antiparticles have the same mass and hence, in thermodynamical equilibrium, any asymmetry between matter and antimatter is forbidden. In a first order phase transition, the out-of-equilibrium conditions are provided by the discontinuous change in the order parameter –the vacuum expectation value of the Higgs field in this case– (Figure 2). The EWPT is indeed a first order transition, where true vacuum bubbles, in the broken phase, are nucleated in a background of false vacuum, which corresponds to the symmetric phase. Here again, the transition has been shown to be too weakly first order (Kajantie et al. 1996).

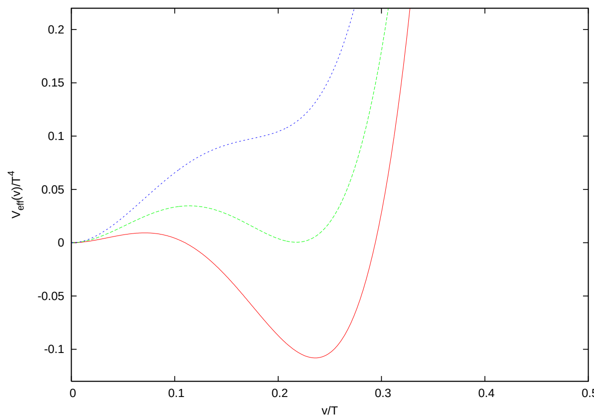

Fig. 2. Effective potential for a first order phase transition (scaled by T^4) as a function of the ratio of the vacuum expectation value of the Higgs field over the temperature. Decreasing the temperature (from the blue to the red line) causes $V_{\rm eff}$ to develop a secondary minimum that becomes degenerate with the original one at a critical temperature (green line).

1.3. *Asymmetry preservation*

Once the asymmetry has been generated, we still have the problem of preserving it from being erased in subsequent processes. To this aim, the rate of the universe expansion H must be greater than the rate of transition between distinct vacua $\Gamma_{\rm sph}$ in the sphaleron configuration:

$$\Gamma_{\rm sph} \propto \exp(-E_{\rm sph}(T)/T) < H \sim g_\star^{1/2} T^2 \,. \tag{1}$$

Notice that we impose this condition at the scale of the EWPT, since afterwards the transition between different topological vacua will be highly suppressed.

This condition leads to a bound for the vacuum expectation value of the Higgs field (vev) and the temperature of the phase transition (PT) (Shaposhnikov 1987):

$$\left(\frac{vev}{T_{\rm C}}\right) \geq 1.0 - 1.5 \,, \tag{2}$$

where the critical temperature $T_{\rm C}$ is taken as the one where the potential presents two degenerate vacua.

However, in the electroweak standard model the maximum value is obtained for a Higgs mass $m_H = 0$ and, even in this case, it is well below the desired value:

$$\left(\frac{vev}{T}\right)_{\rm MS} \leq 0.55 \,. \tag{3}$$

In such a way, if we want to embed the baryogenesis process in the standard model, we have to find the way to enhance this value.

1.4. *Magnetic/hypermagnetic fields*

For temperatures above the EWPT, the symmetry $SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}}$ is restored and the magnetic field corresponds to the group $U(1)_{\mathrm{Y}}$ with the hypercharge Y as the coupling constant. It takes the name of *hypermagnetic* field. An important feature of this field, that we will exploit for generating an asymmetry between the two phases during the PT, is that hypercharges of left and right handed fermions are different. On the other hand, the presence of magnetic fields can change the order of the PT, as it happens in a superconductor (Meissner effect).

The construction of a cosmological scenario with magnetic fields comes from the observation that they seem to be pervading the entire universe. They have been observed in galaxies, clusters, intracluster medium and high-redshift objects (see e.g. Kronberg 1994; Han & Wielebinski 2002). The origin of these fields is nowadays unknown and, although there are no direct detections of purely cosmic magnetic fields (i.e., not associated to gravitationally bounded structures), it is believed that they may be either primordial or associated to the process of structure formation. In the early universe, which is the case of interest here, there are a number of proposed mechanisms that could generate magnetic fields (for a review on the origin, evolution, and cosmological consequences of primordial magnetic fields, see e.g. Enqvist 1998; Grasso & Rubinstein 2001).

