RevMexAA (Serie de Conferencias), **39**, 100–108 (2011)

MUSINGS ON "DENSIDADES, POTENCIALES Y FUNCIONES ASOCIADAS A UNA GALAXIA REDUCIDA" BY POVEDA ET AL. (1960)

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RESUMEN

Presentamos una revisión crítica del artículo "Densidades, Potenciales y Funciones Asociadas a una Galaxia Reducida" por A. Poveda, R. Iturriaga e I. Orozco publicado en 1960 en el Boletín de los Observatorios de Tonantzintla y Tacubaya (BOTT), 2, 20, 3.

ABSTRACT

We present a critical review on the paper "Densities, Potentials and related functions for a spherical reduced galaxy" by A. Poveda, R. Iturriaga, and I. Orozco published in 1960 in the Boletín de los Observatorios de Tonantzintla y Tacubaya (BOTT), 2, 20, 3.

Key Words: galaxies: kinematics and dynamics — galaxies: evolution

1. ARCADIO POVEDA AND HIS RESEARCH PROGRAMS

A complete account of the paramount role that Professor Arcadio Poveda (b., July 15, 1930) has played in the development of Mexican Astronomy would be the subject of a lengthy separate study. Nevertheless, upon being acquainted with the man and his oeuvre, we can safely recognize that Poveda has been gifted with an uncanny curiosity, an acute physical imagination, and an uncommon flair for originality. In the next few pages we reflect upon a paper, which, in our opinion, should be considered one of the most original works in the scientific production of Poveda.

We have revisited the paper *Densidades, Potenciales y Funciones Asociadas a una Galaxia Reducida* published in the Boletín de los Observatorios de Tonantzintla y Tacubaya (BOTT) in 1960 coauthored by Renato Iturriaga and Ismael Orozco (hereafter PIO). Our aim is to provide a thorough review of its contents, impact and scope. PIO reports the results of an early application of digital computers in Astronomy where original analytical expressions and tables with the calculations of dynamical parameters associated to the spherical *reduced galaxy* are presented. The tabulations contain numerical values of the run with radius for density, mass, force, scape velocities, and gravitation potential. The *reduced galaxy* model was an analytical expresion proposed by Gerard de Vaucouleurs (1948) as a fit to the elliptical galaxies' surface brightness distribution. It is more commonly known as the de Vaucouleurs Profile (dVP) or the $R^{\frac{1}{4}}$ Law.

Poveda had just returned from the University of California, after obtaining his Ph.D. under Elizabeth L. Scott and Jerzy Neyman in 1956. Upon his arrival to Mexico Poveda started several research programs. His energetic and charismatic personality attracted very talented students who got engaged in different projects, which included widely separate subjects, ranging from instrumentation to high-energy astrophysics. PIO was an important piece in Poveda's research program aimed at the measurement of galaxies' masses by the application of the Virial Theorem (Poveda 1958). The first articles in Poveda's program were published in the BOTT.

To put PIO into context, we should note that back in the 1960's extragalactic astronomy was slowly acquiring momentum as observations were beginning to accumulate. However, accurate galaxy photometric measurements or radial velocities were scarce, even more so were velocity dispersions. Now, having entered into the XXI century, one can fail to appreciate such limitations, as extragalactic astronomy has become mainstream and vast amounts of data are at our disposal, just a few key strokes away. Neither, can one appreciate the need to publish tables when computer calculations can be done without the need of advanced programing skills on a personal computer. PIO represented a very important attempt to provide a physical interpretation to the dVP. Nevertheless, getting ahead of ourselves, we must recognize that with this paper Poveda and his

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colleagues were marching much ahead of their time; we shall try to elucidate our former statement below.

We also want to provide some flavor of the scientific atmosphere when the work reported in PIO was done. Our view has been drawn from recent informal conversations with Poveda by one of us (OLC) and Poveda (2003). In so doing, we will touch upon some historical aspects, and the impact of PIO on the theory and observations of elliptical galaxies. For the benefit of interested readers who would like to dwell into the formulae provided in the original paper, we have provided explicit mathematical details on the original derivations, but conforming to the current notation employed in papers or textbooks (e.g., Binney & Tremaine 2008). We end by commenting on the persistence of PIO 50 years after its publication.

2. SETTING THE STAGE

The Observatorio Astrofísico Nacional (OANTON) was inaugurated on the 17th of February 1942, occupying the summit of a small hill in the town of Tonantzintla in the State of Puebla. Its main telescope was a $26''/30''$ Schmidt Camera, which for some time was the second largest telescope of its kind in the world. This newly dedicated observatory whose founder and first director was Luis Enrique Erro, represented a decisive move towards the modernization of astronomical research in Mexico.

