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THE MYTH OF THE IMF

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RESUMEN

El mito en la ciencia es la idea de que un fenómeno complejo en la naturaleza es reducible a un conjunto de ecuaciones basadas en las leyes fundamentales de la física. El mito de la IMF se refiere a la noción de que la distribución observada de las masas estelares al formarse (la IMF) puede y debe ser explicada por cualquier exitosa teoría de formación estelar propuesta. En esta contribución se argumenta que la IMF es el resultado de una evolución compleja del medio interestelar en galaxias y por ende, la IMF preserva muy poca o ninguna información sobre la física detallada de la formación estelar. El tratar de inferir la física de la formación estelar de la IMF sería como ¡si se quisiera intentar entender la personalidad de Beethoven del espectro de potencia de la Novena Sinfonía!

ABSTRACT

The Myth of Science is the idea that complex phenomena in Nature can be reduced to a set of equations based on the fundamental laws of physics. The Myth of the IMF is the notion that the observed distribution of stellar masses at birth (the IMF) can and must be explained by any successful theory of star formation. In this contribution I argue that the IMF is the result of the complex evolution of the interstellar medium in galaxies, and that as such the IMF preserves very little information, if any, about the detailed physics of star formation. Trying to infer the physics of star formation from the IMF is like trying to understand the personality of Beethoven from the power-spectrum of the Ninth Symphony!

Key Words: ISM: structure — stars: formation — stars: luminosity function, mass function

1. INTRODUCTION

The coincidence of the main characters of ancient myths across the earth indicates that these myths describe some universal beliefs of primitive human societies. While there is consensus that stars and planets were a critical factor in the genesis of old Egyptian, Greek, and Central and South American myths, some scholars have gone one (important) step further to claim that mythology was the technical language used by ancient civilizations to describe their experience of the starry skies. In particular, in *Hamlet's Mill* the historians Giorgio de Santillana and Hertha von Dechend claimed that the myths of different ancient civilizations in far apart places of the earth actually describe the precession of the equinoxes. This of course implies that the precession was already known several thousand years before Hipparchos, which is difficult to believe but also rather difficult to refute. The first Ziggurats and their astronomer-priests appeared in Mesopotamia more than 5000 years ago, and surely these ancient peoples were keen observers that could have noticed that stars never returned to the same place, but this requires precision measurements over many decades. Closer to Huatulco, the myths and codices of the aboriginal Meso- and South-American indigenous peoples tell us that their social structures and lives were strongly influenced –if not determined– by the stars. Interestingly, the Inca myth of Viracocha (literally the 'Tilted Plane of the Sphere') suggests that indeed the Incas knew about the precession of the equinoxes, but the Incas and also the Zapotecs, Toltecs, and Mayas built their elaborate astronomical observatories on sacred places within a few hundred years (but probably after) of the time when Hipparchos 'discovered' the precession of the equinoxes.

The purpose of this lengthy excursion into the depths of mythology is to introduce the notion that perhaps then as now –always– the human endeavors have been influenced by the stars, not only through astrological pursuits such as Kepler's *Harmonices Mundi*, but at a more subtle cultural level [2]. With this in mind, allow me, oh gentle reader (only gentle readers read conference proceedings anyway), to refer to the main scientific theories as *the modern myths of humanity.* The central *modern* myth of astronomy is of course the standard Big-Bang model with all its dark ramifications — dark matter, dark

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[2]Although it is difficult to conceive a greater revolution of the human spirit than that entrained by Kepler's laws.

energy, and inflation. In this homage to Luis, I would like to talk about yet another myth of astronomy, which I call *The Myth of the IMF*.

The story told by the Myth of the IMF is that we can build from first principles a theory that fully describes the physics of star formation and in particular that predicts exactly the observed mass distribution of stars at birth. In other words, starting from a molecular cloud of a certain mass and chemical composition [3], we can predict from first principles how many stars of a given mass will be formed out of that cloud (assuming of course that the cloud would form stars at all!). While it is tempting to generalize this myth to other branches of astronomy dealing with equally complex processes, I will stay here with the IMF.

The central theme of this presentation is the observation that the mass distribution of stars at birth –the initial mass function (IMF)– appears to be a universal function that within the substantial errors of observation is very well represented by a single power-law for masses larger than a rather ill-determined threshold around $M_L \simeq 1\ M_\odot$. In other words, that for masses larger than M_L, the slope or exponent of the power-law is the same everywhere. Thus,

$$N(m)dm = m^\alpha dm$$

with the same value of α everywhere. Historically this universal IMF is called the Salpeter power-law after the work of Ed Salpeter[4] in the early 50's who found a value of $\alpha = -2.35$. I bring back this well known fact to underline the first property of the Salpeter IMF: More than 50 years after Salpeter's seminal work, it remains essentially unchanged despite the monumental changes in technology that have transformed astronomy: CCD's, Space telescopes, 4m and 8m telescopes, and adaptive optics! In fact, I will show in this presentation that the best determined IMF using the best modern equipment gives a slope $\alpha = -2.25 \pm 0.05$ for the Salpeter slope.

After reviewing the observational evidence for a universal IMF, both in clusters and in the field, I will briefly review some ideas of why the IMF is universal, and what does it tell us about star formation: *The Myth of the IMF*.

2. THE IMF OF CLUSTERS

The steep decline of the IMF toward large masses inevitably leads to the Poisson catastrophe: unless you have a very large number of stars, you will have very large counting errors in the highest mass-bins. One can do two things: either count stars in a large field of a galaxy, or count stars in a very massive cluster. The fundamental limitation of the first approach (which is the one used by Salpeter in the Milky-Way) is that one gets a wide mix of ages, so one must assume a history of star-formation to retrieve the IMF. The fundamental limitation of the second approach is that there are only a handful of massive young clusters that can be resolved using the current generation of ground and/or space telescopes. But a handful is much better than none, so in the second case the limitation is mostly technical, although attention must still be given to the fact that however young these clusters may be, their ages are still not zero.

