RevMexAA (Serie de Conferencias), **38**, 44–47 (2010)

HYDRODYNAMIC WIND THEORY

A. ud-Doula[1]

RESUMEN

Todas las estrellas de secuencia principal pierden masa vía vientos estelares. Los vientos de estrellas frías como el sol son impulsados por el gradiente de presión del gas. Sin embargo, los vientos de estrellas masivas calientes que tienden a ser luminosas son impulsados por la presión de radiación de la estrella. En esta charla, se describe la naturaleza de tal impulso radiativo y se demuestra que las opacidades del contínuo y de las líneas en el viento determinan cuán grande la atmósfera estelar puede aparecer en un interferómetro. Actualmente, los interferómetros pueden detectar sólo las estructuras relativamente grandes. Se describe cómo se inducen estas estructuras por la rotación, pulsaciones, campos magnéticos o interacciones viento-viento.

ABSTRACT

All main sequence stars lose mass via stellar winds. The winds of cool stars like the sun are driven by gas pressure gradient. However, the winds of hot massive stars which tend to be luminous are driven by the star radiation pressure. In this talk, I describe the nature of such radiative driving and show that the continuum and line opacities in the wind determine how large the stellar atmosphere may appear in an interferometer. Currently, interferometers can detect only relatively large scale structures. I will describe how these structures are induced by rotation, pulsations, magnetic fields or wind-wind interactions.

Key Words: stars: magnetic fields — stars: mass loss — stars: oscillations (including pulsations) — stars: rotation

1. INTRODUCTION

There is a big variation in the nature of stars we see in the sky. What distinguishes them the most, is their effective surface temperatures that divide them into two distinct categories: hot and cool stars. What unites all these stars is that they all lose mass. Sun-like cool stars which have convective envelopes lose mass via gas pressure. Massive hot stars which lack convective envelopes lose mass via radiation pressure due to their enormous intrinsic brightness. In this brief review I will discuss the physics behind such mass loss mechanism and how large structures detectable by current interferometers can be formed. It is not my intention to list all the previous works but rather concentrate on basic concepts behind hydrodynamic wind theory by highlighting only a few select articles as examples.

2. LIGHT AS A DRIVING MECHANISM

Intuitively light is not a good source for momentum transfer. This is mainly because the momentum of a photon is determined by division of its energy by the maximum possible speed, c, the speed of light. But hot stars are no ordinary objects. They are very massive and very luminous. In their case, the photons become the dominating factor in controlling the physics of the continuous outflow of material from the stellar surface. The typical flow speeds of hot-star winds can reach as high as 3000 km/s, much faster than the 400–700 km/s speed of the solar wind. At the same time, typical mass loss rates for the hot stars can reach up to $\sim 10^{-6}\ M_{\odot}\ \mathrm{yr}^{-1}$, which are up to a factor of a billion higher than that of the sun.

There are two distinct ways light can impart momentum on gas: photons can scatter off free electrons or bound electrons. The latter case can provide much stronger driving mechanism. We discuss these next.

2.1. *Free Electron Scattering*

The continuum processes in hot stars are presumed to be dominated by free electron scattering. The line force per unit mass (acceleration) due to such electron scattering at a radius r, can be written as,

$$\mathbf{g_e}(r) = \frac{\kappa_e}{c} \oint \int\limits_{\nu=0}^{\infty} I_\nu(r,\hat{n})\, \hat{\mathbf{n}}\, d\Omega\, d\nu\,, \tag{1}$$

where I_ν is intensity at radius r along direction $\hat{\mathbf{n}}$ and κ_e is the mass absorption coefficient of an electron. In the case of spherical symmetry and a point source star, the integrand gives just the total radiation flux. Thus,

$$g_e(r) = \frac{\kappa_e}{c} \frac{L_*}{4\pi r^2}\,, \tag{2}$$

---

[1] Department of Physics, Morrisville State College, Morrisville, USA.

where L_* is the total bolometric luminosity of the star. Here we assumed that the wind is optically thin to the continuum radiation. Note that g_e is inversely proportional to r^2, just like the gravity. Therefore it is useful to compare g_e with gravity g by defining,

$$\Gamma \equiv \frac{g_e}{g} = \frac{\kappa_e L_*}{4\pi G M_* c}\,, \tag{3}$$

where Γ is often referred as the Eddington parameter, G is the gravitational constant and M_* is the mass of the star. Clearly, if the value of Γ exceeds unity, the star cannot remain in a hydrostatic equilibrium. For the OB-type stars we study here $\Gamma \approx 0.5$. Thus g_e essentially reduces the effective gravity by $1-\Gamma$, i.e. $g_{\rm eff} = g(1-\Gamma)$.