Around the 1930's, Mexico was enjoying a renaissance brought about by the reconstruction that followed the triumph of the Mexican Revolution. The sciences and the arts blossomed within the halls of newly erected national institutions concentrated in central Mexico. Luis Enrique Erro and Guillermo Haro shared the firm conviction that first rate science could be generated in Mexico. BOTT represented an effort that reinforced Haro's conviction[3]. A more complete description about the history of Mexican astronomy during this period is provided by Bartolucci (2000). The papers by Bart Bok and Paris Pişmiş in Moreno Corral (1986) are particularly interesting, for they provide firsthand accounts from two key players during the dawn of Mexican modern astronomy.

2.1. *The Origin of Computer Sciences in Mexico*

Computer Sciences in Mexico has its beginnings during the same post-revolutionary period. June 8, 1958 marks the beginning of Computer Sciences in Mexico when the first digital computer was introduced. It was an IBM 650 brought by Nabor Carrillo Flores and Sergio Beltrán López for the Universidad Nacional Autónoma de México (UNAM). However, this initiative wasn't universally welcomed, it faced serious opposition by people who considered digital computers an unnecessary luxury. Those people were wrong.

The IBM 650 (announced in July 14, 1953 by IBM, but the first one was delivered in 1954) was the first mass produced general purpose computer. For its attributes in the programing, processing and storage of information, the IBM 650 is allegedly considered the oldest predecesor of the personal computer. According to Galler (1986) no other computer has been linked so closely to the rise of Computer Sciences than the IBM 650[4]. IBM promoted the Educational Grant Plan (EGP) that made the IBM 650 accesible to the main universities in the US. The success of this program was so great that by 1959 there was at least one IBM 650 in 59 North American universities. The IBM 650 sold very well both in the industrial and academic sector: a total of 2000 IBM 650 were sold from 1954 to 1962. Applications of the IBM 650 developed by Mexican scientists and engineers arose almost immediately. It has been reported that the IBM 650 was used to tackle problems in astronomy, physics, engineering, and influenced Mexico's economy by providing applications to industry, administration, and banking. As a curious fact, we remark that UNAM got its IBM 650 up and running almost two years before one of the modern giants in Computer Sciences: the Massachusetts Institute of Technology.

Poveda was quick to recognize the importance of digital computers[5], these machines were ostentatiously known as *electronic brains* in those early days. Poveda got Renato Iturriaga involved (then a student at the UNAM's Institute of Astronomy) and advised him to learn how to use the IBM 650. Iturriaga would later become one of the first students to receive a Ph.D. in the, then, nascent field of Computer Sciences. He got his degree from the Carnegie Institute of Technology (Carnegie-Mellon University) in 1967. Another achievement that is often overlooked is that Iturriaga was a pioneer in the resolution of the N-body problem using digital com-

[3] Although it was based on a publication which had started in 1890 under the name of Boletín del Observatorio de Tacubaya, but its publication was interrupted during Revolution wartime.

[4] The historical records of IBM report that the IBM 650 could performed, if optimally programmed, 78,000 additions or subtractions per minute, 5,000 multiplications per minute (multiplier = 5,555, 055,555), 3,700 divisions per minute (divisor = 5,555,555,555), and 138,000 logical decisions per minute.

[5] Poveda had used an IBM 650 from Livermore to make calculations for his Ph.D. thesis.

puters. Hénon (1964) cites Iturriaga (1963) along the contributions of von Hoerner (1963), Aarseth (1963), and Sherman & Kinman (1964). Iturriaga and Aarseth used 100 particles while Sherman & Kinman used 380. Iturriaga's B.Sc. thesis was elaborated under the supervision of, non-other than, Poveda. A more detailed account on the beginning of Computer Sciences in Mexico is given by Fernández (2000) and Santillán et al. (2004).

3. DENSIDADES, POTENCIALES Y FUNCIONES ASOCIADAS A UNA GALAXIA REDUCIDA

3.1. *The Reduced Galaxy*

de Vaucouleurs (1953) demonstrated that this model provided a good fit for elliptical galaxies and other stellar systems, including globular clusters and the bulges of spiral galaxies (de Vaucouleurs 1987) . The original expression for the dVP as it was introduced reads: $\log B(R) = -A(R^{\frac{1}{4}} - 1)$, where $B(R)$ is the surface brightness at projected radius R and A is a constant (de Vaucouleurs 1948).

The contribution of de Vaucouleurs improved from previous studies by Reynolds (1913) and Hubble (1930). He conducted a systematic analysis on errors that hamper surface brightness photometry (e.g., sky surface brightness, seeing, detector response, etc.), which allowed him to generate very accurate measurements (de Vaucouleurs 1987). This is what provided the observational basis for the dVP. In addition, the light and the mass in the dVP converge at large radii; while, the Hubble-Reynolds profile turns unphysical as light diverges at large radii.