The most massive young clusters in the Galaxy are of course located close to the galactic plane and therefore suffer substantial foreground (and some internal) extinction. Still, using large telescopes with Adaptive Optics (AO) instruments working in the near-IR it is possible to penetrate the dust clouds and to resolve the clusters from the ground, and of course HST with NICMOS can do a similar job from space. Thus, it has been possible to obtain high quality photometry for the most massive young clusters in the Milky-Way, which together with 30 Doradus in the LMC provide a reasonably large sample of clusters from which the IMF can be reliably determined up to about $\sim 120\ M_\odot$ or so. Here I will present a more detailed review for the 3 clusters in which I have worked, and which illustrate different aspects of the observational challenges one faces in the determination of the IMF in young massive clusters, and I will summarize the relevant parameters

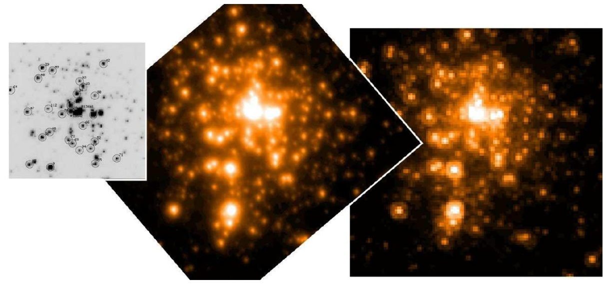

Fig. 1. Comparison of HST WFPC (left), HST NICMOS (right), and MAD VLT (mid) images of the central part of 30 Doradus. Notice that the figure shows only a small portion ($\sim 10'' \times 10''$)0 of the MAD image that covers an area about 6 times larger (Courtesy of C. Evans. See Campbell et al. 2008).

[3]One may also need to specify the initial internal kinematics of the cloud.

[4]Ed Salpeter passed away when I was completing this manuscript, which is dedicated to his memory as well as to Luis Carrasco's Long Walk through Astronomy.

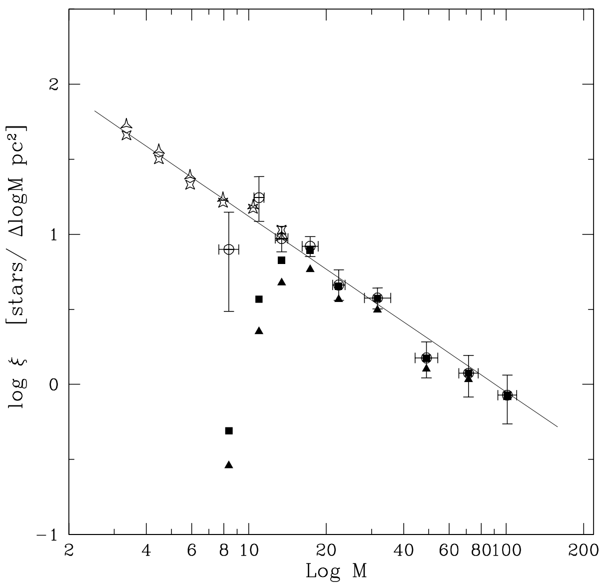

Fig. 2. The IMF of 30 Doradus from a combination of HST and NTT data. The open circles show the data corrected for crowding and magnitude-limit incompleteness biases. The solid triangles show the raw counts, and the solid squares show the raw counts corrected only for crowding. The dominance of the magnitude-limit effect in the lowest mass bins is manifest in this plot. The solid line shows a least squares fit of slope $\alpha = -2.25 \pm 0.05$ (from Selman et al. 1999).

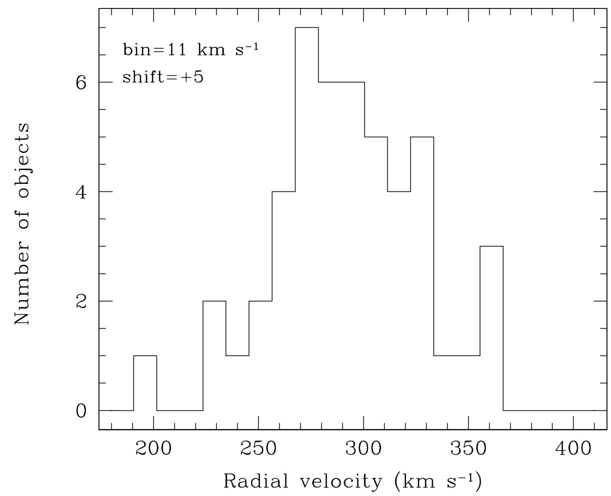

Fig. 3. Radial velocity histogram for 48 O stars in 30 Doradus. The radial velocity dispersion of these stars is ~ 30 km s^{-1}, much larger than the value of < 10 km s^{-1} expected from the photometric mass of the cluster (from Bosch et al. 1999).

of the complete sample based on my own reading of the published observations.

2.1. *30 Doradus*

The ionizing cluster of the Tarantula nebula is the most massive cluster in the sample and also the best studied so it is appropriate to start the discussion with 30 Dor. Figure 1 reproduces the central portion of a multi-conjugate adaptive optics (MCAO) K-band image of 30 Dor taken with MAD on the VLT. The figure illustrates how for some applications the current generation of AO assisted imagers on large ground based telescopes can parallel and even surpass HST. In particular, the full MAD image of 30 Dor (not shown here) covers an area about 6 times larger than NICMOS at a similar resolution. In fact, working at a resolution better than $0.1''$ over a field of almost 4 square minutes poses tough calibration challenges both for the photometry and the astrometry and for that reason the analysis of the MAD data is still in progress, so in what follows I will present mostly published results.

Figure 2 shows the IMF of 30 Dor from Selman et al. (1999) obtained from a combination of HST and NTT data. The figure illustrates the first important observational challenge we encounter when trying to measure the IMF of massive young clusters: the extinction varies significantly from star to star (up to 2 magnitudes in V for 30 Dor) giving rise to statistical completion corrections much larger than those resulting from crowding. Because of variable extinction, photometric completeness does not ensure completeness in mass as two stars of the same mass and spectral type can have widely different magnitudes and colors. We call this effect the *magnitude-limit bias*. The best fit line shown in the figure has a slope of $\alpha = -2.25 \pm 0.05$ for $M > 3\ M_\odot$.