2. AXIAL ASYMMETRY BETWEEN THE TWO PHASES

2.1. *Coexistence of two phases*

As mentioned in § 1.2, during the PT, bubbles of true vacuum live in a background of false vacuum, separated by the bubble *wall*, and the Higgs field vacuum expectation value evolves from zero to some finite value, giving mass to all the particles of the theory. Since it is a tunneling process, this evolution is not well specified, but a simple solution can be used for the kink, in the thin wall approximation, with degenerate minima in the two phases:

$$v = 1 + \tanh(x) \,, \tag{4}$$

where the dimensionless position coordinate x is proportional to z, the direction normal to the bubble wall (Liu et al. 1992). We call v the variable vacuum expectation value for the Higgs field, in contrast with *vev*, that we devote to the vacuum expectation value of the true vacuum, when the PT is over.

2.2. *Movement equations for fermions*

Fermionic modes are coupled differently to the magnetic field in the symmetric and the broken phases.

In the broken phase, fermions obey the Dirac equation for a charged particle:

$$(i\partial\!\!\!/ - \; eA_\mu \gamma^\mu - m(z))\Psi = 0 \,, \tag{5}$$

where e is the fermion charge, $A^\mu = (0, \mathbf{A})$ is the four-vector potential, with null temporal component, in the reference system of the wall. The fermion mass $m(z)$ is proportional to the vacuum expectation value of the Higgs field. We have chosen the field in the z direction: $\mathbf{B} = B\hat{z}$.

In the symmetric phase, the coupling is chiral and we have to write down the Dirac equation for axial fermions, with a hypermagnetic field

$$\begin{aligned}
(i\partial\!\!\!/ - \frac{y_{\mathrm{L}}}{2} g' A\!\!\!/)\Psi_{\mathrm{L}} - m(z)\Psi_{\mathrm{R}} &= 0 \,, \\
(i\partial\!\!\!/ - \frac{y_{\mathrm{R}}}{2} g' A\!\!\!/)\Psi_{\mathrm{R}} - m(z)\Psi_{\mathrm{L}} &= 0 \,,
\end{aligned} \tag{6}$$

where $y_{\mathrm{R,L}}$ stands for right and left handed hypercharges, Ψ_{R} and Ψ_{L} are the right and left handed modes respectively for the spinor Ψ and g' is the coupling constant of $U(1)_{\mathrm{Y}}$.

2.3. *Generation of axial asymmetry*

The solutions Ψ for both equations were found, with analytical and numerical methods, for left and right modes, matching them in the bubble wall ($z = 0$), (Ayala et al. 2002; Piccinelli & Ayala 2004). Then, the reflection (R) and transmission (T) coefficients were derived, for energies near the potential barrier. They are defined as

$$\begin{aligned}
R_{l\to r} &= -J^r_{\mathrm{ref}}/J^l_{\mathrm{inc}} \,, \\
T_{l\to l} &= J^l_{\mathrm{tra}}/J^l_{\mathrm{inc}} \,,
\end{aligned} \tag{7}$$

for a left handed incident particle, and as

$$\begin{aligned}
R_{r\to l} &= -J^l_{\mathrm{ref}}/J^r_{\mathrm{inc}} \,, \\
T_{r\to r} &= J^r_{\mathrm{tra}}/J^r_{\mathrm{inc}} \,,
\end{aligned} \tag{8}$$

for the axial conjugated process, where $J = \Psi^\dagger \gamma^0 \gamma^3 \Psi$ is the current normal to the wall.

Figure 3 shows the coefficients $R_{l\to r}$ and $R_{r\to l}$ for a top quark (the heaviest particle in the broken phase: $m_0 = 175$GeV, and hence the one that has the largest Yukawa coupling) as a function of the magnetic field parameter $b = B/T^2$, for a temperature $T = 100$GeV (a temperature indicative of the

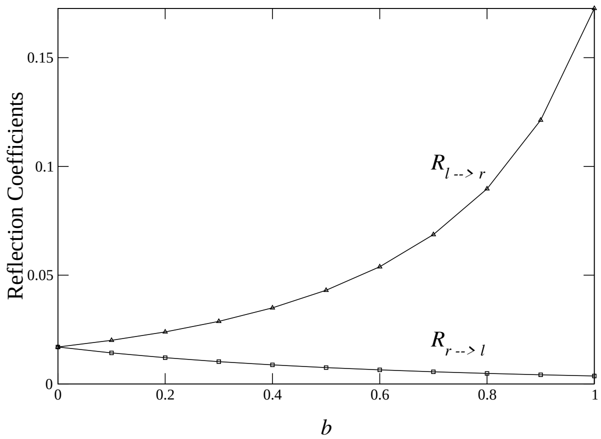

Fig. 3. Reflection coefficients $R_{l\to r}$ and $R_{r\to l}$ for a top quark, as a function of the magnetic field parameter $b = B/T^2$ for an energy slightly larger than the height of the barrier. The coupling constants take the values $y_{\rm R} = 4/3$, $y_{\rm L} = 1/3$ and $g' = 0.344$, as appropriate for the EWPT epoch. The dots represent the computed values.