PIO represented a significant improvement over de Vaucouleurs (1953) who provided tabulations for the space density, luminosity and stellar content for a reduced galaxy at only 18 radial positions.

3.2. *Formulae*

In modern terminology the dVP is given by the following expression:

$$I(R) = I_e \exp\{-b[(R/R_e)^{\frac{1}{4}} - 1]\}, \quad b = 7.66925 \tag{1}$$

where R is the projected radius, R_e is the effective radius, i.e., the radius that encloses half of the light of the galaxy, I_e is the surface brightness at R_e, $I(R)$ often has units of solar luminosities per square parsec [$L_\odot/\mathrm{pc}^2$]. However, observers usually express the surface brightness $\mu(R)$ in terms of magnitudes per square arcsecond; then, equation (1) takes the form $\mu(R) = \mu_e + 8.32675[(R/R_e)^{\frac{1}{4}} - 1]$.

A crucial step was the treatment introduced to integrate the equation for the spatial density (PIO, their equation 8):

$$\rho(r) = -\frac{1}{\pi}\int_r^\infty \frac{\left(\frac{dI}{dR}\right)\, dR}{\sqrt{R^2 - r^2}}, \tag{2}$$

by introducing the change of variables $y = R^{\frac{1}{4}}$ and $\beta^4 = r$, equation (2) takes the following form:

$$\rho(r) = \frac{I_e\, b \exp(b)}{\pi\beta^3}\int_1^\infty \frac{\exp(-b\beta y)}{\sqrt{y^8 - 1}} dy. \tag{3}$$

Equations (2) and (3) diverge as the integrands tend to the lower limit. PIO introduced a transformation to circumvent the singularities, therefore, making it apt for numerical integration. They proceed as follows, first they factorized the denominator in the integrand of equation (3).

$$\int_1^\infty \frac{\exp(-b\beta y)}{h(y)\sqrt{y-1}} dy, \tag{4}$$

where $h(y) \equiv \sqrt{(y+1)(y^2+1)(y^4+1)}$. Then, they integrated equation (4) by parts, setting $u = \frac{\exp(-b\beta y)}{h}$ and $dv = \frac{dy}{y-1}$; therefore, $du = -\frac{\exp(-b\beta y)}{h}(b\beta + h'/h)$, where $h' \equiv \frac{dh}{dy}$, and $v = 2\sqrt{y-1}$. Then equation (4) can be expressed as follows:

$$\int_1^\infty \frac{\sqrt{y-1}\exp(-b\beta y)}{h(y)}\left(b\beta + \frac{h'}{h}\right) dy, \tag{5}$$

and we have

$$\left(\frac{h'}{h}\right) = \frac{d}{dy}\ln(h) = \frac{1}{2(y+1)} + \frac{t}{t^2+1} + \frac{2t^2}{t^4+1},$$

therefore, after substituting this last expression into equation (5) we have the following final expression for the density:

$$\rho(r) = \frac{2I_e\, b \exp(b)}{\pi\beta^3}\int_1^\infty \frac{\sqrt{y-1}\exp(-b\beta y)}{\sqrt{(y+1)(y^2+1)(y^4+1)}} \left(b\beta + \frac{1}{2(y+1)} + \frac{y}{y^2+1} + \frac{2y^3}{y^4+1}\right) dy. \tag{6}$$

Equation (6) is integrable and well behaved within the limits of integration. It provides an operational form for the density for an spherical galaxy whose surface brightness follows a dVP, and a constant M/L.

PIO's equation (18) is also very useful as it provides an analytical expression for the radial dependence of the integrated luminosity. We have that

$L(R) = \int_0^R I(R')2\pi R' dR'$, where $I(R)$ is given by equation (1). By introducing the change of variables $\xi = b(R/R_e)^{\frac{1}{4}}$. The integral takes the general form

$$\int x^m \exp(-x)dx = -\exp(-x)\left[\sum_{i=0}^{m} \frac{m!}{(m-i)!} x^{(m-i)}\right],$$

in our particular case, it is readily found that $m = 7$. Hence, the luminosity at radius R is given by

$$L(R) = L(\infty)\left[1 - \exp(-\xi)\left(1 + \xi + \frac{\xi^2}{2!} + \cdots + \frac{\xi^7}{7!}\right)\right], \tag{7}$$

where

$$L(\infty) = \frac{8! \exp(b)}{b^8}\left(\pi R_e^2 I_e\right) = 7.22\pi R_e^2 I_e$$

is the total asymptotic luminosity, or equivalently the total absolute magnitude is $M_T = \mu_e + 2.5\log(R_e) - 39.96$, here the effective radius R_e is given in kpc. Hence, equation (7) represent the growth curve for the dVP. This equation should be attributed to PIO who published it before anybody else[6]. Observers have modeled the brightness of elliptical galaxies by fitting either the surface brightness profile (e.g., Kormendy et al. 2009; Saglia et al. 1997) or the growth curve (e.g., Prugniel & Heraudeau 1998).