The total photometric mass of the cluster is in principle easy to obtain: just integrate the IMF between the lower and upper mass limits. In practice this is not so easy because we know that (for other clusters, not for 30 Dor) the IMF turns over at a mass $M_L \sim 1\ M_\odot$: the slope *changes*. We still do not know the exact shape of the IMF below this mass limit, or whether it is the same from cluster to cluster, and we will probably never know because low-mass stars easily evaporate from the cluster. Thus, for the present purposes I will determine total masses integrating the power-law from $M_L = 0.5\ M_\odot$ to $M_U = 120\ M_\odot$. The resulting value for 30 Dor is $M_{\rm phot} = 1.6 \times 10^5\ M_\odot$. This number is relevant because the mass is the fundamental property of a cluster, but it also has a second equally relevant application to the study of the IMF.

Figure 3 shows the velocity dispersion of the cluster based on MOS spectra taken with EMMI at the

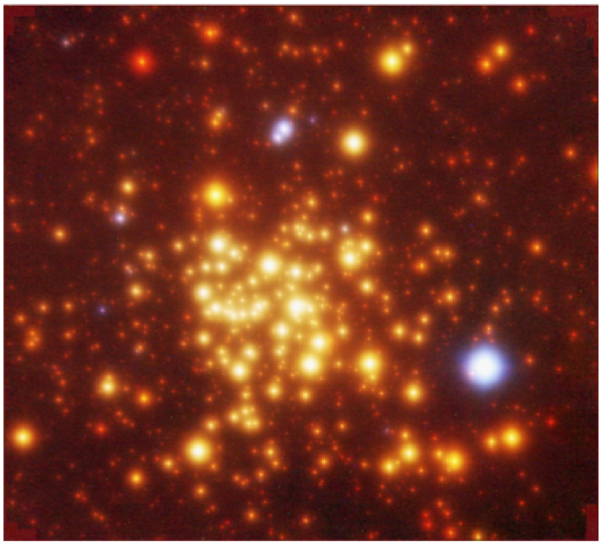

Fig. 4. Three-color (JHK) image of Arches taken with the AO imager NACO on the VLT. The field of view is 28″ (from Espinoza et al. 2009).

NTT (Bosch et al. 1999). After correction for instrumental errors, the velocity dispersion of the cluster comes out to be $\sigma_V = 30.5 \pm 0.2$ km s^{-1}, which is much larger than the Virial value inferred from the photometric mass and the critical radius of the cluster. This means that either the cluster is flying apart at a very large rate i.e. is not virialized, or that the measured velocities are dominated by something else: binaries. Bosch et al. (1999) simulated the effect of binaries and concluded that the observed velocity dispersion can be explained if most of the massive stars in the cluster are binaries. Recently, Bosch et al. (2008) obtained multi-epoch radial velocity observations of 50 O and B stars in the cluster and concluded that about 50% of the observed stars are binaries consistent with an intrinsic binary rate of 100% among massive stars in the cluster. Restricting the sample to their 26 non-variable stars, Bosch et al. (2008) obtained a velocity dispersion of 8.3 km s^{-1} for the cluster. The critical radius of the cluster is not well determined, but assuming a value of 5pc (Selman et al. 1999), the Virial mass comes out to be $M_{\rm vir} \simeq 2.3 \times 10^5\ M_\odot$, consistent within the large uncertainties with the photometric mass. The MAD data will allow us to obtain a better estimate of the critical radius and thus to use the comparison between the dynamical and photometric masses to obtain some constraints on the lower-mass shape of the IMF. The observations of Bosch et al. however, raise a rather fundamental problem: so far, we have not considered binarity in the determination of the IMF of 30 Dor and in fact of any cluster. Now this becomes a must.

In addition to variable extinction and binarity, there is a third ingredient that complicates the observation of the IMF: mass segregation. In 30 Dor the most massive stars appear to be more concentrated towards the center of the cluster. It is not clear whether this effect is due to dynamical evolution — which should not be observed in very young clusters such as 30 Dor, or to primordial segregation (massive stars form predominantly in the densest regions). Whatever the origin, mass segregation will cause radial variations in the IMF. These are not observed in 30 Dor, or at least not strongly, but the MAD data will allow a further insight into the problem. I will return to this issue in the next section where I will also discuss a particularly insidious problem with the photometric calibration of photometry of reddened stars we ran into in our study of 30 Doradus.

2.2. *The Arches cluster*

Arches is one of the 3 famous massive young clusters with a few tens of parsecs from the Galactic Center. The others are the Central Cluster (GCC) itself, and the Quintuplet Cluster. Both Arches and GCC have been claimed to have anomalous (flatter than Salpeter) IMF's, which because they are in a much denser environment than say 30 Dor, could lead to the discovery of the holy grail of star formation: a relation between the IMF and at least some physical parameter, in this case ISM pressure. Of course these three clusters are hidden behind tens of magnitudes of visual extinction and therefore can only be studied in the infrared. Moreover, they are extremely dense so they must be studied either from space, or using state of the art AO imagers. The claim of a non-Salpeter IMF comes from a number of NICMOS and ground-based AO studies pioneered by Figer et al. (1999) using NICMOS. None of these investigations, however, accounted properly for the magnitude-limit bias, which is particularly severe in the Galactic center clusters. Thus, we re-observed Arches using the VLT AO imager NACO taking advantage of the then newly commissioned infrared wavefront sensor that is particularly important for highly reddened objects. Figure 4 reproduces our three-color (JHK) NACO image of the Arches and provides a full color illustration of the complexities of the field in terms of dynamic range and crowding.