2.2. *Bound Electron Scattering*

The force due to bound electron scattering can be significantly larger due to significantly larger cross sections per unit mass of bound electrons, κ_L. This was originally formulated by Castor et al. (1975) and extended in terms of the "quality" of resonance lines, Q by Gayley (1995). Given that metals comprise only 10^{-4} fraction of total number of ions in a hot star wind, on average mass absorption coefficient for thin lines is $\kappa_L = \bar{Q} \times \kappa_e \approx 1000 \times \kappa_e$. Thus, the lines force due to thin lines is $g_{\rm thin} = 1000 \times g_e$ and the corresponding Eddington parameter for thin lines is $\Gamma_{\rm thin} = 1000 \times \Gamma$.

Such a strong force would drive an unrealistic amount of mass off the stellar surface. However, out of thousands of lines that a typical hot star wind has, many of them are optically thick as these lines tend to self-shadow. If we take into account the combination of all these thin and thick lines, the net line force is roughly,

$$g_{\rm lines} = \frac{1}{(1-\alpha)}\,\frac{\bar{Q}\kappa_e L_*}{4\pi r^2 c}\left(\frac{\partial v/\partial r}{\rho c \bar{Q}\kappa_e}\right)^{\alpha}\,, \tag{4}$$

where α represents the fraction of optically thick lines.

It turns out that, overall, the Eddington parameter for lines is of order unity, i.e. $\Gamma_{\rm lines} \sim 1$. This enables the radiation to drive an enormous amount of material off the stellar surface approximated by,

$$\dot{M} \sim \frac{L}{c^2}\left[\frac{\bar{Q}\Gamma}{1-\Gamma}\right]^{(1-\alpha)/\alpha}\,. \tag{5}$$

The terminal speed of the outflowing gas is comparable to the escape speed from the stellar surface, $V_\infty \sim V_{\rm esc}$. The solution is not usually obtainable analytically, and numerical codes must be adopted to find a self-consistent solution. However, in general, the velocity can be approximated by a "β-law",

$$V(r) \approx V_\infty (1 - R_*/r)^{\beta}\,, \tag{6}$$

where typically $\beta \approx 0.8$ for O-star winds. Lower β implies a shallower acceleration of the wind near the base.

3. THICKNESS OF THE WIND

Now, we may ask, how optically thick is this wind? Can we detect it with our current interferometers in the continuum? In order to estimate the size of the expanding photosphere of the winds, we need to compute optical depth along the line of sight z-axis, and impact parameter, p,

$$\tau_\nu(p,z) = \int_z^\infty \kappa_\nu[r(p,z')]\rho[r(p,z')]dz'\,. \tag{7}$$

For a rough estimate and simplicity, we can integrate this along the central ray where $p = 0$,

$$\tau_\nu(0,r) = \int_r^\infty \kappa_\nu \rho dr' = \frac{\kappa_\nu \dot{M}}{4\pi v_\infty}\int_r^\infty \frac{dr'}{r'^2} = \frac{R_1}{r}\,. \tag{8}$$

Here, we define a characteristic radius, R_1 with $\tau(r = R_1) = 1$,

$$R_1 \equiv \frac{\kappa_\nu \dot{M}}{4\pi v_\infty}\,. \tag{9}$$

For typical a O-star wind ($\dot{M} = 10^{-6}\ M_\odot\ {\rm yr}^{-1}$ and $V_\infty = 2000\ {\rm km\ s}^{-1}$), $R_1 = 0.12\ R_\odot$ only, implying that expanding photosphere of the wind is very close to the stellar surface. To make hot star winds optically thick to continuum radiation requires an exceptionally thick winds such as the ones seen in Wolf-Rayet stars ($\dot{M} \sim 10^{-4}\ M_\odot\ {\rm yr}^{-1}$). For such winds the values of R_1 can be few tens of solar radii. Thus, Wolf-Rayet star winds are optically thick to continuum opacity, but ordinary OB hot star winds are optically thin.

4. LARGE-SCALE STRUCTURES

The above discussion assumed a spherically symmetric smooth wind. However, we know from observations, especially from discrete absorptions components (DACs), that hot star winds are not steady and spherically symmetric. A number factors can give rise to such an asymmetry and large scale structures, e.g. rotation, stellar pulsation, wind-wind collision and magnetic fields. We discuss these next.