3.3. *The Computations*

The code employed in PIO was written in Assembly Language[7]. The Symbolic Optimal Assembly Program (SOAP) was the language processor of the IBM 650, it was written in 1955 by Stan Poley at Columbia University. The IBM 650 could also be programed in FORTRAN (The IBM Mathematical FORmula TRANslating System, the first high-level, machine-independent programming language). A compiler for FORTRAN was introduced in 1957[8], which compiled FORTRAN into SOAP. However, it seems that a FORTRAN compiler was not available to Iturriaga. To perform the computations indicated in the paper, POI adapted the routines from Livermore and the integrations were done using a pseudocode for the Simpson integration rule.

All the physical quantities are derived using the above expression avoiding projected radius $R < 0.01$. Having no observational evidence at hand, PIO pointed out that the reduced galaxy model could fail at such small radii. This assumption was proved to be correct, HST observations revealed that at very small radius, bright early-type galaxies could have a break in the surface brightness turning into shallower slopes, while less luminous galaxies could be coreless (Lauer et al. 1995). They also examined other cases where they observed departures from the theoretical model (PIO used observational data provided by William Baum in a private communication).

The tables in PIO provide the calculations (with seven significant figures) for 160 conveniently distributed points around the center of the reduced galaxy. The tabulated quantities include the cumulative mass

$$M(r + \Delta r) = \int_r^{r+\Delta r} 4\pi\rho(r')dr',$$

the gravitational force

$$\mathbf{F}(r) = G\frac{M(r)}{r^2},$$

the gravitational potential

$$\mathbf{\Phi}(r + \Delta r) = \mathbf{\Phi}(r) + \frac{GM(r)}{r} - \frac{GM(r + \Delta r)}{r + \Delta r} + \int_r^{r+\Delta r} 4\pi G\rho(r')r'dr',$$

the scape velocity $v_{\rm esc}^2 = -2\mathbf{\Phi}(r)$, and the Potential Energy

$$W(r) = \frac{1}{2}\int \rho(r)\mathbf{\Phi}(r)dr^3 = \frac{1}{2}\int_0^r \mathbf{\Phi}(r)dM(r) - \frac{GM^2(r)}{2r} - \frac{M(r)\mathbf{\Phi}(r)}{2},$$

where $\mathbf{F} = -\nabla\mathbf{\Phi}$ and $\nabla^2\mathbf{\Phi} = 4\pi G\rho$ (cf., Binney & Tremaine 2008; Aguilar 2008).

[6]This equation was presented by de Vaucouleurs (1962) who in previous publications indicated the operational definition for integrated aperture magnitudes, but did not provide anything close to equation (7). Poveda had communicated his results to de Vaucouleurs; however, de Vaucouleurs (1962) did not provide any reference to PIO. The derivation of equation (7) is left as exercise in the popular textbook by Binney & Merrifield (1998, exercise 4.4) (cf., Saglia et al. 1997, Appendix).

[7]This is the most basic programming language (low-level), it is the native language of computer processors; therefore, assembling languages provide great freedom and speed by taking advantage of the processor's architecture.

[8]FORTRAN represented a tremendous advance in computing for it demanded less skill to program, and the resulting programs were portable. It quickly became the language of choice for all scientific applications and has remained the primary language for some of the most intensive supercomputing tasks.

4. APPRAISAL

The main contribution of PIO was that it provided a bridge to connect dynamics with observations. It was found that the even the simplest profiles proposed by the observers to model the surface brightness profiles didn't generate analytical expression for the density, the mass, the potential, etc. Even in the case of the Hubble-Reynolds profile the corresponding physical formulae are difficult to derive. The modified Hubble profile is an analytical expression where their both spatial and projected luminosity densities are analytical (Binney & Tremaine 2008). The tabulations in PIO could be used to interpolate values for observational or dynamical parameters at any intermediate radius (e.g., Burbidge et al. 1961; Morton & Chevalier 1972; Rood et al. 1972). The tabulated quantities also served as inputs for numerical or analytical calculations (e.g., White 1979; Binney 1982, see below). These are in essence the main contributions of PIO.