But the problems only start there: the deeper one goes in the analysis of the data, the subtler the problems become. Figure 5 shows a Voronoi diagram

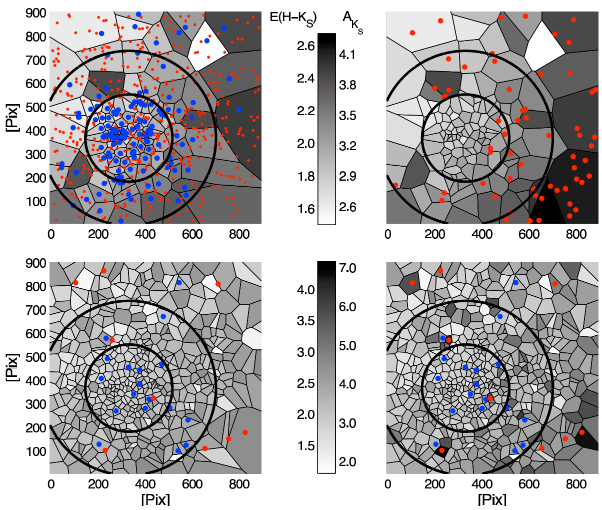

Fig. 5. Voronoi diagrams of the cluster reddening obtained with two different methods as described in the text. The extinction is seen to vary by as much as 5 magnitudes in K from star to star (from Espinoza, Selman, & Melnick 2009).

depicting the distribution of reddening in the cluster constructed as follows: the area around each star was *tesselated* in such a way that the borders of each cell are equidistant to the nearest star for which photometry is available. Two sets of tesselations are shown depending on whether three (JHK; top) or only two (HK; bottom) bands are used to compute the reddening. Each cell in these Voronoi diagrams is shaded by a tone of gray proportional to the reddening of its parent star as indicated in the central bars. The cluster stars are shown in the upper-left panel where the blue dots represent stars with JHK photometry and the red dots stars with only HK measurements. The extinction is seen to vary up to 5 magnitudes in K from star to star. (Espinoza, Selman, & Melnick 2009). Of course these are variations of the foreground extinction to the cluster and reflect the Fractal structure of the ISM: large fluctuations are observed at essentially all spatial scales.

30 Doradus lies at a high galactic latitude so the extinction variations are mostly internal to the cluster and caused by the dust that is intimately mixed with the gas in the Tarantula Nebula. Thus, in 30 Dor the distribution of extinction has a full width at half maximum of $\Delta A_V \simeq 1$ mag with maximum fluctuations of about 2 magnitudes. In Arches the K-band distribution is significantly broader, so we expect the magnitude-limit bias to be more severe, and indeed it is. In order to correct for this bias we need to know the reddening distribution in the cluster,

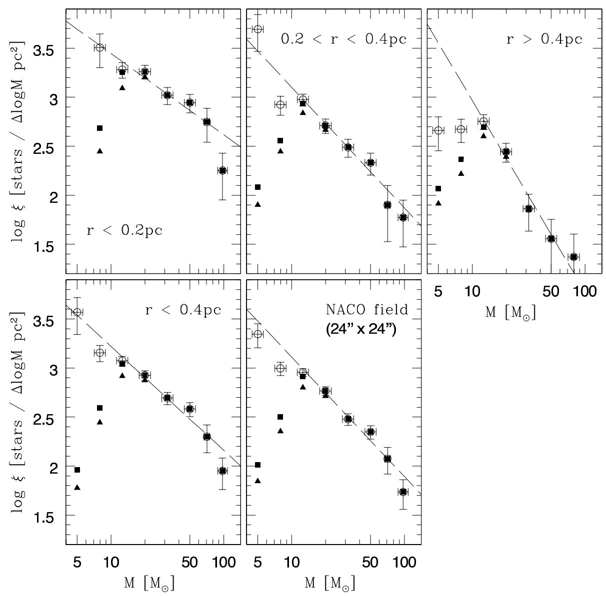

Fig. 6. The IMF of the three 'annuli' in the cluster defined in Figure 5. The different symbols show the raw counts (triangles); counts corrected for crowding (squares) and counts corrected for crowding and magnitude-limit bias (open symbols). For the faintest bins the magnitude-limit corrections can be more than a factor of 10! The overall IMF corresponds to the region $r < 0.4pc$ for which the field contamination is minimal (from Espinoza et al. 2009).

which in principle can be a function of position (our Voronoi diagrams suggest that the reddening may be systematically larger in the direction of the Galactic Center – lower-right). And we also need to know the extinction law. All this is compounded with the fact that the transformation to any standard photometric system is based on unreddened stars, which introduces a subtle but noticeable effect on very reddened clusters that must be considered. So it is much cleaner to work in the natural photometric system of the instrument, which of course leads to other problems, in particular with the theoretical models.

To make a long story short, once all these problems are carefully and some times painfully taken into account, and the corrections for crowding are computed as a function of magnitude, color, and distance to the cluster center, one finally gets the IMF's shown in Figure 6.

As observed by previous authors, the IMF slope gets flatter towards the cluster center, providing strong evidence of dynamical evolution, but our overall slope (r < 0.4 pc) is steeper than previously thought, and consistent with Salpeter (30 Doradus): $\alpha = -2.1 \pm 0.2$, albeit with a significant error.

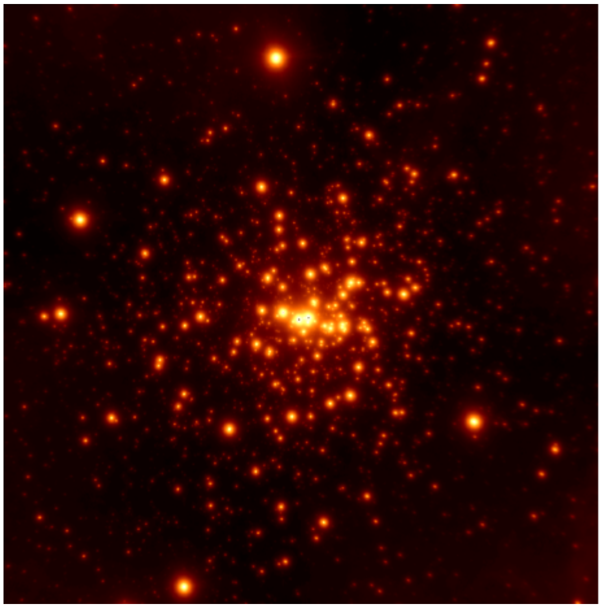

Fig. 7. MAD K-band image of NGC3603. The resolution of this image if comparable to that of WFPC and NICMOS on HST. (Courtesy of Joao Alves.)