EWPT scale) and a fixed energy slightly larger than the height of the barrier. The coupling constants take the values $y_{\rm R} = 4/3$, $y_{\rm L} = 1/3$ and $g' = 0.344$, as appropriate for the EWPT epoch. Notice that when $b \to 0$, these coefficients approach each other and that the difference grows with increasing field strength. A left handed particle is expected to be reflected as a right one and vice versa, although we find here some mixing effect due to the interaction with the energy barrier, with different coupling constants. Though not explicitly shown here, the antiparticle contribution has also been worked out, exploiting CPT invariance and unitarity, obtaining that the total effect is twice the one we are depicting here.

It is interesting to notice that, with this mechanism, we are not generating a net excess of one type of particle (left- or right- handed) over the other; we are merely generating an axial charge segregation between the two phases.

It has been shown (Nelson et al. 1992) that this axial asymmetry can be translated into a CP violation in the broken phase, giving a preferential direction in the sphaleron configuration.

3. ELECTROWEAK PHASE TRANSITION IN PRESENCE OF HYPERMAGNETIC FIELDS

3.1. *Temperature effective potential of the standard model*

We will analyse here another aspect of the baryogenesis process: the generation of out-of-equilibrium conditions required for developing the baryon asymmetry. As we mentioned in § 1.2, this depends on

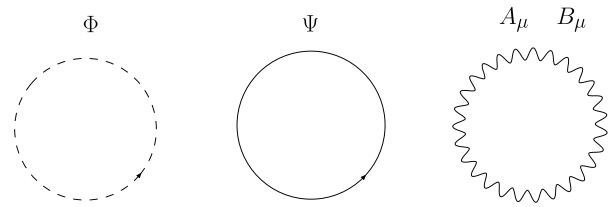

Fig. 4. 1-loop Feynman diagrams for Higgs bosons Φ, fermions Ψ and gauge bosons A_μ, B_μ.

how strongly first-order can the EWPT be, and we want to explore the influence that a primordial magnetic field can have on this process. To quantize this, we may resort to the analysis of the shape of the effective potential. This quantity considers all the quantum corrections of the theory but, unfortunately, it doesn't have a closed form, as the classical potential, and has to be analyzed order by order.

The effect of magnetic fields on the EWPT has been studied both classically (Giovannini & Shaposhnikov 1998) and to 1-loop order (Elmfors et al. 1998), as well as by means of lattice simulations (Kajantie et al. 1999). All these calculations agree that the strength of the PT is enhanced by the presence of hypermagnetic fields (see however Skalozub & Demchik 1999), although to different extents and with different restrictions. A general conclusion is that for high values of the Higgs mass, as dictated by present experimental bounds, the desired value for $vev/T_{\rm C}$ is not reached.

The Feynman diagrams for the construction of the effective potential to 1-loop, before introducing the external field, are depicted in Figure 4.

In this figure and the next ones, dotted lines represent Higgs fields Φ, solid lines, fermions Ψ, and wavy lines are for gauge bosons A_μ, B_μ.

The expression and the leading order terms for this quantity can be found in the literature (Le Bellac 1996; Carrington 1992).

Nonetheless, the next order, i.e., ring diagrams, has been shown to be necessary in order to have a self-consistent theory, both because it gives contributions of the same order and because it cancels some imaginary terms raised in the 1-loop part (Carrington 1992). These are depicted in Figure 5.

We calculated all the corrections due to the interaction with plasma (finite temperature) and to coupling to a constant hypermagnetic field, up to ring diagrams, in the weak field limit $yB \ll m^2 \ll T^2$, where m is a generic mass of the problem at the electroweak scale (Sánchez et al. 2007). Figure 6 shows the contributions to the effective potential of the standard model that are modified by the presence

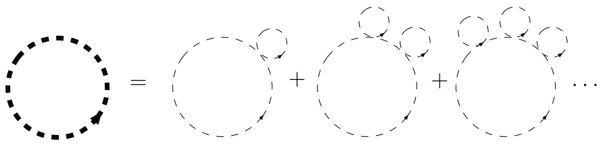

Fig. 5. Schematic representation of ring diagrams, that consist in the ressumation (thick line) of successive insertions of self-energies in vacuum bubbles.