The results presented in PIO remained unchallenged for almost 16 years. In 1976 by direct suggestion of de Vaucouleurs, Peter Young[9], a very bright Ph.D. student at the University of Texas drew heavily from PIO by following the notation and associated expressions (Young 1976). Young (1976) also showed that PIO numerical results and his own were in good agreement. Young improved on the numerical precision over PIO and provided some very useful asymptotic approximations for very small and large projected radii. He provided the following approximation, for small radii ($r \to 0$):

$$\rho(r) \approx I_0 \frac{\exp(-b\beta)}{\beta^3}, \quad I_0 = \frac{1}{8}\int_0^{\frac{\pi}{2}} \sec^{\frac{1}{4}}\theta d\theta \sim 0.24099; \tag{8}$$

while for large radii ($r \to \infty$), the following approximation shows a change in the slope:

$$\rho(r) \approx \frac{\exp(-b\beta)}{2\beta^3}\left(\frac{\pi}{8b\beta}\right)^{\frac{1}{2}}\left[1 + \frac{7}{8b\beta} + ...\right], \tag{9}$$

where as in § 3.2, $\beta = r^{\frac{1}{4}}$ and $b = 7.66925$.

4.1. *Further Analytical Approaches*

An alternative approach to model elliptical galaxies was to seek for simple analytical models whose derived physical properties remained analytical, but whose projected properties resemble what is observed in real galaxies. In some cases models generalize or extend the dVP.

After the approximations provided by Young (1976) to the spatial density associated to the dVP. A general analytical expression for the density was provided by Jaffe (1983) in which $\rho \propto r^{-4}$ for $r \to \infty$, the proposed expression reproduced the dVP. Mellier & Mathez (1987) provided an analytical two-power law fit to the tabulations in Young (1976). Another analytical expression for the spatial density that closely reproduced and generalized the $\rho \propto r^{-4}$ behavior was provided by Hernquist (1990). Nevertheless, a most significant advancement was introduced by Dehnen (1993) who suggested a general two-power law profile which included both the Jaffe and the Hernquist models as special cases. The spatial density in Dehnen models are cuspy at small radii but fall as $\rho \propto r^{-4}$ as $r \to \infty$; additionally, their phase-space distribution functions can be expressed in closed form in terms of energy and angular momentum. These two-power density models seem to reproduce the distribution of dark matter haloes which results after numerical simulations (Binney & Tremaine 2008), as well. The general family of two-power density models reads:

$$\rho(r) = \frac{\rho_0}{\left(\frac{r}{a}\right)^{\gamma}\left(1 + \frac{r}{a}\right)^{\alpha-\gamma}}, \tag{10}$$

where a is a characteristic scale length, ρ_0 is the central density. Associated surface brightness profiles for some interesting values of γ are given in Binney & Merrifield (1998, § 4.2.3). When $\alpha = 4$ and $\gamma \in [0, 3)$ we recover the Dehnen models, which turn into the Hernquist model when $\gamma = 1$ or the Jaffe model when $\gamma = 2$, respectively. The dVP is reproduced better when $\gamma = \frac{3}{2}$ (Dehnen 1993). Although, good fits for the centers of elliptical galaxies are provided when $\gamma \in (0.6, 2.0)$. When $\alpha = 3$ and $\gamma = 1$, we recover the model advanced by Navarro et al. (1996, hereafter NFW). We have seen that the mass converges in the de Vaucouleurs profile, it can also be shown that mass converges in the more general Dehnen models. However, in the NFW model the mass diverges logarithmically with radius, this is an undesirable property, for arbitrary truncation radii are introduced to make it manageable. Moreover, the universality of the NFW profile is not well understood (Binney & Tremaine 2008). The properties of the NFW model have been revised by Aguilar (2008).

In the mean time, the search for integrations of the dVP has continued. Recently Mazure &

[9]Peter Young is remembered for his genius and his engrossing research style. He made important contributions to the study of black holes, gravitational lensing, and AGN. Young was appointed assistant Professor at Caltech at the age of 25 in 1979. His brilliant scientific career ended abruptly when he took his own life in 1981 (Keel 2006).

Capelato (2002) arrived at a solution for equation (2) using the computer algebra package Mathematica. They found that solutions for equation (2) and the other quantities derived in PIO could be expressed by *Meijer G functions*. These functions are generalizations of the hypergeometric series.

4.2. *The Formation of Elliptical Galaxies and the de Vaucouleurs Profile*

For a long time the dVP guided the exploration of the formation of elliptical galaxies using N-body simulations, for example it has been showed that ellipticals could be formed through dissipationless collapse from a variety of initial conditions (e.g., van Albada 1982; Aguilar & Merritt 1990). The dVP also results in mergers of spiral galaxies (e.g., Barnes 1988; Schweizer 1996, for a review of observational evidence), this has been taken as an indication that a large fraction of the present day ellipticals might have originated by mergers of spiral galaxies (Toomre 1977). It was also found that dPV is quite robust, as galaxies affected by heavy dammage produced by tidal truncation keep on following the dVP or will conform to it, when the initial distribution was a King Profile (Aguilar & White 1986).