The fundamental reason for our steeper slope is the magnitude-limit bias.

2.3. *NGC 3603*

To conclude the tour of massive young clusters I will make a short stop at NGC 3603 to illustrate yet another insidious effect in the observation of the IMF of young clusters: the pre-main sequence population. Figure 7 shows a MAD image of the cluster that illustrates the power of modern AO-assisted imagers on large telescopes: NICMOS resolution with 10 times the field.

As we saw for 30 Dor and Arches, it is rather difficult to break the 'one solar-mass barrier' and to reach stars considerably fainter than a couple of solar masses. This is not only a problem of crowding and exposure time. Figure 8 shows the JK color-magnitude diagram of NGC 3603 obtained by Harayama, Eisenhauer, & Martin (2008) using NACO on the VLT. The various lines show main-sequence and pre-main sequence tracks of different ages as indicated in the figure. We see that in young clusters the low mass stars are still in the pre-main sequence, and it seems that tracks of a single age do not fit the locus of these stars: different ages are required to fit stars of different masses, in the case of NGC 3603 ages ranging from 0.3 Myr to 3.0 Myr. This is a big problem for which there is no good solution yet. The slope of the IMF depends on what age one assumes, and Harayama and collaborators

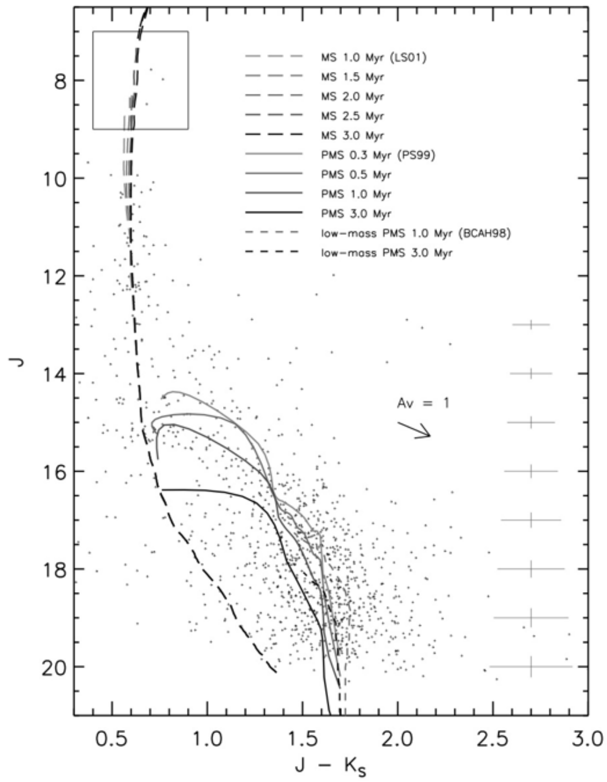

Fig. 8. The JK color-magnitude diagram of NGC 3603. Stars of mass $M \leq 2\ M_{\odot}$ are still in the pre-main sequence phase and seem to have different ages (from Harayama et al. 2008).

have resorted to tabulating their results as a function of age. The value I have chosen, $\alpha = -2.0 \pm 0.2$ corresponds to an age of 1 Myr.

2.4. *The IMF of Clusters*

The previous sections show that even for the best studied young massive clusters in the Galaxy and the Magellanic Clouds, which have thousands of massive stars, and after tackling the formidable observational challenges with state of the art instruments and computational techniques, we are still left with a number of fundamental sources of uncertainty: binaries, dynamical evolution, and pre-main sequence tracks. So the compilation of the results for the most massive young clusters from the literature presented in Table 1 must be taken with a grain of salt.

The only cluster besides 30 Dor for which an estimate of the dynamical mass has been attempted is Westerlund 1 for which Mengel & Tacconi-Garman (2008) give a value of $M_{\rm Dyn} = 1.5 \times 10^5\ M_{\odot}$ based on spectroscopy of 10 (red) supergiants. This is 5 times larger than the photometric mass given in Table 1, and provides a strong indication that the population

TABLE 1

GLOBAL PROPERTIES OF YOUNG MASSIVE CLUSTERS

Cluster	Dist. [kpc]	Age [Myr]	Mass[a] [$M_\odot$]	IMF [$-\alpha$]
30 Dor	52	2-3	1.6×10^5	2.25 ± 0.05
Arches	8	2-3	2.0×10^4	2.1 ± 0.2
NGC 3603	6-8	0.3-3	2.5×10^4	2.0 ± 0.2
Trumpler 14	2.8	0.5-6	10^4	2.4 to 3.0
W49A	11.4	0.5 ± 1	1.4×10^4	2.6 ± 0.3
Westerlund 1	3.6	4-5	3×10^4	2.3
Westerlund 2	3-8	2 ± 1	5×10^3	2.20 ± 0.16

[a]Photometric mass assuming a single power-law between 0.5 and 120 $M_\odot$.

of massive stars in Westerlund 1 may also be dominated by binaries. Notice that because Westerlund 1 is significantly older than 30 Dor, we are probing a population of lower-mass binaries.