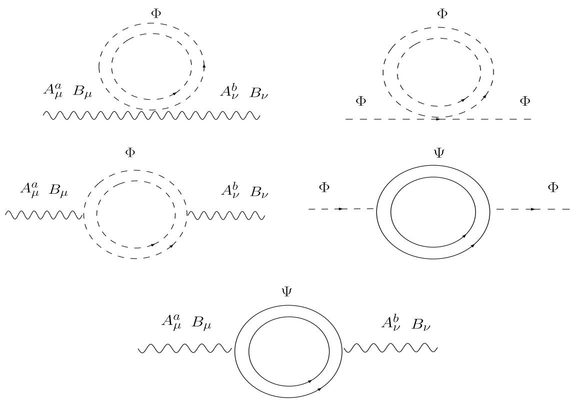

Fig. 6. Self-energy Feynman diagrams that contain loop particles affected by the hypermagnetic field. These particles are represented by double lines. In (a), (c) and (e), we have gauge bosons A_μ and B_μ with loops of Higgs Φ and fermions Ψ. In (b) and (d), we have Higgs bosons, with loops of Higgs and fermions.

of the external hypermagnetic field, represented by a double line. Gauge bosons do not couple to hypermagnetic fields, whereas fermions do, but they have negligible contributions at ring level.

3.2. *Symmetry breaking*

Since the Feynman diagrams calculations are rather involved, we only report here the effective potential shape. In Figure 7, we see the effective potential, at the same temperature, for three different strengths of the hypermagnetic field. It was determined summing the contributions of all the fields involved, at tree (classical), one-loop and ring level. We refer the interested reader to the original work (Sánchez et al. 2007).

In Figures 7 and 8, the coupling constants and masses involved in the effective potential take the appropriate values for the standard model. In particular, we consider a high value for the Higgs mass, in accordance with the present experimental bound: $m_{\rm H} \geq 116$GeV.

Comparing the three cases, we can see that the PT is delayed as the magnetic field strength is increased, thus enhancing the ratio $vev/T_{\rm C}$.

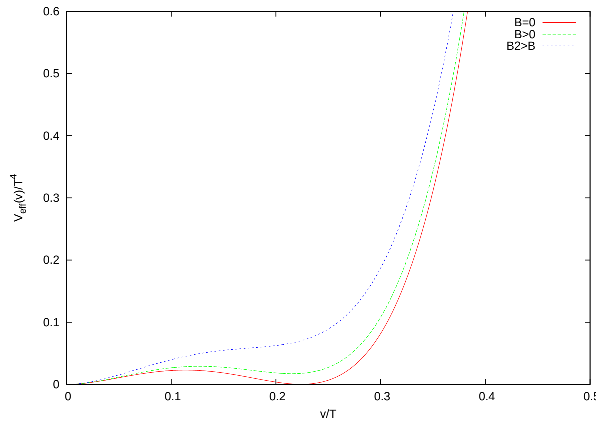

Fig. 7. $V_{\rm eff}$ as a function of v for constant T and different hypermagnetic field strengths. For higher values of B, the phase transition is delayed, favoring higher values of $vev/T_{\rm C}$.

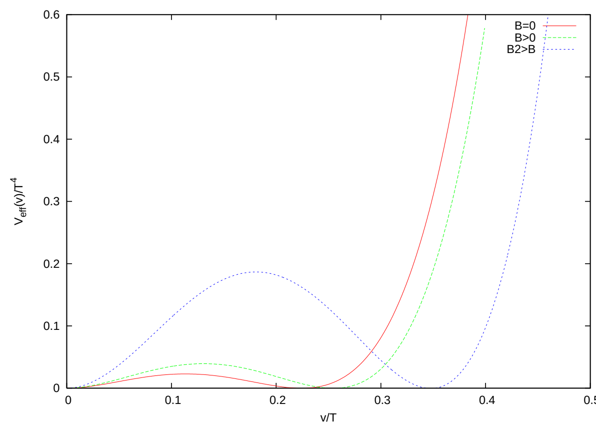

Fig. 8. $V_{\rm eff}$ as a function of v for different hypermagnetic field strengths, at their corresponding critical temperatures. When the intensity of the field is increased, the barrier between minima becomes higher and the ratio $vev/T_{\rm C}$ becomes larger.