We have seen above, that $\rho \propto r^{-4}$ at large reproduces qualitatively the properties of the dVP. What is so special about $\rho \propto r^{-4}$? Does this behavior arise naturally in all spherical galaxies? Aguilar (2008) has answered these questions by providing a proof to the following proposition:

> If a spherical galaxy with finite mass, no rotation and isotropic velocity distribution, develops a finite, non-zero population of particles at $E = 0$ (where E is the total energy); then, the tail of the density profile at large radii will exhibit a $\rho \propto r^{-4}$ behavior.

Reinforcing the view that dynamical systems will naturally follow the dVP, Binney (1982) has shown that for galaxies that follow a $R^{\frac{1}{4}}$ law are described by a distribution functions $f(E)$ in which the number $N(E)dE$ of stars with binding energy near E is well described by a Boltzmann distribution $N(E) \propto N_0 \exp(-\eta E)$, where $\eta = 2.08$. Moreover, Hjorth & Madsen (1991) have shown that the equilibria of violently relaxed self-gravitation many-particle system will resemble the dVP in a sufficiently deep central potential. Hence, the shape of the potential might play a very important role in the generation of the dVP; maybe, this was set by initial conditions (Binney 1982). Ellipticals can then formed in at least three ways, by dissipationless collapse, by mergers of disks, and by tidal truncation, this is now called galaxy harassment.

5. THE DE VAUCOULEURS PROFILE GOES ON: NEW OBSERVATIONAL RESULTS

The dVP was originated empirically after the analysis of the surface brightness profile of bright elliptical galaxies. In general, the dVP provide adequate fits for the inner regions of bright galaxies($M_B \sim -21$ mags.), which in some cases, these could be remarkably accurate over a 9 mag range (e.g., Capaccioli et al. 1990). The universality of the dVP was assumed as a property of early-type galaxies; however, significant departures from the dVP were found, specially in lower luminosity ellipticals (e.g., Caon et al. 1993). Most of the recent work has concentrated on exploring the departures from the dVP and in the consideration of other structural components in elliptical galaxies.

With the advent of charge coupled devices (CCDs) galaxy photometry experienced a revival (Kormendy & Djorgovski 1989). Painstaking surface brightness measurements taken with large telescopes and photographic plates were improved by CCD observations and, relatively, small telescopes. The dVP was applied as a fitting function the surface brightness of early-type galaxies at any accesible redshift. A generalization to the dVP originally suggested by Sérsic (1968) was reintroduced by Caon et al. (1993) who build on previous work by Capaccioli. The Sérsic profile allows the radial index n to vary, it reads:

$$I(R) = I_e \exp\{-b_n[(R/R_e)^{\frac{1}{n}} - 1]\}, \qquad (11)$$

where as in § 3.2 (cf., equation 1), R_e and I_e are the effective radius and the effective surface brightness, respectively; b_n is constant that depends on the "shape" of the light profile n. To find the values of n the following equation has to be solved: $\Gamma(2n) = 2\hat{\gamma}(n, b_n)$, where Γ and $\hat{\gamma}$ are the gamma and the incomplete gamma functions, respectively. There are several approximations to the previous equation; for example, $b_n \approx 1.9992n - 0.3271$ for $0.25 < n < 10$. An advantage of the Sérsic profile is that it smoothly morphs, by varying n, from a Gaussian profile ($n = 0.5; b_n = 0.6725$) into an exponential profile ($n = 1; b_1 = 1.678$), or into a dVP ($n = 4; b_4 = b$), but in principle n is a positive real number. See Graham & Driver (2005) for a review of the properties of the Sérsic profile.

Blanton et al. (2003) used the Sloan Digital Sky Survey (SDSS) and studied the surface brightness behavior for a sample of 183,487 galaxies. They generated synthetic fits using aperture photometry and templetes for a single-component Sérsic Profile; they found that most galaxies have $n = 1$ and also found a dependance with luminosity, in which brighter galaxies have larger n (also seen in Caon et al. 1993). Kormendy et al. (2009) have analyzed the surface brightness profile for sample of 43 galaxies in Virgo using high quality data that allowed them to fit the innermost regions (HST) and the outer regions; they also found that the Sérsic index is correlated with the brightness of the galaxy.