The table shows that the best observed massive young clusters in the Galaxy and the Magellanic-Clouds have IMF's formally indistinguishable from the Salpeter power-law. So within the still large uncertainties discussed below, the observations are consistent with the hypothesis that the IMF of clusters is a universal function, which is best determined for 30 Doradus at least in the mass range $M > 3\ M_\odot$. In fact, there is evidence from NICMOS photometry (Zinnecker, private communication) that the power-law IMF of 30 Dor does not show a turn-over down to at least $M_L \sim 0.5\ M_\odot$, (but remember that in that mass-range most, if not all, stars are still in the pre-main sequence where the uncertainties are severe). It is therefore appropriate to revise the 'Salpeter' slope from $\alpha = -2.35$ to the 30 Doradus slope, $\alpha = -2.25$.

3. THE FIELD

If all (or at least most) stars form in clusters then most field stars must be remnants of dissolved clusters. Since clusters have a power-law distribution of slope $\beta \sim -2$ [$N(M)dM = M^\beta dM$ where M is the mass of the cluster] some authors have claimed that the IMF of the field should be steeper than the IMF of clusters. In other words, galaxies should have steeper IMF's than stellar clusters (e.g. Kroupa & Weidner 2003). Their reasoning is that since clusters cannot contain stars more massive than the clusters themselves, and since there are many more low-mass clusters than massive ones, the dissolution of a population of clusters should build a field with more low mass stars. I will show below that this argument is wrong, but even if it were right, the cluster

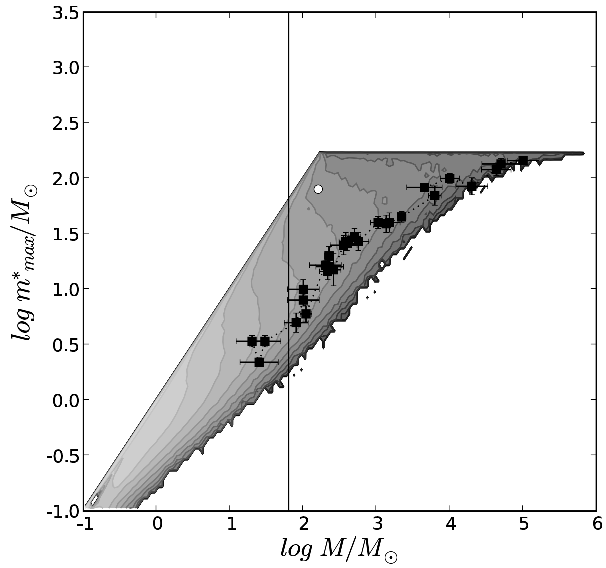

Fig. 9. The probability distribution of maximal stellar mass for $\beta = -2$. The points correspond to the sample of clusters by Weidner & Kroupa (2006) with a few additions (see Melnick & Selman 2008 for more details).

slope β would have to be much steeper than observed ($\beta \sim -2$) for the effect to be noticeable at all (e.g. Elmegreen 2006). And of course, the observations of Salpeter himself were done in the field! In fact, the IMF of the field is observed to be indistinguishable from Salpeter (Selman & Melnick 2005), but remember the grain of salt!

We can explore this issue further by asking a different and probably more fundamental question: do clusters sample a Universal stellar IMF? This question was asked by Weidner & Kroupa (2006) who concluded that the statistics of the most massive stars in clusters was *not* consistent with the view that clusters randomly sample a Universal stellar IMF. The problem, however, turns out to be rather tricky. Figure 9 shows the Weidner-Kroupa plot recalculated by Fernando Selman and myself (2008) using a different algorithmic approach.

We obtain the same result as Weidner & Kroupa: the observations (dots) do not scatter throughout the figure as expected if clusters randomly sample a Universal IMF, but tend to concentrate towards the lower maximal star masses allowed by the model. To illustrate this point better we have plotted a vertical line at log $M/M_\odot = 1.8$. The contours show clusters of equal numbers of stars and the contour corresponding to a cluster of log $M = 1.8$ has two maxima: one at log $m^*_{\rm max} \simeq 0.9$ and the other at log $m^*_{\rm max} \simeq 1.7$ close to the total mass of the cluster. The clusters from the compilation of Weidner & Kroupa (2006) shown in the figure concentrate in the region defined by the first (lower mass) peak and do not cover the full mass range allowed by the models. In other words, small clusters dominated by one or at most a few high mass stars do not seem to exist, or at least are not included in Weidner & Kroupa's sample. (Note that the ridge defined by the observations is not a physical correlation but a size of sample effect: the most massive stars live in the most massive clusters so the contours get shorter and shorter for more massive clusters.)

Fernando and I have looked at the problem from a different perspective: instead of characterizing clusters by their total mass, we chose to characterize them by their total number of stars n. The important advantage of this approach is that for small clusters (which are the crucial objects in this problem) the functional form of the IMF must depend on the cluster mass because the IMF cannot sample stars more massive than the clusters themselves, whereas if clusters are characterized by total number of stars, only the normalization of the IMF depends on n, but not the functional form. So the IMF of the field built by the disruption of clusters is simply given by,

$$P(m) = \sum_{m=0}^{\infty} P(m|n)P(n) \qquad 1$$

where $P(m)$ is the field IMF (the probability of finding a star of mass m), and $P(m|n)$ is the cluster IMF, that is the probability of finding a star of mass m in a cluster of n stars. $P(n)$ is the distribution of clusters by number of stars, which is also a power-law of slope β (Oey et al. 2004). Since only the normalization of $P(m|n)$ depends on the number of stars but not the slope, if $P(m|n)$ is Salpeter, $P(m)$ is also Salpeter. The same conclusion does not hold if clusters are characterized by mass. In that case,

$$P(m) = \int_0^{\infty} P(m|M)P(M)dM \qquad 2$$

but now while $P(M)$ is still a power-law, $P(m|M)$ is Salpeter only for very massive clusters (because it must have a cut-off for $m > M$). If $P(m)$ is Salpeter, then we can calculate $P(m|M)$ inverting Equation 2, and indeed we get functions which are not power-laws, but rather complicated functions (that in fact explain why the contours in Figure 9 have two maxima; Selman & Melnick 2008).