4.1. *Rotation*

Rotation can naturally provide latitudinal variation in wind structure. Since mass loss rate is dependent on radiation flux and local gravity,

$$\dot{m}(\theta) \sim \frac{F(\theta)}{c^2}\left[\frac{Q\Gamma}{1-\Gamma}\right]^{(1-\alpha)/\alpha} . \tag{10}$$

Here, due to gravity darkening, the radiation flux is latitude dependent and is highest at the poles (Zeipel 1925). The denominator term in the square bracket represents the effective gravity, $1-\Gamma = g_{\rm eff}/g_{\rm grav}$.

$$\dot{m}(\theta) \sim \frac{F(\theta)^{1/\alpha}}{g_{\rm eff}(\theta)^{1/\alpha-1}} \sim \frac{F^2(\theta)}{g_{\rm eff}(\theta)} . \tag{11}$$

In the last equality, we assumed $\alpha = 1/2$ for illustrative purposes.

If we ignore the effects of gravity darkening and consider only the effects of rotation then,

$$\dot{m}(\theta) \sim \frac{1}{g_{\rm eff}(\theta)} . \tag{12}$$

This implies that the highest mass flux is at the equator where the gravity is the weakest. However, if we do include the gravity darkening wherein radiation flux $F(\theta) \sim g_{\rm eff}(\theta)$ then,

$$\dot{m}(\theta) \sim F(\theta) , \tag{13}$$

which in turn implies the highest mass flux at the poles. Since the flow velocity $V_\infty(\theta) \sim V_{\rm eff}(\theta) \sim \sqrt{g_{\rm eff}(\theta)}$, we expect dense and fast wind along the polar regions. This is exactly what we observe in the case of Eta Carinae (Smith 2002) wherein we see dense equatorial skirt with two large and dense lobes along the poles. Slit-spectroscopy confirmed that indeed the polar wind is faster than the equatorial flow.

Stellar rotation effects can also lead to Corotating Interaction regions (CIRs) (Cranmer & Owocki 1996). These structures are created when slow moving dense material is slammed from behind by fast moving low density wind. Lobel & Blomme (2008) showed this can be well explained by co-rotating dark and bright spots on the stellar surface that are separated by certain angle. This leads to differential mass loss rates in the equatorial region resulting in fast and slow winds that collide leading to observed DACs.

4.2. *Stellar Pulsation*

However, based on observations of early-type B supergiant HD64760 (B0.5 Ib), Kaufer et al. (2006) showed that bright and dark spots can be a result of interference of non-radial stellar pulsations and they do not need to co-rotate with the stellar surface. The differences in flux magnitudes are large enough to raise spiral co-rotating interaction regions (Cranmer & Owocki 1996). Kaufer et al. (2006) showed that simple rotating wind model can in fact explain Hα wind-profile variability.

4.3. *Wind-Wind Collision*

In a binary system wind-wind collision can also lead to large scale structures that are readily detectable by current interferometers. In particular, the first images of Wolf-Rayet star WR 104 revealed a dust plume that stretched hundreds of AU in an Archimedean spiral pattern (Tuthill et al. 1999). The origin of such large-scale structure seemed obvious: at the center of the system the hot stars drive dense spherical winds that collide and shock, cool enough temperature within the shocked material allows dust formation, and the rotation of the binary causes the pattern to move in a spiral pattern. Such a structure is coined as "pinwheel" nebula.

A host of other similar systems of pinwheel nebulae have been detected in recent years, These include WR98a (Monnier et al. 1999), WR 112 (Marchenko et al. 2002) and WR 140 (Monnier et al. 2002). A few more systems have been discovered near the center of our Galaxy, at the heart of the massive Quintuplet cluster (Tuthill et al. 2006).

4.4. *Magnetic Fields*

Finally, magnetic fields can lead to formation of large scale structures as well. They can influence hot star winds significantly. Their overall influence on the wind dynamics can be characterized by a single magnetic confinement parameter,

$$\eta_* \equiv \frac{B_{eq}^2 R_*^2}{\dot{M} v_\infty} , \tag{14}$$

which characterizes the ratio between magnetic field energy density and kinetic energy density of the wind (ud-Doula & Owocki 2002).

Extensive magnetohydrodynamic (MHD) simulations show that, in general, for the stellar models with weak magnetic confinement ($\eta_* < 1$) field lines are stretched dynamical timescale into radial configuration by the strong outflow. However, even for magnetic confinement as weak as $\eta_* \sim 1/10$ the field can influence the wind density by diverting the wind material from higher latitudes towards the magnetic equator.