We have modeled the surface brightness using a bidimensional approach for 1606 galaxies in 18 Abell LOCOS (López-Cruz 1997, 2001) clusters at $0.02 \leq z \leq 0.08$. We have developed a Driver for GALFIT (Peng et al. 2002) on Cluster Galaxies (DGCG, Añorve et al. 2009; C. Añorve 2011, in preparation). DGCG automatically finds, masks, model and deblends galaxies in crowded fields, We have modeled bulges, disks and bars, using the Sérsic and the Exponential profiles. The accuracy and precession of the fits have been tested using simulated data (Añorve 2011, in prep.). This study has helped us to establish quantitative approach to galaxy morphology based on the distribution of the bulge-to-disk ratio and the isophotal ellipticity of galaxies (Añorve et al. 2009).

We have fit the entire sample with a single Sérsic profile, the sum of an exponential for the disk component and a Sérsic for the bulge component, and in the presence of bars we have a third Sérsic to model the bar. We then have combined the results and decide on the best fit between a single or two components fits by examining the value of the χ^2 and the degeneracy of the fits. Figure 1 shows the frequency distribution of n, the mean error in n is 0.2, but the most probable error in n is $\Delta n = 0.1$. We find that there is a maximum in the distribution at $n \sim 1$. This is qualitative agreement with Blanton et al. (2003), Coenda et al. (2005), and Fisher & Drory (2008); however, our results systematically differ from those of de Jong et al. (2004), who have used a similar bidimensional fitting scheme but based on another fitting package. In Figure 2 we show the dependence of the n with total magnitude (assuming, $H_0 = 73$ km/s/Mpc, $\Omega_M = 0.27$, and $\Omega_\Lambda = 0.73$), there is a mean error in absolute magnitude of $\Delta M_R = 0.1$ mags. Here, again, there is agreement with Blanton et al. (2003) and Kormendy et al. (2009). In our analysis we have allowed the Sérsic index to vary in the range $0.02 \leq n \leq 10$, we found that brighter galaxies tend to have higher indexes; however, the solid thick line in Figure 2, which was generated by a robust locally weighted scheme (Cleveland 1979), shows that the growth of the index with brightness levels off, tending towards 4 ($n \to 4$), this is also seen in Blanton et al. (2003). Figures 1 and 2 strengthen the distinction between bulges and pseudobulges (Kormendy & Kennicutt 2004; Fisher & Drory 2008). Pseudobulges ($n < 2$) pick around $n = 1$, while true bulges ($n > 2$) weakly cluster around $n = 3$. Pseudobulges are preferentially found in spirals, they are thought to grow by secular processes (Kormendy & Kennicutt 2004). True bulges, which are commonly found in ellipticals, S0, and spirals, having $n \to 4$ might have formed by dissipationless collapse, mergers or tidal stripping, the processes that we have identified to form elliptical galaxies (see above). This dichotomy is stressed by the coevolution of bulges and black holes. Kormendy et al. (2011) have found that while black hole mass correlates with bulge luminosity; black holes in pseudobulges, on the other hand, are uncorrelated. Kormendy et al. have suggested that coevolution of black holes and bulges is set globally by major rapid mergers and activity. In contrast, coevolution between black holes and pseudobulges is absent, this might be the result of local and intermittent black

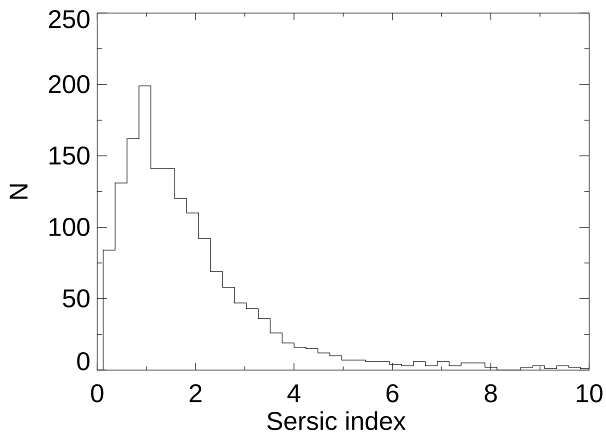

Fig. 1. The distribution of the Sérsic index (n) for a sample of 1606 galaxies in the fields of 18 low-z clusters of galaxies observed in R band. The fits have been done with a Driver for GALFIT (Peng et al. 2002) on Cluster Galaxies (DGCG, Añorve et al. 2009; Añorve 2011, in prep.). DGCG automatically finds, masks, models and deblends galaxies in crowded fields. We have modeled bulges, disks and bars, using the Sérsic and the exponential profiles. There is a maximum in the distribution at $n \sim 1$. This distribution is in qualitative agreement with Blanton et al. (2003), Coenda et al. (2005), and Fisher & Drory (2008).