So if clusters are Salpeter, the field resulting from the dissolution of clusters must also be Salpeter; not approximately, exactly. Let us now return to the random-sampling question. The trouble with the maximal mass statistics is that cluster selection effects are acute, and the results depend on how clusters are defined. The white dot in Figure 9 show an example of a different definition (see Selman & Melnick 2008 for details). So we decided to use an independent test for validating or falsifying the random sampling model. We used the complete sample of young embedded clusters by Lada & Lada (2003) to test our models.

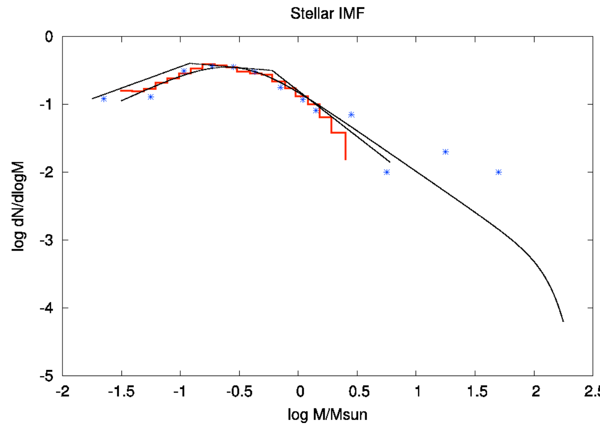

Fig. 10. The analytical stellar IMF used in the Monte-Carlo experiments compared with the IMF of the Trapezium cluster from the literature (from Melnick & Selman 2008).

Lada & Lada compiled a catalog of embedded clusters with more than 35 stars from the literature, so their sample is ideal for our purposes as described above. Moreover, the Lada & Lada clusters sample the critical range of masses, that is, clusters with masses in the single-star range (i.e. smaller than a few hundred solar masses). Our test consists in reproducing the observed mass function of embedded clusters by sampling randomly a universal stellar IMF. Since we are dealing with (small) clusters dominated by low-mass stars, it is critical to sample the full IMF, and not only the power-law massive star regime. Therefore we fitted an empirical analytical formula to the observed IMF of the Orion Trapezium cluster forcing it to be Salpeter between $1 < M/M_\odot < 120$. Our 'analytical' universal IMF is shown in Figure 10.

Using this IMF we computed a large number of Monte-Carlo models drawing each time 72 clusters of more than 35 stars to match the selection parameters of the Lada & Lada catalogue. The results are shown in Figure 11 using the same representation as Lada & Lada, which gives the total mass of the clusters rather than the total number of stars. In this representation, a power-law cluster mass function of slope -2 gives a flat distribution. We find excellent agreement between our models and the observations except for the mass bins at log $M \sim 0.95$ and log $M \sim 3.5$, which are totally de-populated in the Lada & Lada catalogue. Lada & Lada proposed that the downturn they observed at smaller masses was evidence for a favored cluster formation mass scale at around $M_{\rm cl} \sim 50\ M_\odot$, but our models indicate that this downturn is naturally explained by the cutoff in n ($n \geq 35$) they introduced in an otherwise scale-free spectrum. There is no need therefore to invoke a special cluster formation scale. In fact, Figure 11 indicates that the data is best modeled by a cutoff somewhat larger than the $n > 35$ criterion of Lada & Lada to select clusters (see Selman & Melnick 2008 for more details). So not only do our models reproduce the observations very well, but they also show considerable predictive power, leading us to conclude that indeed clusters form by randomly sampling a universal stellar IMF. This leads to an apparent impasse between our results and the maximal mass star statistics discussed above. Weidner & Kroupa resorted to the ad-hoc assumption that stars from in an ordered fashion (with less massive stars forming first) to reproduce the observations, but there are other alternatives. Besides the selection effects introduced by the definition of clusters and cluster-boudaries discussed above, another possibility is that small clusters dominated by one or a few massive stars can be gravitationally unstable. The jury is still out, but this is an important issue that new observations and/or new n-body simulations should help to answer.

An interesting corollary of our random-sampling models is that the IMF may be universal over the whole range of stellar masses, not only for massive stars.

4. FRACTALS

Power-laws are intimately related to Fractals and molecular clouds have Fractal structures. Elmegreen (1997) noticed that the mass distribution of clumps in a Fractal molecular cloud is a power-law of slope exactly equal to -2 and explored this remarkable similarity with the Salpeter law. The Fractal slope is also very close to distribution function of stellar clusters. The left panel of Figure 12 shows a cartoon of a Fractal cloud: the entire cloud is made of three big clumps each of which is made of three smaller sub-clumps that in turn consists of three even smaller clumps each and so on all the way down to the smallest structures in the cloud. We may imagine that we

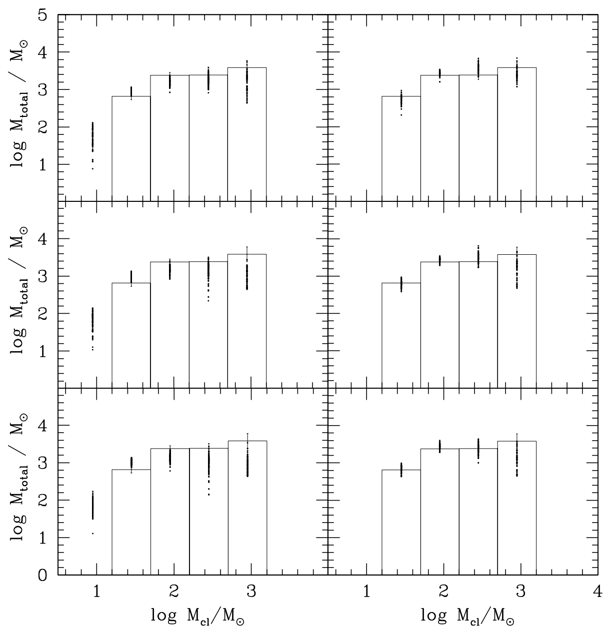

Fig. 11. The Lada & Lada (2003) distribution of masses of embedded clusters together with our Monte-Carlo simulations in which we randomly sample the stellar IMF assuming cluster probability distributions of slope $\beta = -1.8$ (top); $\beta = -2.0$ (middle); and $\beta = -2.2$ (bottom). The left-panels are for $n \geq 35$ and the right-panels for $n \geq 75$ stars (from Melnick & Selman 2008).