On the other hand, if we draw the potential for $B = 0$ and $B \neq 0$, when the minima are degenerated for each case (i.e., different temperatures; see Figure 8), we can see directly that the barrier between the two vacua grows with the strength of the magnetic field, thus making the PT more strongly first-order. In Figure 8, the blue line corresponds to the highest magnetic field strength allowed by our hierarchy of scales $yB \ll m^2 \ll T^2$.

4. CONCLUSIONS AND PERSPECTIVES

Working in the weak field approximation and in the degrees of freedom of the symmetric phase, we have shown that the presence of hypermagnetic primordial fields contributes to satisfy two of the three basic ingredients for baryogenesis:

- It generates an axial asymmetry between the two phases, that results, in the broken phase, in a preferential direction for the transition between different topological vacua of the sphaleron that are associated to baryonic number violation, acting as a CP violation.
- It induces an EWPT more strongly first order, reducing the PT temparature and enhancing the ratio $vev/T_{\rm C}$, as the strength of the hypermagnetic field is enhanced. Nonetheless, for realistic values of the Higgs mass, the desired value for this ratio is not reached yet.

On the other hand, it should be noted that another consequence of the presence of a magnetic field is that the sphaleron bound becomes more restrictive due to the interaction between the sphaleron's magnetic dipole moment and the external field (Comelli et al. 1999).

Present and future work:

- To study the PT with the degrees of freedom of the broken phase and check consistency between the two scenarios (Tejeda-Yeomans et al. 2008)
- To study the rate of decay between the two vacua, in the presence of a (hyper)magnetic field
- To work with arbitrary magnetic field strength
- To explore the effect of the (hyper)magnetic field and finite temperature contributions on other processes of the early universe, as leptogenesis, inflation and dark energy.

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REFERENCES

Ayala, A., Piccinelli, G., & Pallares, G. 2002, Phys. Rev. D, 66, 103503
Carrington, M. E. 1992, Phys. Rev. D, 45, 2933
Comelli, D., Grasso, D., Pietroni, M., & Riotto, A. 1999, Phys. Lett. B, 458, 304
Elmfors, P., Enqvist, K., & Kainulainen, K. 1998, Phys. Lett. B, 440, 269
Enqvist, K. 1998, Int. J. Mod. Phys. D, 7, 331
Gavela, M. B., Hernandez, P., Orloff, J., & Pene, O. 1994, Mod. Phys. Lett. A, 9, 795
Giovannini, M., & Shaposhnikov, M. E. 1998, Phys. Rev. D, 57, 2186
Grasso, D., & Rubinstein, H. R. 2001, Phys. Rep., 348, 163
Han, J.-L., & Wielebinski, R. 2002, Chinese J. Astron. Astrop., 2, 293
Kajantie, K., Laine, M., Peisa, J., Rummukainen, K., & Shaposhnikov, M. 1999, Nuclear Phys. B, 544, 357
Kajantie, K., Laine, M., Rummukainen, K., & Shaposhnikov, M. 1996, Nuclear Phys. B, 466, 189
Klinkhamer, F. R., & Manton, N. S 1984, Phys. Rev. D, 30, 2212
Kolb, E. W., & Turner, M. S. 1990, The Early Universe (New York: Addison-Wesley)
Kronberg, P. P. 1994, Rep. Prog. Phys., 57, 325
Le Bellac, M. 1996, Thermal Field Theory (Cambridge: Cambridge Univ. Press)
Liu, B. H., McLerran, L., & Turok, N. 1992, Phys. Rev. D, 46, 2668
Nelson, A. E., Kaplan, D. B., & Cohen, A. G. 1992, Nuclear Phys. B, 373, 453
Piccinelli, G., & Ayala, A. 2004, Lect. Notes Phys., 646, 293
Sakharov, A. D. 1967, J. Exp. Theor. Phys. Lett., 5, 24
Sánchez, A., Ayala, A., & Piccinelli, G. 2007, Phys. Rev. D, 75, 043004
Shaposhnikov, M. E. 1987, Nuclear Phys. B, 287, 757
Skalozub, V., & Demchik, V. 1999, arXiv:hep-th/9912071
Stecker, F. W. 2002, arXiv:hep-ph/0207323
Tejeda-Yeomans, M. E., Navarro, J., Sánchez, A., Piccinelli, G., & Ayala, A. 2008, arXiv:0804.3433
t' Hooft, G. 1976, Phys. Rev. Lett., 37, 8
__________. 1976, Phys. Rev. D, 14, 3432