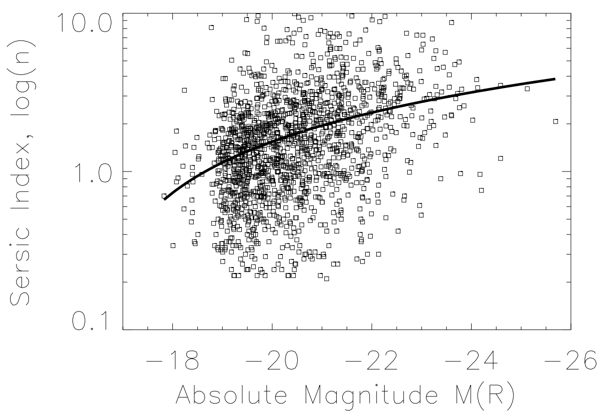

Fig. 2. The dependence of the Sérsic index (n) with total host luminosity in the R band for the same galaxies as in Figure 1. The general tendency shown in this figure is also found in Blanton et al. (2003) and Kormendy et al. (2009). Brighter galaxies tend to have higher indexes; however, the solid thick line, which was generated by a robust locally weighted scheme (Cleveland 1979), shows that the growth of the index with brightness levels off, tending towards 4 ($n \rightarrow 4$).

hole growth, which may not be in phase with the secular evolution of their pseudobulge hosts.

These are, indeed, exciting times in the exploration of elliptical galaxy formation. The dVP keeps on guiding recent developments.

6. CLOSING REMARKS

We have shown that PIO is an original and relevant paper, for it contains elements that makes it unique. Its results have impacted both the theory and observation of elliptical galaxies.

Guided by number of citations reported in the astronomical data service (ADS), we might be inclined to believe that PIO has not been properly recognized. There are few reasons for this to be the case. At the beginning, BOTT used to subdivide each volume into numbers. For example, Volume 2 runed from 1954 through 1960, is composed of Numbers 11 to 20, in each number the pagination starts all over again. The scheme that ADS uses entering year, name of journal, volume number, and first page number; this makes papers in volume 2 hard to cite in ADS. However, a full text search ADS reports some 50 citations to PIO. Another factor that could have prevented PIO from being cited is that it was published in Spanish in an observatory bulletin rather than in a main journal. We believe that is only a minor effect, however. Scientific research is not free from the human fashions. When PIO was published, Poveda was only a young researcher guided into galactic dynamics by mere curiosity, his Ph.D. work was on the statistical analysis of the large scale structure as traced by galaxies. We might be inclined to believe that not having an established name in this field may have affected the recognition of PIO's results. There were very few extragalactic astronomers in 1960s, but most of them seem to have known about Poveda's work; therefore, being a young researcher entering a new field didn't affect the recognition of the work in PIO. Sadly, the work in PIO did not receive much attention in Mexico in the immediate years after its publication. The staff at Tonantzintla Observatory was very small, most of their studies focused on stars, star formation, metallicity of galactic and extragalactic gaseous nebulae, and surveys for blue objects, planetary nebulae, etc. Some researchers or groups of researchers applying stellar photometry techniques attained recognition through their sustained work (e.g., E. E. Mendoza V. and B. Iriarte); however, we fail to identify any Mexican astronomer doing surface brightness photometry of galaxies during the 1960s and early 1970s. Poveda after recognizing how hard it was to get reliable observational data for galaxies, published very little on the subject and moved to other fields. We conclude that PIO results were widely known by most astronomers in 1960s. However, neither de Vaucouleurs nor Poveda received much citations during the 1960s and 1970s: there were less than 50 astronomers, worldwide, working on this field during those two decades. However, we believe that the lack of continuity in Mexico of PIO's line of work was the main factor that limited its impact. The work of Peter Young published in 1976 was timely – CCDs were introduced in astronomy a few years later.

PIO proved Erro's thesis: first rate science has been generated in Mexico. Mexican scientists have been able to take advantage of state-of-art equipment and made important contributions, rising to the occasion with talent and vision. Fortunately, this phenomenon was not particular to astronomy. With this, BOTT rose as one of the first and most influential scientific journals of, its time, in Latin America.

Long gone are the days when elliptical were considered simple stellar systems made of single stellar populations, with no gas, no stellar formation. Our models and observations are revealing a more comprehensive view on galaxy formation and evolution. The field is a stage of rapid development with new components such as black holes, pseudobulges and velocity fields. The field is certainly reaching maturity.

While writing this review, we had some very enlightening conversations with Arcadio Poveda, Renato Iturriaga, Silvia Torres-Peimbert, Luis Carrasco, Elsa Recillas, and Alfredo Santillán.

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