For stronger confinement ($\eta_* > 1$), the magnetic field remains closed over a limited range of latitude and height about the equatorial surface, but eventually opens into a nearly radial configuration at large radii. Within closed loops, material is channeled toward loop tops into shock collisions, leading to X-ray emission that is generally consistent with that derived in the original "magnetically confined wind shock" (MWCS) model first developed by Babel & Montmerle (1997). But in MHD simulations, once shocked material cools and becomes dense, it eventually is pulled by gravity back onto the star in quite complex and variable inflow patterns. Within open field flow, the equatorial channeling leads to oblique shocks that eventually forms a thin, dense, slowly outflowing disk at the magnetic equator.

Such large scale wind structures are inferred most directly from time variability in the blueshifted absorption troughs of UV P Cygni profiles. But they can also be a source of x-ray emission. Fully dynamic MHD models of θ^1 Ori C, are able to explain both the hardness and location of x-ray emission from the star (Gagné et al. 2005).

5. CONCLUSION

We have discussed only briefly how large scale structures can be created in hot star winds. Here are the main conclusions of this talk:

- Radiation is an efficient way to drive mass loss.
- Thick winds limit how deep we can see through an interferometer.
- Only large scale structures can be detected.
- Rotation, pulsation, wind-wind collision and magnetic field can create large scale structures.

DISCUSSION

G. Meynet: *It seems that the geometry of the magnetic field is radial. I would expect that the matter would follow the magnetic field lines. How then can we obtain matter concentrated in the equatorial plane?* — Dipole magnetic field energy decreases as $\sim 1/r^6$ whereas the wind kinetic energy falls off as only $\sim 1/r^4$. So, far away from the stellar surface the wind will always win, and stretch the field lines into a radial configuration. However, near the stellar surface field can be stronger and it can guide the wind towards the magnetic equator as long as the magnetic confinement parameter $\eta_* \leq 1$.

J. Groh: *Have you done any radiative transfer calculations to have an idea on the brightness contrast between the structures that you have shown and the central star?* — The short answer is no. However, shocked material can heat up to millions of K which in turn can emit X-rays. We can certainly estimate X-ray emission measure from such shocked regions.

E. Trunkovsky: *There is some paradoxical concept by Igor Veselovsky from Moscow State University about some deep hidden connection between situations of accretion of matter onto the Sun and the origin of solar wind. What can you tell us about this?* — I am afraid I am not familiar with this paradoxical concept. As such, I am unable to make any comments on this at this moment.

D.S. Gunawan: *Many observations of non-thermal emission (in radio) from massive stars can be well explained by interaction of stellar winds of massive star components in binaries. Model requires significant magnetic field (0.2–0.5 mG). How is magnetic field preserved in the wind to remain important at radio photosphere distances?* — Magnetic flux is conserved. At the interaction region, the field will be compressed and its magnitude will be amplified. As such, the magnetic field will continue to be important in the wind-wind interaction region although it may not be able to dominate the dynamics.

REFERENCES

Babel, J., & Montmerle, T. 1997, A&A, 323, 121
Castor J. I., Abbott D. C., & Klein R. I. 1975, ApJ, 195, 157
Cranmer, S. R., & Owocki, S. P. 1996, ApJ, 462, 469
Gagné, M., Oksala, M. E., Cohen, D. H., Tonnesen, S. K., ud-Doula, A., Owocki, S. P., Townsend, R. H. D., & MacFarlane, J. J. 2005, ApJ, 628, 986
Gayley, K. G. 1995, ApJ, 454, 410
Kaufer, A., Stahl, O., Prinja, R. K., & Witherick, D. 2006, A&A, 447, 325
Lobel, A., & Blomme, R. 2008, ApJ, 678, 408
Marchenko, S. V., Moffat, A. F. J., Vacca, W. D., Côté, S., & Doyon, R. 2002, ApJ, 565, L59
Monnier, J. D., Tuthill, P. G., & Danchi, W. C. 1999, ApJ, 525, L97
_________. 2002, ApJ, 567, L137
Smith, N. 2002, MNRAS, 337, 1252
Tuthill, P. G., Monnier, J. D., & Danchi, W. C. 1999, Nature, 398, 487
Tuthill, P., Monnier, J., Tanner, A., Figer, D., Ghez, A., & Danchi, W. 2006, Science, 313, 935
ud-Doula, A., & Owocki, S. P. 2002, ApJ, 576, 413
Zeipel, H. V. 1925, MNRAS, 85, 678