observe the cloud with higher and higher resolution that reveals finer and finer structures. Clumps may be stars or clusters. The right panel of Figure 12 shows a representation of our Fractal cloud as a Hierarchical tree. Elmegreen showed that sampling the tree at random (to convert gas into stars) steepens the resulting mass distribution of stars. The steepening arises from two effects: (1) when a clump is selected all the hierarchy further down from that clump is erased; and (2) smaller clumps are denser and collapse faster so must be sampled more frequently. The first effect steepens the resulting stellar mass function from -2 to ~ -2.15, while the second requires some assumption about the collapse time and the Fractal dimension of molecular clouds. Using standard values, Elmegreen recovered the Salpeter IMF with almost no physics: pure statistics!

If the Fractal clumps are clusters instead of stars, then the Fractal slope gives directly the observed mass distribution of clusters without any assumption about the physics of star formation except of course that the mass of the cluster be proportional to the initial mass of gas. Bruce Elmegreen has pointed out that fragmentation of the smallest protostellar molecular cores may explain the turn-over in

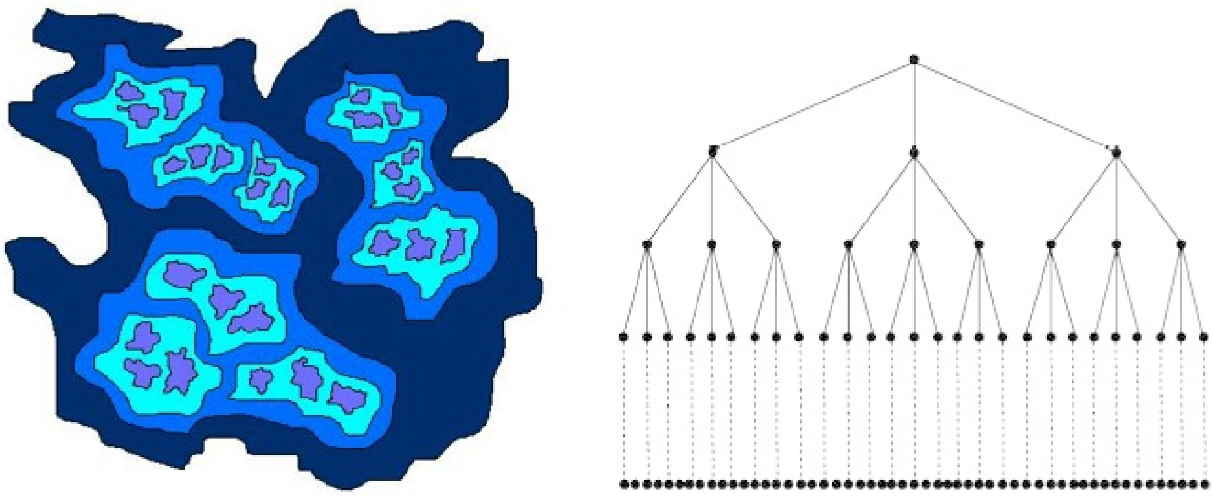

Fig. 12. (left) Cartoon of a fractal molecular cloud. (right) Hierarchical tree of the Fractal cloud shown in the left-panel. For simplicity the hierarchy was drawn with three branches per level for every level, but the results discussed in the text are valid for an arbitrary number of branches per level varying from level to level and from branch to branch.

the IMF below some threshold mass. But already in the gas-phase the mass distribution of molecular cores is observed to be a log-normal function and not a power-law (or perhaps a log-normal with a power-law tail; e.g. Enoch et al. 2008, and references therein). And again log-normal functions arise from the statistics of fragmentation trees, although in this case some assumptions need to be made about the probability of fragmentation at each level of the hierarchy.

5. CONCLUSION

The central premise of this presentation is that the universality of the IMF stems from the Fractal structure of the interstellar medium, and therefore does not depend (or depends very weakly) on the physics of star formation. The Fractal structure of the ISM results from the complex process of formation and evolution of galaxies and the recurrent feedback of many generations of stars on the ISM, and of the ISM on the mass distribution and spatial structure of stars. In a famous *Scientific American* article, Martin Gardner (1978) popularized the work of Voss and Clarke on the power spectrum of classical music. The spectral power density of classical music follows a $1/f$ law, which is now well understood (it was less known in 1975, but still known) to reflect fractality in the time-domain (e.g. Mandelbrot 1977). Thus, trying to learn about the physics of star formation from the IMF is like trying to understand the personality of Beethoven from the power spectrum of the Ninth Symphony!

Apparently every form of beauty in Nature or Art has an underlying Fractal structure (Mandelbrot cited by Gardner). This may also apply to astronomical objects: the beauty of star clusters and galaxies could be related to their underlying Fractal structures. Stars form in a hierarchy of structures with different numbers and masses. Some of these structures end-up forming large clusters and some don't: they become small associations formed in neighboring regions almost by chance. All stars form in Fractals, but only some form in clusters.

The Long Walk

I met Luis Carrasco in the early 80's when I was beginning to think about the IMF. Luis was spending a sabbatical in Heidelberg and had come to Munich to give a seminar on his work on angular momentum. After the talk and the many questions was over, I introduced myself and we instantly became friends, a friendship that continues until know. We have even authored a paper together that was based on some work we did during that short visit of Luis to Munich, and after surviving an incredible adventure in the frozen highways of Bavaria when we drove to Heidelberg to fetch the plates of M82 that were used for that paper. Already then Luis was thinking of IR instrumentation, and some time later he began to think of a large mm-wave telescope for Mexico, which eventually became the GMT. I thank the organizers for inviting me to participate in the celebration of Luis's long walk through astronomy. It has been a pleasure and a privilege to be here.

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